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EXPERIMENTAL AND NUMERICAL ANALYSIS OF THE MUSICAL BEHAVIOR OF TRIANGLE INSTRUMENTS

Pascal Bestle, Michael Hanss and Peter Eberhard

Institute of Engineering and Computational Mechanics, University of Stuttgart,
Pfaffenwaldring 9, 70569 Stuttgart, Germany
{pascal.bestle, michael.hanss, peter.eberhard}@itm.uni-stuttgart.de
www.itm.uni-stuttgart.de

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Abstract. In the last centuries, the typical processing of musical instruments was pushed forward empirically based on the personal experience of the instrument manufacturer. This paper presents steps to use methods of computational mechanics to optimize musical instruments. These first steps are the investigation of the musical properties of radiated sound and the derivation of criteria to evaluate and rate this sound. Therefore, the sound and the structural behavior of two different triangle instruments are analyzed using experimental modal analysis. The chosen instruments exhibit only small geometrical differences, but a clearly different radiated sound. Furthermore, due to the simple geometry and the isotropic material behavior, numerical models can be obtained for comparison of computational results and measurements.

1 INTRODUCTION

The history of musical instruments dates back nearly as long as the humanity itself. During the last centuries, the development of the quality as well as of the play of musical instruments was pushed empirically and experimentally by instrument manufacturers and players. This current stage of development can hardly be surpassed. At first sight, the scientific study of instruments does not seem to allow any further significant improvement. It is, therefore, very important to gain a mechanical understanding for these complex systems and to provide numerical models to illustrate the behavior of sound. At a second sight this deepened understanding may be the chance for systematic improvements.

2 PSYCHOACOUSTICS

To evaluate and rate the sound of an instrument, it is necessary to take the psychoacoustic effects of the human perception of sounds into account. Hearing cannot be considered only as the phenomenon of the transfer of mechanical waves from the air to the inner ear. Caused by the mechanical behavior of the human ear, the sound intensity is weighted depending on the frequency. However, even more important is that the perception of sound strongly depends on the person and the musical experience of the listener.

Basically, a sound is made up of a multitude of different tones which are heard at the same time. A single tone is defined by a frequency and an amplitude. The frequencies of tones depend on the mechanical behavior of the musical instrument. Tones of instruments which are excited only one time, e.g. percussion instruments or the piano, additionally exhibit a damping ratio. This means that the amplitude of the tone fades away.

For technical applications the radiated sound can be described by its spectrum. The spectrum is transformed within the inner ear into neural pulses. This leads to different effects for the cognition of the sound. In the following, these effects are explained for two simultaneously sounding tones.

2.1 Overtone series

For the cognition of a sound one global distinction can be done between dissonance and consonance. Here it is intending what happens when two tones with different frequencies are played simultaneously. In this context, dissonance describes the effect that the difference of the frequencies of two simultaneously sounding tones is smaller than the critical bandwidth. The listener than only notices one single tone. If the difference is bigger than the critical bandwidth, the listener hears two distinct tones which can be harmonious or disharmonious. This distinction is not part of this paper because the triangle instrument can be played without respect to the tonality. In the following, three effects are explained which occur depending on the difference of the frequencies.

The characteristics of the sound of an instrument significantly depends on the overtones. These overtones originate from the structural behavior, e.g. the string of a bow instrument oscillates with the full length, the half length, the third of the length and so on. The frequency series of the overtones f_i can be described by

$$f_i = (i+1) f_0 c_i \tag{1}$$

depending on the fundamental frequency f_0. For an absolutely harmonic sound, all overtones are multiples of f_0 where the factor $c_i = 1$ for all overtones. The constant c_i is an indicator for the brightness of the sound. The less c_i deviates from an average value $\bar{c}$, the more pleasant the sound appears.

