TOWARDS AN EFFICIENT SIMULATION OF CAVITATING FLOWS WITH REAL GAS EFFECTS AND UNCERTAINTY QUANTIFICATION

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Abstract. A recent development of the discrete equation method (DEM) for the simulation of compressible multiphase flows including real-gas effects is illustrated. A reduced five equation model is obtained starting from the semi-discrete numerical approximation of the two-phase model. A simple procedure is then proposed for using a more complex equation of state, thus improving the quality of the numerical prediction. Classical test-cases well-known in literature are performed featuring a strong importance of thermodynamic complexity for a good prediction of temperature evolution.

1 INTRODUCTION

Modeling two-phase flows is of primary importance for engineering applications. Two aspects are fundamental: (i) how to model the interface between two fluids with different thermodynamic properties and (ii) to characterize the mechanisms occurring at the interface as well as in zones where the volume fractions are not uniform.

Instead of the traditional approaches to multiphase modeling, where an averaged system of (ill-posed) partial differential equations (PDEs) discretized to form a numerical scheme are considered, the discrete equation method (DEM) results in well-posed hyperbolic systems. This allows a clear treatment of non-conservative terms (terms involving interfacial variables and volume fraction gradients) permitting the solution of interface problems without conservation errors. This method displays several advantages, such as an accurate computation of transient flows as the model is unconditionally hyperbolic, boundary conditions solved with a simple and accurate treatment, an accurate computation of non-equilibrium flows as well as flows evolving in partial or total equilibrium. With the DEM, each phase is compressible and behaves according to a convex equation of state (EOS). In many works of interface problem, the *Stiffened Gas* (SG) EOS was usually

used [1, 2, 3]. As explained in Saurel *et al.* [2], this EOS allows an explicit mathematical calculations of important flow relation thanks to its simple analytical form. Moreover, in mass transfer problem it assures the positivity of speed of sound in the two-phase region, under the saturation curve.

When complex fluids are considered, such as cryogenic, molecularly complex and so on, the use of simplex EOS can produce imprecise estimation of the thermodynamic properties, thus deteriorating the accuracy of the prediction. This amount of uncertainty should be taken into account for assessing the variability of the quantity of interest.

An effort for developing a more predictive tool for multiphase compressible flows is underway in Bacchus Team (INRIA-Bordeaux). Within this project, several advancements have been performed, *i.e.* considering a more complete systems of equations including viscosity [4], working on the thermodynamic modeling of complex fluids [6], and developing stochastic methods for uncertainty quantification in compressible flows [5].

The aim of this paper is to show how the numerical solver based on a DEM formulation has been modified for including a complex equation of state for the vapor region, and how the cavitation term can be treated when non-convex equations of state are used for describing the vapor region.

2 PROBLEM STATEMENT

2.1 General introduction

Let us introduce here the general formulation of a seven equation model and the assumptions for yielding a five equations models, with heat and mass transfer :

$$
\begin{cases}
\dfrac{\partial \alpha_1}{\partial t} + v_I \dfrac{\partial \alpha_1}{\partial x} = \mu(P_1 - P_2) + \eta Q + \dfrac{\rho}{\rho_I} \dot{Y} \\[2mm]
\dfrac{\partial(\alpha_1 \rho_1)}{\partial t} + \dfrac{\partial(\alpha_1 \rho_1 v_1)}{\partial x} = \rho \dot{Y} \\[2mm]
\dfrac{\partial(\alpha_1 \rho_1 v_1)}{\partial t} + \dfrac{\partial(\alpha_1 \rho_1 v_1^2 + \alpha_1 P_1)}{\partial x} = P_I \dfrac{\partial \alpha_1}{\partial x} + \lambda(v_2 - v_1) + v_I \rho \dot{Y} \\[2mm]
\dfrac{\partial(\alpha_1 \rho_1 E_1)}{\partial t} + \dfrac{\partial(\alpha_1(\rho_1 E_1 + P_1)v_1)}{\partial x} = P_I v_I \dfrac{\partial \alpha_1}{\partial x} + \lambda v_I(v_2 - v_1) + \mu P_I(P_1 - P_2) + Q + E_I \rho \dot{Y} \\[2mm]
\dfrac{\partial(\alpha_2 \rho_2)}{\partial t} + \dfrac{\partial(\alpha_2 \rho_2 v_2)}{\partial x} = -\rho \dot{Y} \\[2mm]
\dfrac{\partial(\alpha_2 \rho_2 v_2)}{\partial t} + \dfrac{\partial(\alpha_2 \rho_2 v_2^2 + \alpha_2 P_2)}{\partial x} = P_I \dfrac{\partial \alpha_2}{\partial x} - \lambda(v_2 - v_1) - v_I \rho \dot{Y} \\[2mm]
\dfrac{\partial(\alpha_2 \rho_2 E_2)}{\partial t} + \dfrac{\partial(\alpha_2(\rho_2 E_2 + P_2)v_2)}{\partial x} = P_I v_I \dfrac{\partial \alpha_2}{\partial x} - \lambda v_I(v_2 - v_1) - \mu P_I(P_1 - P_2) - Q - E_I \rho \dot{Y},
\end{cases}
\tag{1}
$$

