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VIBRO-ACOUSTIC WAVE INTERACTION IN CRACKED PLATE MODELED WITH PERIDYNAMICS

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Abstract. The work is devoted to simulation of the phenomenon of vibro-acoustic wave interaction for a 2D model of a plate with a crack. The authors investigate a possibility of simulation of sidebands generation for high frequency excitation in presence of low frequency model deformation corresponding to the planar normal mode with closing and opening a crack. A nonlocal approach, i.e. the peridynamics, has been applied to discretize mechanical continua and therefore to take an advantage of more convenient introduction of geometric discontinuities resulting from cracks.

1 INTRODUCTION

Recently, waves and crack-wave interaction based simulation and experimental techniques have been widely applied for damage detection [1, 2, 3, 4, 5]. They are considered very attractive since making possible the detection of short and narrow flaws. These techniques are very often applied in the area of structural health monitoring and offer reliable concluding on the crack propagation using on-line systems. A comprehensive review on nonlinear approaches for the crack-wave interaction analyses is given in [6]. Amongst others the following issues are known in the field of the study: bi-linear stiffness, breathing cracks, clapping, hysteresis, thermo-elasticity and the Luxemburg-Gorky effect. Also in case of simulations there are known applications based on the finite element and finite difference methods, also with indirect modeling of periodic changes for model stiffness in the area of crack for crack-wave interaction [7]. The work presented in the paper makes use of a nonlocal technique of modeling mechanical continuum to simulate waves interactions while opening and closing a crack.

The idea of a nonlocal modeling for mechanical continuum in not a new one [8, 9]. Originally it was applied for continuous models with integro-differential analytical problem description. The authors of these original works proved that the integral based governing equations also offer accurate description of the behavior of solids, simultaneously satisfying all necessary conditions regarding force equilibrium, displacement compatibility and constitutive relations. The advantage of the applications of the mentioned methods is less requirements on geometric discontinuities. One of recently proposed nonlocal modeling technique is peridynamics [10].

The peridynamics is an integral based modeling and simulation technique with a spatial partial derivative-free governing equation. It introduces the definition of pairwise forces acting between subsequent subregions of a solid, within a horizon of nonlocal interactions [10, 11]. The method may allow easily for spontaneous crack growth. The authors apply the peridynamics to study the phenomenon of wave interaction in presence of crack in a two-dimensional model of aluminum plate.

The agenda of the paper is as follows. Section 2 serves an overview on the theory of peridynamics, section 3 describes numerical analysis of vibro-acoustic wave interaction performed in the work, including the model formulation, applied excitation and introduced crack. Section 4 presents and discusses the results, and final section 5 summarizes the paper and gives the overall conclusions.

2 PERIDYNAMICS

The peridynamics was introduced by Silling in 2000 [10]. It defines a nonlocal definition of mechanical continuum for solids, by their division into infinite number of pieces of matter (*particles*) with attached pairwise forces $\mathbf{f}$ spread within *a horizon* H, as presented in Figure 1. The governing equation takes the following integral-based form

$$\rho\ddot{\mathbf{u}}\left(\mathbf{x},t\right)=\int_{H}\mathbf{f}\left(\mathbf{u}(\hat{\mathbf{x}},t)-\mathbf{u}(\mathbf{x},t),\hat{\mathbf{x}}-\mathbf{x}\right)dV_{\hat{\mathbf{x}}}+\mathbf{b}(\mathbf{x},t) \tag{1}$$

where: ρ is mass density, $\mathbf{u}$ is vector of particle displacements, localized in $\mathbf{x}$, i.e. for an actual central particle, and $\hat{\mathbf{x}}$, for a neighboring particle, respectively. The horizon H is determined with the radius δ and defines the area for all nonlocal interactions between an actual central and neighboring particles.

Given integral aggregates the products of the pairwise function $\mathbf{f}$ and the volume $V_{\hat{\mathbf{x}}}$ to find the overall volumetric density of a reaction force acting on a central particle. $\mathbf{b}$ defines the vector of volumetric density of external force acting on the central particle.