From the overtone series, the intervals of two simultaneously or successively sounding tones is derived. Intervals are the basic modules of music. They build melodies or chords. The interval of two tones with the frequencies f_1 and f_2 is defined by the proportion $p = f_1/f_2$, e.g. the proportion of an octave is $p_{\mathrm{oct}} = 2/1$, a minor second $p_{\mathrm{minSec}} = 16/15$, a major second $p_{\mathrm{minSec}} = 10/9$, and a minor third $p_{\mathrm{minTh}} = 6/5$. These intervals can also be dissonant or consonant, see Figure 1.

2.2 Critical bandwidth

The critical bandwidth is used as a technical term from psychoacoustics describing a frequency bandwidth of auditory filters created by the inner ear. If the difference of the frequencies of two simultaneously sounding tones f_1 and f_2 is smaller than the value of the critical bandwidth Δf_{crit}, than the listener hears only one tone and the effect of interference between these frequencies arises, [1]. The bandwidth depends on the average frequency $\overline{f} = (f_1 + f_2)/2$.

Figure 1 shows the dependency of the critical bandwidth on the average frequency $\overline{f}$, [4]. The blue line represents the critical bandwidth Δf_{crit} at each average frequency $\overline{f}$ in the frequency range from 100 Hz to 20 kHz. This line splits the plot into an area of consonance (upper area) and the area of dissonance (lower area).

In addition, the frequency differences between the tonal intervals of a minor second, a major second and a minor third depending on the frequency are presented. It can be shown that any minor or major second, independent of the frequency, is dissonant because in the whole frequency range the frequency differences are smaller than the critical bandwidth. The first consonant interval in the music is the minor third. The fact of consequent dissonant intervals is used in music to create chords with small or large disharmonies.

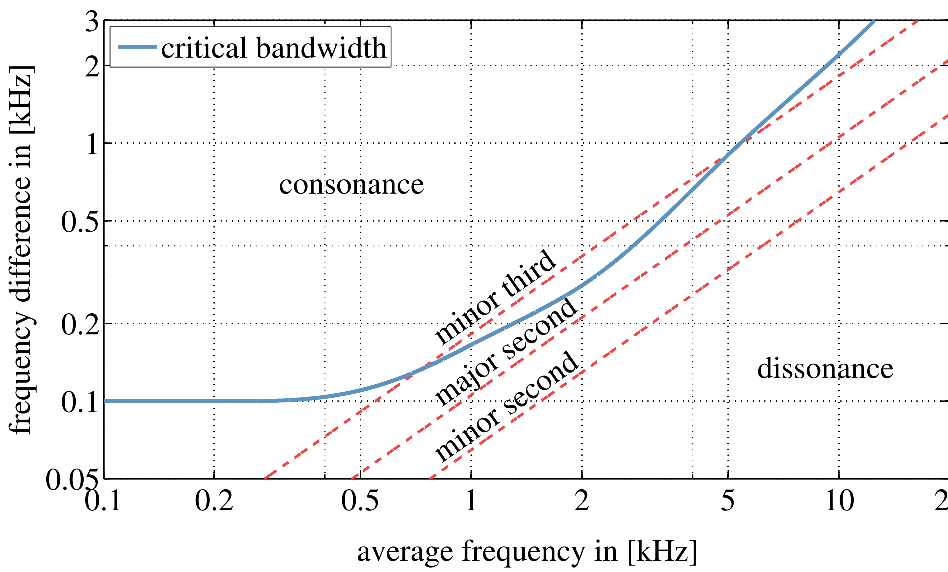

Figure 1: Critical bandwidth depending on the average frequency of two tones.

A dissonance is perceived as roughness because the listener only notices one tone, but with a not clear sound. Different experiments have shown the dependency of the roughness on the frequencies. The roughness of two simultaneously sounding frequencies can be graded. However, this is a subjective classification depending on the musical experience of the listener.

2.3 Beat frequency

A more detailed look to the critical bandwidth shows that tis range can be further split into the dissonant areas and the area of beat frequency, see Figure 2. The beat frequency occurs in a small range of the critical bandwidth around the average frequency $\overline{f}$. Actually, it is not dissonant because the listener only hears one single tone with a beating amplitude, [3].

