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THE WEAK SUBSTITUTION METHOD – A NEW APPROACH FOR THE CONNECTION OF NURBS SURFACE PATCHES IN ISOGEOMETRIC ANALYSIS

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Isogeometric analysis uses the geometry description of the design system for the numerical simulation in order to allow an integrated design and analysis work flow. The most common geometry description both in the design industry and the current isogeometric research are NURBS surfaces. One NURBS surface patch is created from two NURBS curves by a tensor-product multiplication. Thus, the topology of NURBS surface patches is always quadrilateral and refinement propagates throughout the whole patch. Real-world geometrical models of complex systems always consist of multiple NURBS patches. The parametrization along common edges is in general neither conforming, nor do the degrees of the NURBS basis functions match. Thus, neighboring patches cannot be connected by linking the degrees of freedom of control points with corresponding coordinates. The ability to handle non-conforming meshes is essential if mutual refinement of meshes shall be avoided. A possible strategy is to enhance the weak form of the potential with additional terms to constrain the deformations of adjacent patches. The Lagrange multiplier method and Nitsche's method – adapted to isogeometric analysis in [1] and [2] – base on this approach. In this contribution a different strategy is chosen. A relation between the deformations of adjacent patches is established. With the help of this relation the edge degrees of freedom of the slave patch are replaced by the edge degrees of freedom of the neighboring master patch. Thus, the patches are naturally connected without adding additional terms to the weak form. Superfluous degrees of freedom are condensated out of the system. The same holds for the redundant control points of the slave patch. For adjacent edges with hierarchical knot vectors and the same degree of the NURBS basis functions an exact relation is provided in [3]. For general cases an exact relation does not exist. This work presents a new method to derive this relation numerically. Basing on the weak form of the equality of mutual deformations a relation between the degrees of freedom of adjacent patches is established. This relation is computed once as a preprocess before the numerical simulation. The method is applicable for nonlinear computations without additional effort for the equilibrium iteration. The implementation is straightforward. The element - control point relation has to be adapted and the stiffness of replaced control points is distributed to the master control points on element level. The method reproduces the exact solution of [3] for hierarchical knot vectors. In the general case of non-conforming meshes the proposed method performs as follows. The initial geometry is described exact to numerical precision using the substitution relation for the control points. The error of mutual deformation of the current configuration decreases exponentially with mesh refinement and does not affect the overall solution significantly. The condition of the stiffness matrix does not deteriorate in comparison to conforming meshes. This is shown with the help of numerical studies. Furthermore, the new method is compared to computations with the Lagrange multiplier method and results provided in [1]. Linear and nonlinear examples are shown.

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