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Computational model of seismic wave propagation in prestressed formation

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Analysis of seismoacoustic wavefields is the basic tool for the study of internal structure of Earth and rock masses in the mining technology. The seismic wave properties can depend not only on the material characteristics of the formation (density and speeds of sound), but in addition on the existence of zones of non-hydrostatic stress field. These prestressed zones can be caused by many factors, such as geotectonic processes, gravity, temperature gradients, etc. In the mining area these zones, for example, can be caused by the underground excavation during shaft sinking. The impact of prestressed zones on seismic waves is a poorly studied problem and one can expect that the account of initial stress can have an influence on interpretation of the results of solution of inverse problems and seismic imaging.

The basis of the theory of elastic waves in prestressed elastic media goes back to the pioneer work of M.Bio [1]. Its application to seismic problems was not systematic (see, for example, [2],[3] and references therein) and there is still an open area for research work.

We propose a new computational model for small amplitude wave propagation in the prestressed medium, the simplified version of which is presented in [4]. The derivation of the model is based on the general theory of finite deformations and as a result, the governing equations in terms of velocities, stress and small rotations are formulated in the form of the first order hyperbolic system. The method of derivation is applicable for an arbitrary dependence of elastic energy on the invariants of the strain tensor and the smallness of the initial strain tensor is not required in general.

As an example, we consider a quadratic dependence of elastic energy on the strain tensor in the form $E=\frac{\lambda}{2\rho_{00}}\left(\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33}\right)^2+\frac{\mu}{\rho_{00}}\left(\varepsilon_{ij}\varepsilon_{ji}\right)$ and come to the following system of governing equations:

$$\rho_0\frac{\partial u^i}{\partial t}=\frac{\partial\sigma^{ij}}{\partial x^j}+\left(\varepsilon_{j\alpha}+\omega_{j\alpha}\right)\frac{\partial\Sigma^{i\alpha}}{\partial x^j},$$

$$\frac{\partial\sigma}{\partial t}=-\Sigma U-U^T\Sigma-trW\,\Sigma+\lambda trW\,I+2\mu W-$$
$$-\lambda tr\varepsilon^0 trW\,I-2\mu tr\varepsilon^0\,W-2\lambda tr\left(\varepsilon^0U\right)I-2\lambda trW\,\varepsilon^0-4\mu\varepsilon^0\,W-2\mu W\,\varepsilon^0,$$

where λ,μ - are Lame parameters, ρ_{00},ρ_0 - are densities in the unstressed and Lagrangian configurations, u^i - velocities, $\sigma=\left[\sigma^{ij}\right]$ - is a perturbation of initial stress tensor $\Sigma=\left[\Sigma^{ij}\right]$, $\varepsilon^0=\left[\varepsilon^0_{ij}\right]$ - is the strain tensor corresponding to Σ and $\varepsilon=\left[\varepsilon_{ij}\right]$ - is its perturbation, $U=\left[\partial u^i/\partial x^j\right]$ - is the velocity gradient, $W=\left(U+U^T\right)/2$ - is the strain rate tensor. These equations should be supplemented by the equations for small deformation tensor and rotation

$$\frac{\partial \varepsilon_{ij}}{\partial t} = \frac{1}{2}\left(\frac{\partial u^i}{\partial x^j} + \frac{\partial u^j}{\partial x^i}\right), \qquad \frac{\partial \omega_{ij}}{\partial t} = \frac{1}{2}\left(\frac{\partial u^i}{\partial x^j} - \frac{\partial u^j}{\partial x^i}\right).$$

It turns out that the initial stress produces anisotropy of the medium . On Figure 1 one can see the plane waves velocity distribution for unidirectionally stretched with $\Sigma^{11} = \rho V_p^2 / 50,\ \Sigma^{ij} = 0\left(ij \neq 11\right)$ (left) and compressed with $\Sigma^{11} = -\rho V_p^2 / 50,\ \Sigma^{ij} = 0\left(ij \neq 11\right)$ (right) medium with material parameters $V_p = 3000\, m/s,\ V_s = 2000\, m/s,\ \rho_{00} = 2000\, kg/m^3$.

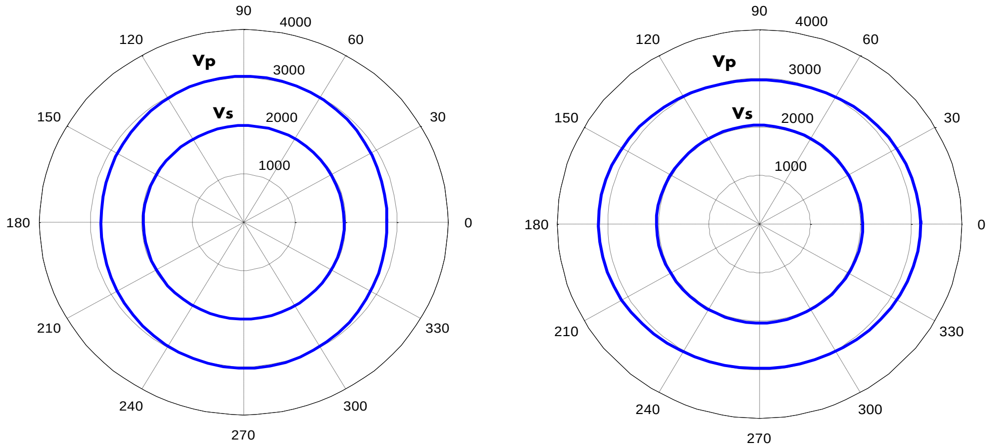

Figure 1. Velocity distribution for stretched (left) and compressed (right) media.

The staggered grid finite difference method has been developed to solving the presented differential equations. The numerical study of interaction of seismic waves with zones of prestressed rock demonstrate their significant influence on the wave field. Thus the existence of prestressed zones must be taken into account for forward modelling and inversion of seismic waves.

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