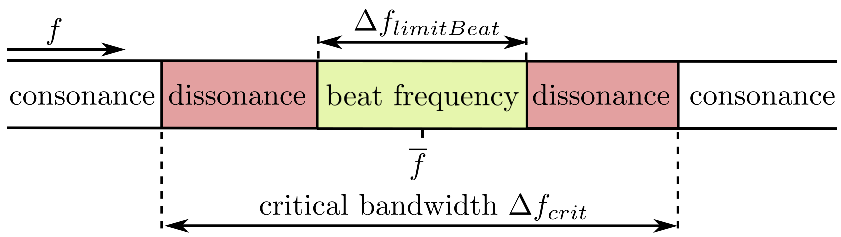

Figure 2: Partition of the critical bandwidth into the dissonant area and the area of beat frequency.

The bandwidth of beat frequency does not depend on the average frequency of the two tones. It is in the range of 10 Hz to 15 Hz and depends on the musical experience of the listener. The phenomenon can be explained mathematically by two simultaneously sounding tones with the frequencies f_1 and f_2 with identical amplitudes as

$$\cos(2\pi f_1 t) + \cos(2\pi f_2 t) = 2\cos\left(2\pi\frac{f_1+f_2}{2}t\right)\cos\left(2\pi\frac{f_1-f_2}{2}t\right). \tag{2}$$

The first factor on the right-hand side describes the carrier signal which the listener hears as one tone. The carrier frequency corresponds to the average frequency $\overline{f}$ of f_1 and f_2. It is modulated by the second factor with the frequency $f_\Delta = (f_1 - f_2)/2$, which is perceived as a pulsation of the amplitude. Figure 3 shows the superposition of two cosine oscillations with a frequency difference $f_1 - f_2 = 2$ Hz.

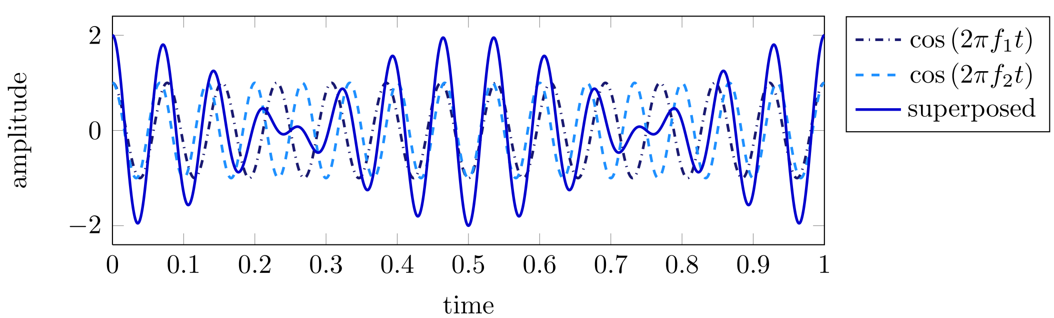

Figure 3: Beat frequency occurring by the superposition of two tones with a frequency difference of 2 Hz.

Mathematically, this frequency f_Δ exists independent on the frequency difference. However, if f_Δ is smaller than 10 Hz to 15 Hz the roughness of the dissonant sound turns into the perceptible pulsation of the amplitude.

3 EXPERIMENTAL MODAL ANALYSIS

The experimental analysis of each of the two triangle instruments is divided into two parts. First, the results of the experimental modal analysis using a Laser-Doppler Vibrometer (LDV) provide the eigenfrequencies and eigenforms of the structure. In a second step, a measurement is done using a microphone in order to obtain information related to the transmission behavior from the structural vibration to what the listener hears.

3.1 Suspension and excitation

In musics usage, the triangle is suspended by one string and it is excited by a small metal stick. In the experiment the triangle has to be suspended at two points to avoid a twist and a large motion during the measurement. With the very soft springs of the suspension, the triangle can be decoupled from the experimental rig. Furthermore, this suspension facilitates the comparison between measurement and numerical analysis, as the structure can be assumed as free moving, see Figure 4.

