

# NUMERICAL SIMULATION OF CAVITATING FLOWS BY A MIXTURE-ENERGY-CONSISTENT TWO-PHASE FLOW MODEL WITH THERMODYNAMIC RELAXATION

Marica Pelanti<sup>1\*</sup> and Keh-Ming Shyue<sup>2</sup>

<sup>1</sup> Department of Mechanical Engineering, École Nationale Supérieure de Techniques Avancées - ENSTA ParisTech, 828, Boulevard des Maréchaux, 91762 Palaiseau Cedex, France, [marica.pelanti@ensta-paristech.fr](mailto:marica.pelanti@ensta-paristech.fr), <http://www.ensta-paristech.fr/~pelanti>

<sup>2</sup> Department of Mathematics, National Taiwan University, Taipei 106, Taiwan, [shyue@ntu.edu.tw](mailto:shyue@ntu.edu.tw), <http://www.math.ntu.edu.tw/~shyue>

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Compressible multiphase flow models stemming from the original model of Baer–Nunziato [1] have proven to be effective tools to simulate the complex wave patterns and thermodynamic mechanisms that characterize cavitating flows, e.g. [4, 5]. In this work we present a numerical model for liquid-gas flows with cavitation and evaporation waves based on a variant of the six-equation single-velocity two-phase compressible flow model with stiff mechanical relaxation of Saurel *et al.* [5]. Heat and mass transfer contributions are included in the model system as temperature and chemical potential relaxation terms to model liquid-vapor transition. The original idea of our method is to employ in the numerical discretization an alternative mathematical formulation of the model system that uses the total energy equations of the two phases instead of the internal energy equations employed in the classical six-equation model [5]. Our new approach [3] allows us to easily conceive a simple numerical method that ensures consistency with conservation of the mixture total energy at the discrete level and agreement of the relaxed equilibrium pressure with the correct mixture equation of state. The model equations are solved in two dimensions by a fully-discretized high-resolution scheme. A wave propagation method based on a hybrid HLLC/Roe Riemann solver is employed for the approximation of the homogeneous hyperbolic portion of the system. Thermal and chemical source terms are handled through efficient stiff relaxation solvers that at metastable interfaces drive the two-phase mixture to thermodynamic equilibrium via the solution of simple algebraic systems of equations. We illustrate several numerical experiments that show the ability of the proposed numerical method to simulate the dynamics of cavitation pockets and evaporation fronts. Moreover, results show that the propagation speeds predicted by the numerical model for different levels of activation of the stiff mechanical and thermochemical relaxation processes agree with the theoretical results for the characteristic equilibrium speeds of two-phase relaxation models [2].

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