

# A PARALLEL ALGORITHM FOR THE COMPUTATION OF INVARIANT TORI IN LARGE-SCALE DISSIPATIVE SYSTEMS

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The invariant manifolds of a dynamical system are organizing centers which drive its behaviour around them. Therefore, their computation and the study of their dependence on the parameters of the system is necessary in order to understand its global dynamics. The continuation of steady solutions with respect to parameters is now a common tool in Science and Engineering. The computation of other invariant manifolds as periodic orbits and invariant tori is not so usual in the case of large-scale systems, although all of them can be cast into a common framework. Their computation can be reduced to the calculation of fixed points of a map  $G(x, p)$  ( $x$  being the phase space variables and  $p$  a parameter) (see [1-3]). A parallelizable algorithm to compute invariant tori of high-dimensional dissipative systems, obtained upon discretization of PDEs will be presented [4]. The size of the set of equations to be solved is only a small multiple of the dimension of the original system. The sequential and parallel implementations will be compared with a previous method [3], showing that important savings in wall-clock time can be achieved. In order to test it, a thermal convection problem of a binary mixture of fluids has been used.

## REFERENCES

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