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BRIDGING SINGULARITIES ACROSS SCALES

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Interactions among multiple spatial scales is pervasive in many engineering applications. Structural failure is often caused by the onset of localized damage like cracks or shear bands that are orders of magnitude smaller than the structural dimensions. Another class of engineering problems involving multiple scales of interest is the case of heterogeneous materials. Damage evolution in these materials is governed by micro-structural details that are several orders of magnitude smaller than structural dimensions of interest. The mathematical homogenization theory has been used extensively since the 1970s as a tool for analyzing multiscale responses of materials. This approach involves the computation of effective material properties (coefficients of governing equations) and the numerical solution of a macro-scale boundary value problem using these homogenized properties. The solution u_H of the homogenized equations is a good approximation to the solution u of the original problem in the L^2 norm but not in the energy norm [2]. Thus, quantities such as strains and stresses computed from u_H have in general a large error, in particular, near singularities and boundary points since the derivatives of u are very large and oscillatory in these regions. Understanding the behavior of local stress and strain fields across these regions is necessary for quantifying failure initiation and evolution in heterogeneous materials and structures.

The generalized FEM (GFEM) is an instance of the partition of unity method which has its origins in the works of Babuška et al. [1] and Duarte and Oden [3]. The GFEM has been successfully applied to the simulation of dynamic propagating fractures, polycrystalline and fiber-reinforced micro-structures, porous materials, etc. All these applications have relied on closed-form enrichment functions that are known to approximate well the physics of the problem. However, for many classes of problems–like those involving multiscale phenomena–enrichment functions with good approximation properties are not amenable to analytic derivation.

In this talk, we present a Generalized FEM based on the solution of interdependent macro/global and fine/local scale problems. The local problems focus on the resolution of fine-scale features of the solution in the vicinity of regions with singularities while the global problem addresses the macro-scale behavior. Fine-scale solutions are accurately solved in parallel using the h-version of the GFEM and thus the proposed method does not rely on asymptotic solutions. These solutions are embedded into the global solution space using the partition of unity method. Figure 1 illustrates the methodology applied to an L-shaped domain with heterogeneous material properties.

The boundary conditions for the fine-scale problems are provided by the solution u_H of the homogenized equations. The solutions of these problems are used, in turn, to build enriched GFEM approximation spaces using the partition of unity property of the finite element shape functions. The proposed methodology enables accurate modeling of problems involving multiscale phenomena on macro-scale meshes with elements that are orders of magnitude larger than those required by the FEM. Numerical examples demonstrating the approximation properties of the proposed GFEM are presented.

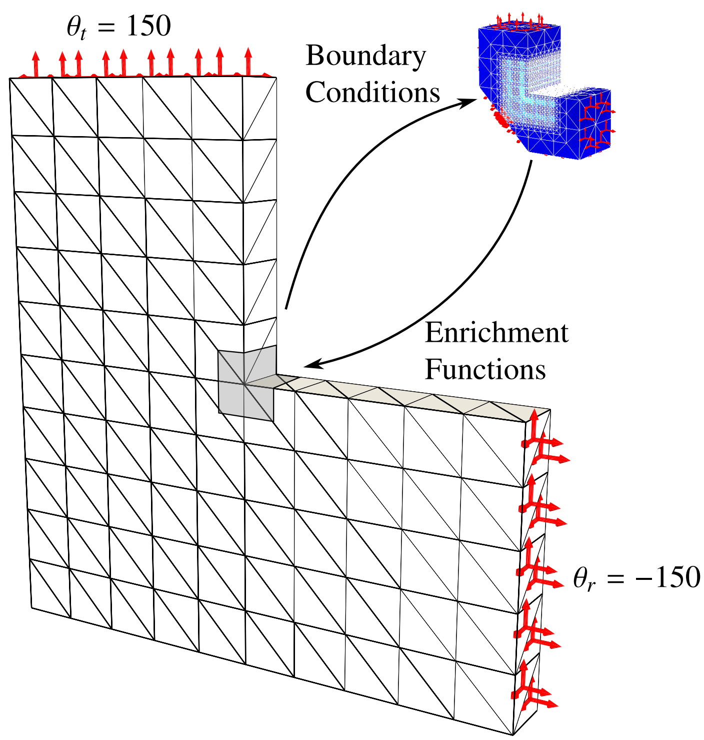

Figure 1: Analysis of an L-shaped domain with heterogeneous material. The assumptions of the homogenization theory are not valid in the neighborhood of the edge due to the presence of singularities.

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