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THERMAL CONDUCTION IN FGM AND MLC SHELL STRUCTURES

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Shell structures made of **F**unctionally **G**raded **M**aterials (FGM) or **M**ulti - **L**ayer - **C**omposites (MLC) show arbitrary continuous or discontinuous variations of material properties. This work focuses on the efficient evaluation of temperature distributions in such structures (see e.g. [1]). Consider a curved FGM or MLC shell geometry of thickness h where the geometrical mid-surface is discretized with four noded shell elements. We assume that the thermal conductivity k is element-wise constant in the membrane directions $\hat{x}$ - $\hat{y}$ and varies arbitrary in transverse direction $\hat{z}$. A convection boundary condition is applied on the bottom (b) and the top (t) surface of the shell structure, with h_{ct} and h_{cb} denoting the convection coefficients on the top and bottom surface, while T_{Bt} and T_{Bb} refers to the corresponding fluid temperatures. It is a main issue of this paper that the temperature field $T(\hat{x}, \hat{y}, \hat{z})$ is evaluated using a decomposition according to

$$T(\hat{x}, \hat{y}, \hat{z}) = \bar{T}(\hat{x}, \hat{y}) + \theta_1(\hat{z}) = \bar{T}(\hat{x}, \hat{y})\theta_2(\hat{z}). \tag{1}$$

There, $\bar{T}(\hat{x}, \hat{y})$ denotes the mean temperature while $\theta_1(\hat{z})$ and $\theta_2(\hat{z})$ represent the unknown temperature distribution in transverse direction. The solution strategy is iterative while each iteration consist of two steps: Within the first step we evaluate the mean temperature field $\bar{T}(\hat{x}, \hat{y}) = \frac{1}{h}\int_h T(\hat{x}, \hat{y}, \hat{z})\, d\hat{z}$ in membrane direction, while within the second step the temperature distribution in transverse direction $\theta_1(\hat{z})$ and $\theta_2(\hat{z})$ based on a mean temperature $\bar{T}$ is estimated. An iterative procedure is required since the shell's surface temperatures $T(\hat{z} = \pm\frac{h}{2})$ defining convection are not known within the first step. The framework to calculate the mean temperature field is not included here since it is rather classical using a standard finite element approach. Once the mean temperature $\bar{T}(\hat{x}, \hat{y})$ is evaluated at every point of the shell's structure, the temperature distribution with respect to the thickness direction is calculated next. Thereby, we analyze a one-dimensional problem in transverse direction with the strong form of

$$\frac{d}{d\hat{z}}\left(k(\hat{x}, \hat{y}, \hat{z})\frac{d}{d\hat{z}}\theta_1(\hat{z})\right) + Kk(\hat{z}) = 0\ , \tag{2}$$

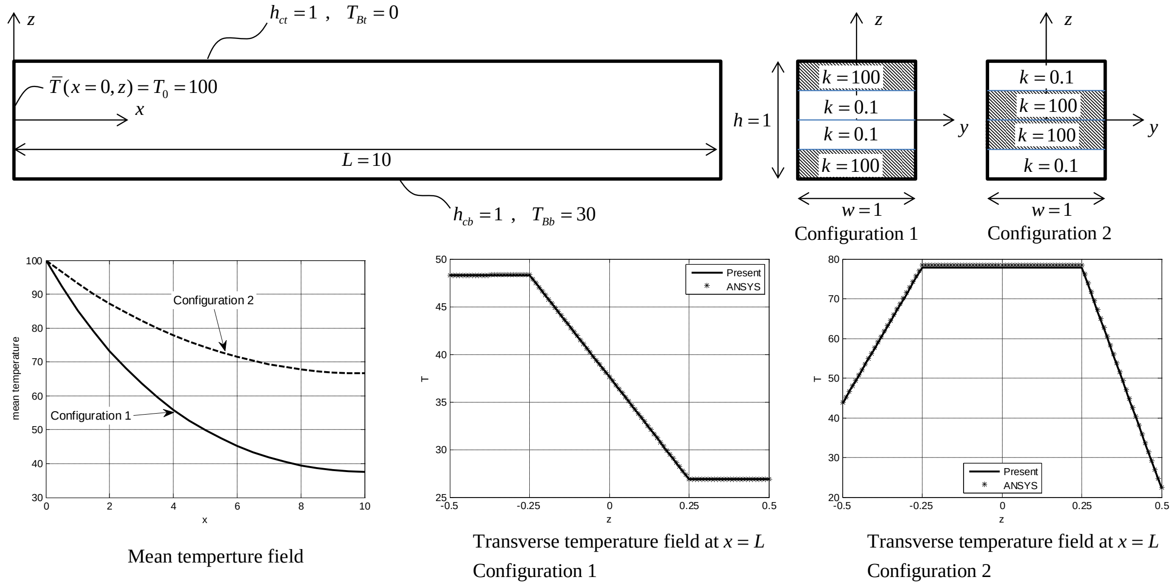

Figure 1: Rectangular MLC fin

$$\hat{z} = \frac{h}{2} : \ -k(\hat{x}, \hat{y}, \hat{z} = \frac{h}{2}) \left. \frac{d\theta_1(\hat{z})}{d\hat{z}} \right|_{\hat{z}=\frac{h}{2}} - h_{ct} \left(\bar{T}(\hat{x}, \hat{y}) + \theta_1(\hat{z} = -\frac{h}{2}) - T_{Bt} \right) = 0 \ , \tag{3}$$

$$\hat{z} = -\frac{h}{2} : \ -k(\hat{x}, \hat{y}, \hat{z} = -\frac{h}{2}) \left. \frac{d\theta_1(\hat{z})}{d\hat{z}} \right|_{\hat{z}=-\frac{h}{2}} - h_{ct} \left(\bar{T}(\hat{x}, \hat{y}) + \theta_1(\hat{z} = -\frac{h}{2}) - T_{Bb} \right) = 0 \ , \tag{4}$$

$$\int_h \theta_1(\hat{z}) \, d\hat{z} = 0 \ . \tag{5}$$

There, a crucial step is the introduction of $Kk(\hat{z})$ in (2) where K denotes a constant. Only the inclusion of this term (missing in [1]) leads to accurate results. The problem (2) - (5) is solved using a discretization with n linear 1D elements of length $\frac{h}{n}$. In order to show the good predictive quality of the proposed algorithm a rectangular MLC fin of Fig. 1 is analyzed. In Configuration 1 highly conductive layers are placed on the outside while nearly isolating layers are around the mid-surface. In Configuration 2 the order of the layers has changed, however, both configurations have the same mean value of thermal conductivity. Fig. 1 shows a comparison of the present approach compared to continuum solutions evaluated in ANSYS. All results indicate high accuracy.

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