The pairwise interaction force $\mathbf{f}$ depends on the relative position in the reference coordinate system $\xi=\hat{\mathbf{x}}-\mathbf{x}$ and the relative displacement $\eta=\mathbf{u}(\hat{\mathbf{x}},t)-\mathbf{u}(\mathbf{x},t)$. By definition

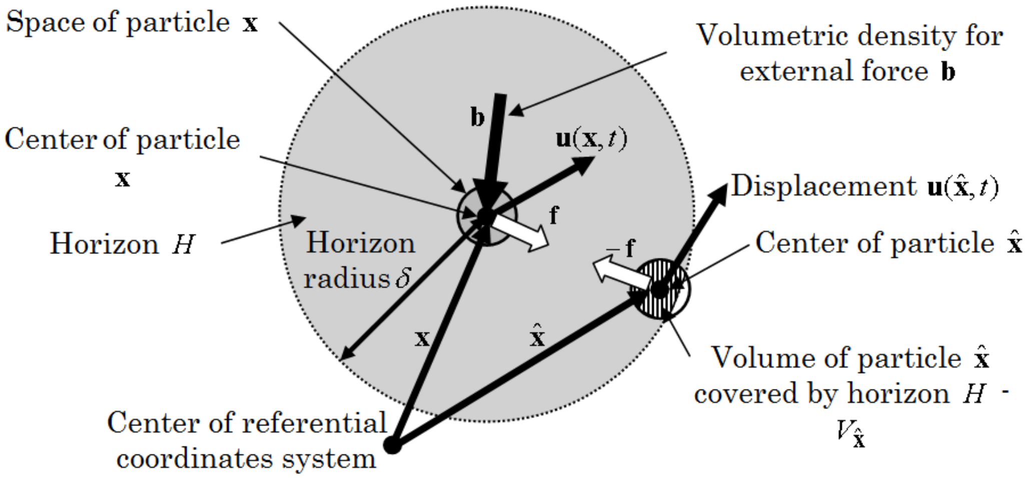


Figure 1: Definition of particles in peridynamics.

it also includes all the elastic properties of the materials. These properties are considered with *the micromodulus function c*

$$\mathbf{f}(\eta, \xi) = \begin{cases} \mathbf{e}(\eta, \xi) c(\xi) s \text{ if } ||\xi|| \leq \delta \\ 0 \text{ if } ||\xi|| > \delta \end{cases} \tag{2}$$

where $\mathbf{e}$ is a unit vector along the direction linking a pair of particles and s is strain. For a 2D case and given thickness of an anisotropic plate T, the constant micromodulus function c can be found with the following formula [12]

$$c = \frac{6E}{\pi \delta^3 (1 - \nu) T} \tag{3}$$

where the material properties are: E - the Young's modulus and ν - the Poisson ratio.

In the peridynamics there is an assumption on nonlocality of interactions between particles. Hence, in contrast to local methods e.g. classical formulations of finite element method or finite difference method, a region where pairwise force acts exceeds the area of a nearest neighbor. The greater horizon radius δ the more interaction forces are considered and integrated for an actual central particle. When a model undergoes deformation, the criterion on the maximum strain is applied separately to each link between particles within the horizon, to decide on its further existenceand possible crack growth.

The most important advantage of the application of peridynamics is its ability of an easy introduction any geometric discontinuities into a model. Regardless of the shape of the model and applied boundary conditions the integral-based formulation of the governing equation (1) determines a solution. Moreover long-range interactions enable model upscaling, e.g. with van der Waals forces or the results obtained with molecular dynamics. The peridynamics seems a versatile modeling technique which offers convergence to both local and nonlocal methods depending on the horizon radius.

3 NUMERICAL MODEL

In the work the phenomenon of wave interaction is studied in a cracked plate of dimensions 10mm, 40.5mm and 1mm. A rectangular aluminum plate undergoes a periodic uniaxial stretching and compression resulting from external low frequency forces (LF) attached to the models edges, as presented in Figure 2. The frequency of the LF forces is arbitrary chosen to be $65kHz$ and corresponds to the natural frequency for the normal mode with in-plane opening and closing crack observed at 65.7kHz. Moreover there is applied an additional, asymmetrically localized high frequency excitation (HF). It characterizes approximately ten times greater frequency ($500kHz$) and is considered as a wave modulated by the LF wave in presence of a crack. Nodal displacements for given measurement point are taken into account for the analysis of wave interaction.