The eigenmodes of the triangle can be split into in-plane (IP) modes and out-of-plane (OP) modes. The measurement by the used LDV is only one directional. That is why it is necessary to separate it into the IP-direction and the OoP-direction. In Figure 4 the two excitation directions are shown.

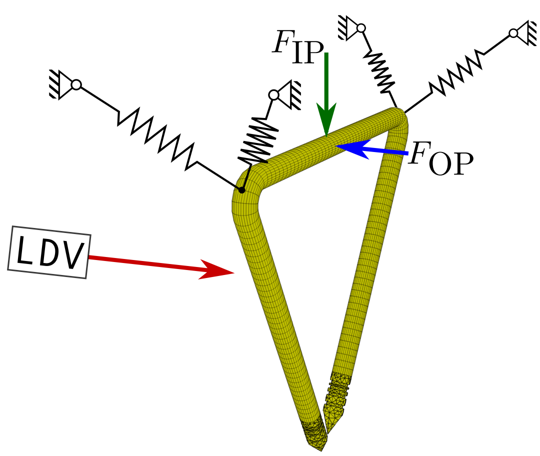

Figure 4: Suspension and excitation directions of the triangle instrument.

Similar to the original play, the excitation in the experiment is performed by an impact hammer with a metal tip. The sound does not differ significantly from the one by an excitation with the metal stick. To ensure that every excitation takes place at the same location, in the same direction and with approximately the same energy, the impact hammer is mounted as shown in Figure 5.

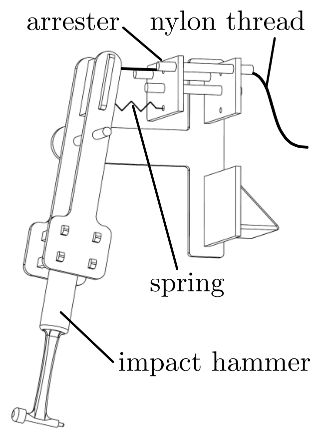

Figure 5: Mounting of the impact hammer.

3.2 Measurement and analysis

The measurement of the velocity for the modal analysis is done using a Polytec Scanning-Laser-Doppler Vibrometer PSV-500. In contrast to alternative measuring methods with acceleration sensors, the contactless measurement using Laser-Doppler Vibrometry has some advantages. At first, the vibration properties of the structure are not influenced as when adding additional accelerometer masses. Furthermore, the optical support of the PSV-500 greatly simplifies the set up of the scan grid. The measurement is done at 74 different points. In order to obtain a good signal quality of the backscattered laser beam, a reflection foil has to be applied to the chrome-plated surface of the triangle.

The measurement of the sound is performed by a microphone, placed at a distance of 40 cm from the triangle. On the one hand the distance is limited by the spatial dimension of the test rig, and on the other hand, there must be a minimum distance to measure outside the acoustic near field. The suspension geometry and the excitation of the structure remain unchanged.

4 NUMERICAL ANALYSIS

The global geometry of the two different triangle instruments is nearly the same. Both are specified as 8″ triangle. This means that the sides of the triangle have a length of about 8″. The exact geometries of the instruments can be found by measurement, the material of both is specified as steel.

The numerical analysis is performed using the finite element program Ansys. To identify the real material properties, a parameter identification is done by updating the properties density, Young's modulus and the Poisson ratio. The criterion to find the optimal parameters is the minimization of the maximum of the relative error between the eigenfrequencies of the measurement and the numerical eigenvalues calculation. The triangle exhibits 42 eigenfrequencies in the audible range from 20 Hz to 20 kHz. Table 1 shows the first 15 frequencies of the high quality triangle in measurement and simulation as well as the relative error between them.

