

## TEMPERATURE INFLUENCE ON SMART STRUCTURES: A FIRST APPROACH

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The partial differential equations of elasticity show that, when an elastic structure  $\Omega$  is subjected to a system of external loads, it undergoes a passive deformation, without being able to control its strain state. In the case of the so-called *smart structures*, the strain state is constantly under control with the help of sensors and actuators, made of piezoelectric materials, which are integrated within the structure – for this reason, their conception and use have undergone a major development over the past few years. In some practical situations, the influence of the temperature can be relevant. The goal of this work is to enrich the classical piezo-elastic model, by adding an additional scalar equation (i.e., the energy balance), in order to take into account the influence of the temperature. For a description of the coupling between the physical quantities and of the multiphysical phenomenologies occurring in such structures, as well as of their applications, the reader can refer, e.g., to [1].

A distinctive feature of the problems encountered in applications is the presence of several parameters, which show the coexistence of different scales when performing a *non-dimensionalization* procedure: for instance, the thickness of the piezoelectric layer may be small with respect to the other dimensions of the structure, the temperature influence may be relevant only on certain unknowns or on certain parts of the multi-structure, etc.

Imperiale and Joly [3] obtained a mathematical model of ultrasonic piezoelectric sensors by taking into account both the electric and the magnetic effects, but neglecting the temperature effects. The superimposition of two wave propagation phenomena characterized by completely different velocities, as is the case with elastodynamic and electromagnetic waves, entails an unworkable numerical treatment of the problem. This issue can be addressed by resorting to a *quasi-static* pyro- and piezoelectric model, which is justified by means of a non-dimensionalization procedure. Such a procedure was performed in [3] without considering the temperature effects; on the other hand, an *a priori* quasi-static assumption on the electric field was made in [2] and [4]. In our model, the thermo-electromechanical state is defined by three unknowns, i.e., the electric field, the displacement field and the temperature, whose corresponding evolution equations are fully coupled in a linear mixed parabolic-hyperbolic system. Furthermore, we carry out a formal non-dimensionalization of the equations, so as (i) to extend the results by [3] and (ii) to justify the quasi-static assumption in [2] and [4]. These results will then be used to study efficient numerical methods for smart structures taking into account the temperature effects.

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