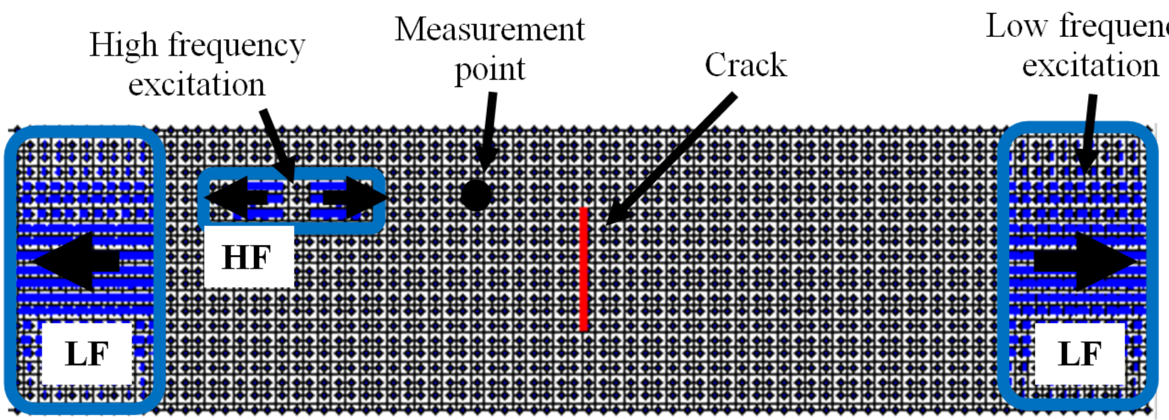


Figure 2: Model of an aluminum plate modeled with peridynamics

A numerical model of the plate is created according to the theory of peridynamics. For a discrete case the governing equation (1) may be rewritten to the following form, determined for the i-th particle

$$\begin{cases} \rho\ddot{u}_i\left(t\right) = \sum_{j\epsilon H_i} \left\{\left(\xi_{Xi,j} + u_j\left(t\right) - u_i\left(t\right)\right) F_{i,j}\left(t\right) cA_{i,j}T\right\} + b_{Xi}(t) \\ \rho\ddot{v}_i\left(t\right) = \sum_{j\epsilon H_i} \left\{\left(\xi_{Yi,j} + v_j\left(t\right) - v_i\left(t\right)\right) F_{i,j}\left(t\right) cA_{i,j}T\right\} + b_{Yi}(t) \end{cases} \tag{4}$$

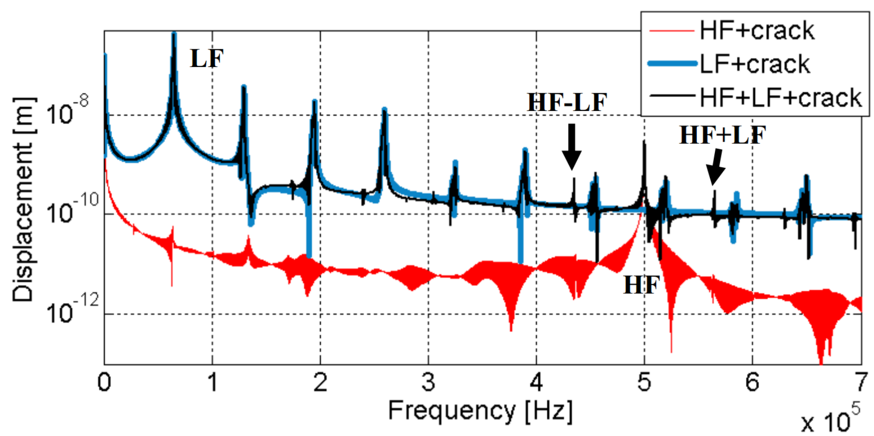
where u_i and v_i are in-plane components for the particle displacements along the axes OX and OY. H_i is the horizon determined by the list of indexes j of all neighboring particles for the i-th particle. The relative particle position and the external force acting on a particle are represented with the two pairs of transversal components: $\xi_{Xi,j}$, $\xi_{Yi,j}$ and b_{Xi}, b_{Yi}, respectively. The auxiliary function $F_{i,j}$ becomes

$$F_{i,j} = \frac{1}{\sqrt{\xi_{Xi,j}^2 + \xi_{Yi,j}^2}} - \frac{1}{\sqrt{(\xi_{Xi,j} + u_j - u_i)^2 + (\xi_{Yi,j} + v_j - v_i)^2}} \tag{5}$$