Table 1: Comparison of the eigenfrequencies between measurement and simulation.

	measurement in [Hz]	simulation in [Hz]	relative error in [%]
f_1	148.4	148.5	0.09
f_2	152.3	153.6	0.85
f_3	251.9	252.4	0.16
f_4	724.6	725.9	0.18
f_5	892.6	893.6	0.11
f_6	1349.6	1359.8	0.75
f_7	1388.6	1395.3	0.48
f_8	1517.6	1531.6	0.93
f_9	1539.1	1532.1	0.45
f_{10}	2509.8	2520.1	0.41
f_{11}	2845.5	2828.9	0.58
f_{12}	3250.2	3265.3	0.46
f_{13}	3505.9	3503.6	0.06
f_{14}	4039.8	4076.2	0.90
f_{15}	4064.7	4089.0	0.59

The maximum deviation between measurement and simulation is less than one percent. That indicates that the numerical model and the associated material properties provide a very good approximation of the real instrument. The same analysis is also done for the low-quality triangle instrument, where results have a similar quality as for the high-quality triangle.

5 RESULTS

It is worth to note that the triangle is a percussion instrument and it can be played without regard to the tonality. Therefore, the series of frequencies should not exhibit one fundamental tone. That is why the criterion of harmoniousness according to the overtone series cannot be applied.

Much more important is the behavior in time domain. This can be represented in a spectrogram with the amplitude over frequency and time as shown in Figure 6 for the OP-direction of the high-quality triangle (left) and the low-quality triangle instrument (right).

The amplitude represents the acoustic pressure normalized to the excitation energy over a time of five seconds and a frequency range of 0 kHz to 12 kHz.

In this diagrams two significant effects can be illustrated. The first is the loudness of the radiated sound represented by the acoustic pressure at the measurement point of the microphone. At the same excitation energy, the sound of the high-quality triangle is much louder than the sound of the low-quality triangle. The second characteristic is the damping behavior. The sound of the high-quality triangle appears much longer than the one of the low-quality triangle.

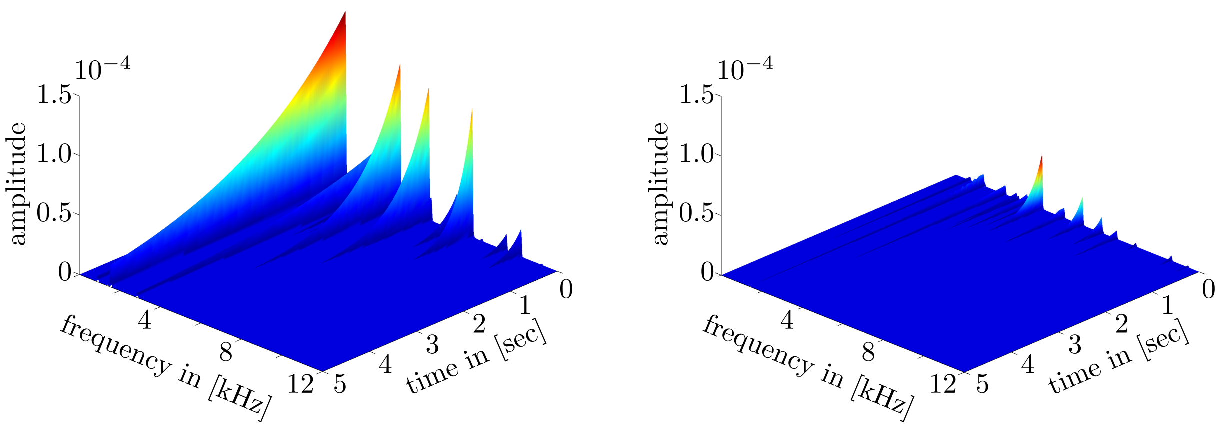

Figure 6: Spectrograms for the high-quality triangle (left) and the low-quality triangle instrument (right).

Generally, the sound of a triangle instrument appears pleasant and agreeable if it contains a fundamental sound of a couple of frequencies which are significantly louder than the remaining sound. This remaining sound is without a tonality and the sound level is significantly lower than the one of the dominant sound.

The spectrum of the high-quality triangle contains four tones which are significantly louder than the others, see Figure 6 (left). The corresponding frequencies are listed in Table 2.

Table 2: Dominant frequencies of the fundamental sound of the high-quality triangle.

	measurement in [Hz]
f_9	1539.1
f_{16}	4239.6
f_{17}	5658.9
f_{21}	7795.9

Important for a pleasant and agreeable sound is, that these louder tones are not dissonant. This can be estimated by the criterion of the critical bandwidth. Two sequenced frequencies of the spectrum of the dominant sound are compared with reference to the critical bandwidth criterion. Here, the average frequency of two respective frequencies is calculated and the related critical bandwidth is obtained. This can be represented in a diagram shown in Figure 7.

The red bars represent the location of the eigenfrequencies from Table 2. In this case there are three pairs of frequencies. The blue bars show the width of the critical band between two frequencies. In this diagram it can be seen, that the frequency difference of each pair of the dominant sound is bigger than the critical bandwidth. Therefore, the dominant sound is not dissonant.

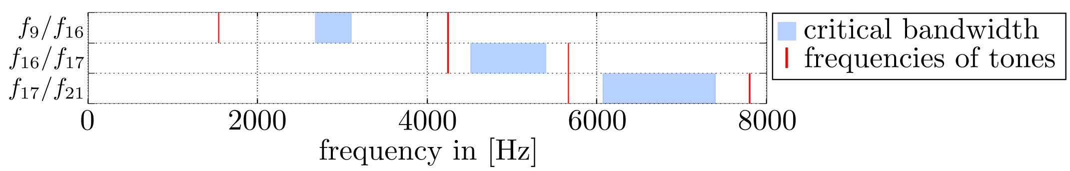

Figure 7: Critical bandwidth between two neighboring frequencies of the dominant sound of the high-quality triangle.

The same diagram can be presented for the frequencies of the non-dominant sound. Therefore, it is necessary to select only the audible frequencies of the sound. Due to the excitation in the OP-direction only the corresponding frequencies to the OP-eigenmodes are remaining, because the modes of the IP-direction are not excited. But, not all of the OP-eigenmodes are excited or they do not create an audible tone. This can be determined by a threshold of the acoustic pressure created by the inner ear, [2]. The relevant frequencies of the audible tones are listed in Table 3.

Figure 8 shows dissonant and consonant frequency pairs of the non-dominant or subliminal sound. In contrast to the fundamental sound, this spectrum exhibits a couple of dissonant frequency pairs. A dissonance occurs if the difference of two frequencies is smaller than the corresponding critical bandwidth. Caused by the fact that the dominant sound is much louder than these dissonances, the non-dominant sound does not influence the consonance of the whole sound. These dissonances can also be called an imperfection what makes the sound interesting and gives the triangle a recognition value.

These imperfections in the sounds of instruments are the reason, why people are able to distinguish between a large variety of instruments. It is also the reason why persons prefer certain different instruments. If the imperfection is too dominant, the sound appears disagreeable.

Compared to the high-quality triangle, the sound of the low-quality triangle appears very dissonant. This can be explained from different effects. The first is that the spectrum exhibits a significantly fewer number of eigenfrequencies than the other triangle.

Table 3: Audible frequencies of the high quality triangle not contributing to the fundamental sound.

	measurement in [Hz]
f_5	892.6
f_{11}	2845.5
f_{12}	3250.2
f_{13}	3505.9
f_{18}	5779.3
f_{19}	7038.9
f_{24}	9485.8
f_{25}	10205.8
f_{27}	12605.1