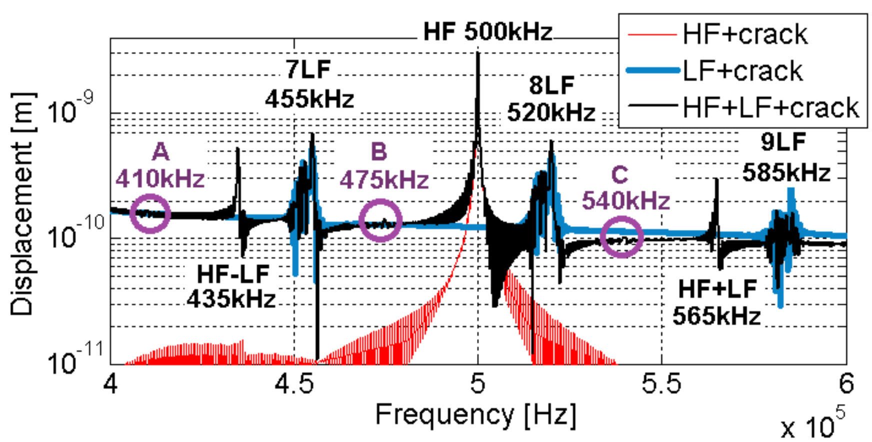
The parameter $A_{i,j}$ determines the part of the area of the j-th particle, which is covered by the horizon related to the i-th particle. A centrally localized crack is introduced in the model via breaking all the links between particles which cross the 4.5mm long line of the crack. The model introduces contact. Hence the phenomenon of the wave interaction can be successfully modeled. All broken links in the area of the crack are temporarily recovered for the period when the distances between respective particles are less than their initial values. This leads to a bilinear stiffness in the model for the region with the crack, which finally results in wave modulations.

4 RESULTS FOR VIBRO-ACOUSTIC WAVE INTERACTION

Figure 3 presents an example of the FFT plots for a cracked plate for three different excitation configurations: HF, LF, and simultaneous existence of HF and LF.



(a)



(b)

Figure 3: FFT plots for wave modulation and sidebands generation - horizontal displacement: (a) entire frequency spectrum up to 700kHz, (b) magnified plots for the range 400-600kHz

The results show that the phenomenon of wave interaction and modulation can be effectively simulated with the application of peridynamics for a cracked plate. There are visible higher harmonics for the LF wave, e.g. marked as 7LF, 8LF, etc. The HF wave gets modulated by the LF excitation as well as its multiples. Hence, there are present the main sidebands localized symmetrically with respect to the fundamental harmonics for the HF wave at 500kHz, i.e. marked as HF-LF and HF+LF. Moreover there are identified additional resonance peaks resulting from the interaction between the HF wave and consecutive multiples of the LF wave. In Figure 3b there are found the following additional minor sidebands for the HF wave: at 410kHz (marked as A), at 475kHz (B) and at 540kHz (C). It seems that for the case A the resonance peak is generated due to the interaction between HF and 7LF. The resultant frequency can be found as follows: $f_A = f_{7LF} - (f_{HF} - f_{7LF})$. Similarly for the case C: $f_C = f_{8LF} + (f_{8LF} - f_{HF})$. The frequency for the case B can be easily found as: $f_B = f_A + f_{LF}$ or $f_B = f_C - f_{LF}$. Finally there are expected many other resonance peaks, of significantly less amplitude, due to all possible higher harmonics interactions and modulations between already generated sidebands as in the cases A, B or C.

5 SUMMARY AND CONCLUDING REMARKS

The paper presents the results of the analysis of wave interaction present in a two-dimensional model of cracked plate. Modeled plate undergoes the two external in-plane excitations: a low frequency force used to generate a mode with opening and closing the crack and a high frequency force which gets modulated in presence of the crack and the model deformation resulting from the latter force. A bilinear stiffness in the model, which is introduced by the contact mechanism, allows for the wave sidebands generation.

The model of a cracked plate created with the theory of peridynamics, additionally with introduced contact, gives the opportunity of effective modeling the phenomenon of wave interaction. As clearly seen in the work, periodic sidebands appear as the result of nonlinearity (bilinearity) from imposed opening and closing the crack. Generated sidebands can be tracked in order to assess the crack growth.

It should be noted that, with the application of peridynamics, the authors take an advantage of a very convenient method for introduction of geometric discontinuities (cracks) into a numerical model. It helps to determine more reliable and realistic formulation for the governing equation compared to the classical derivative-based problem description.

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