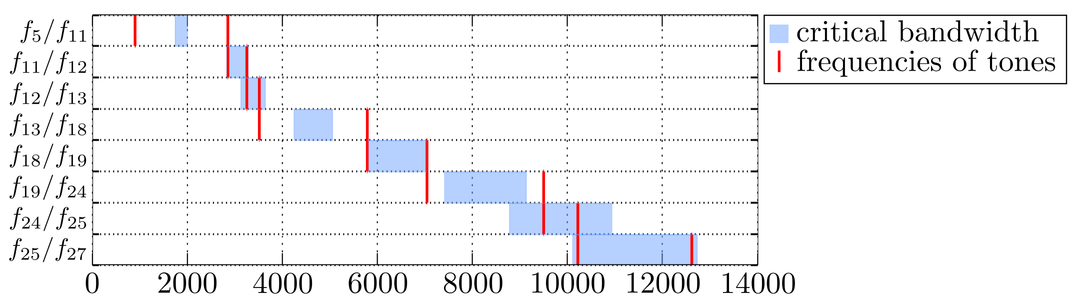

Figure 8: Critical bandwidth between two neighboring frequencies of the none dominant sound of the high-quality triangle.

In addition, the spectrum of the low-quality triangle has only one tone which is significantly louder than the remaining tones. This single tone is very dominant but does not form a sound. Due to this, the sound cannot be split into a fundamental sound and a non-dominant sound. Therefore, all tones have nearly the same audibility thus the same weighting in the spectrum.

In Figure 9 the audible tones of the spectrum of the low-quality triangle are represented. All these tones are perceived with nearly the same audibility. The pairs of tones exhibit a couple of dissonances which are distributed over the whole frequency range.

6 CONCLUSIONS

The investigation of the two different triangle instruments has shown that it is possible to provide numerical models representing the structural behavior of the real instrument. The relative error of the eigenfrequencies and their corresponding eigenforms between

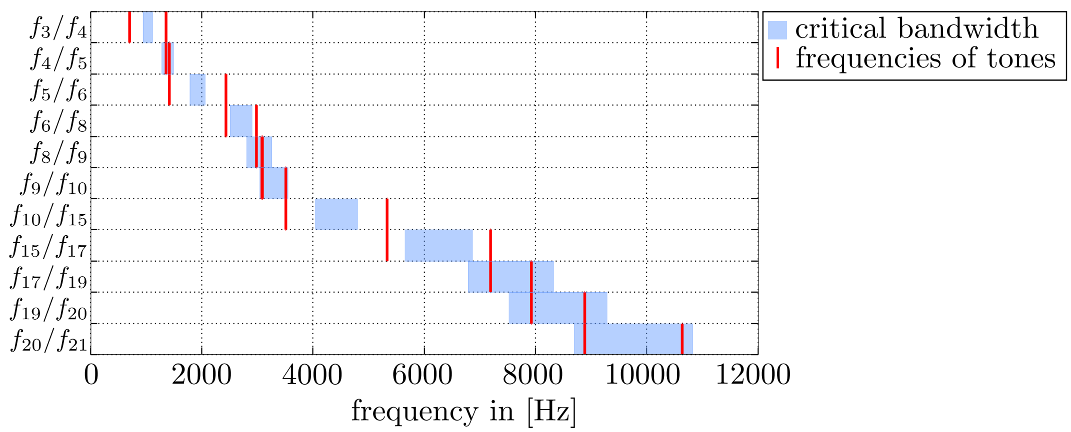

Figure 9: Critical bandwidth between two neighboring frequencies of the spectrum of the low-quality triangle.

the experimental modal analysis and the numerical simulation is less than one percent. This constitutes a very good initial basis for further investigations of both triangles on the sound radiation in time domain and the analysis of the geometrical features of the high-quality triangle.

Furthermore, methods of psychoacoustics were introduced to evaluate and rate the sound of instruments. With the help of these criteria it has been show why the sound of the high-quality triangle is sensed as much more pleasant than the one of the low-quality triangle. The main criterion is that the overall sound exhibits a separation in a dominant sound, consisting of a few frequencies which are not dissonant, and a non-dominant sound which can exhibit a couple of dissonances to give the sound an imperfection.

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