where:

- $Q = \theta(T_2 - T_1)$ is the heat transfer term, T_2 and T_1 are the liquid and the gas temperature, respectively

- $\dot{Y} = \nu(g_2 - g_1)$ is the mass transfer term, g_2 and g_1 are the liquid and the gas Gibbs free energy, respectively.

The symbols ρ_k, α_k, v_k and P_k represent the density, the volume fraction, the velocity and the pressure for each phase, respectively. The symbols, ρ_I and E_I are the interface density and interface energy, respectively and $\rho = \alpha_1\rho_1 + \alpha_2\rho_2$ is the mixture density. $\eta = \frac{\alpha_1\alpha_2}{\alpha_2\rho_1 c_1^2 + \alpha_1\rho_2 c_2^2}\left(\frac{\Gamma_1}{\alpha_1} + \frac{\Gamma_2}{\alpha_2}\right)$. The pressure and velocity at the interface are indicated with P_I and v_I. Several definitions of interface variables exist in literature.

The coefficients $\lambda = C/\varepsilon$ and $\mu = C'/\varepsilon$ are the relaxation velocity parameter and the dynamic compaction viscosity, respectively, where the coefficients C and C' depend on the acoustic impedance of the fluids. These are the so-called *relaxation terms* which yield a process inducing a pressure and velocity equilibrium.

System (1) can be, also, expressed in vectorial form as follows:

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial x}F(U) + B(U)\frac{\partial \alpha_1}{\partial x} = \frac{\partial U}{\partial t} + FT(U) = \frac{R(U)}{\varepsilon} \tag{2}$$

where

$$U = \begin{pmatrix} \alpha_k \\ \alpha_k\rho_k \\ \alpha_k\rho_k v_k \\ \alpha_k\rho_k E_k \end{pmatrix}, \quad F(U) = \begin{pmatrix} 0 \\ \alpha_k\rho_k v_k \\ \alpha_k(\rho_k v_k^2) + P_k \\ \alpha_k(\rho_k E_k + P_k)v_k \end{pmatrix}$$

$$B(U) = \begin{pmatrix} v_I \\ 0 \\ -P_I \\ -P_I v_I \end{pmatrix}, \quad \frac{R(U)}{\varepsilon} = \begin{pmatrix} \mu(P_k - P_{k^*}) \\ 0 \\ \lambda(v_{k^*} - v_k) \\ \lambda v_I \cdot (v_{k^*} - v_k) - \mu P_I(P_k - P_k^*) \\ -\mu(P_1 - P_2) \end{pmatrix}.$$

When a mechanical equilibrium is considered, the equality of pressure and velocity can be obtained through a stiff mechanical relaxation of the system (1), or an asymptotic development, where relaxation parameters, λ and μ, are assumed as tending to infinity (thus supposing a very large interface area), *i.e.* $\epsilon \to 0^+$. As a consequence, this asymptotic procedure allows to find a final solution where relaxation terms are equal to zero. Thus, the system (1) becomes:

$$\begin{cases} \dfrac{\partial \alpha_1}{\partial t} + v\dfrac{\partial \alpha_1}{\partial x} = \dfrac{\alpha_1\alpha_2(\rho_2 c_2^2 - \rho_1 c_1^2)}{(\alpha_1\rho_2 c_2^2 + \alpha_2\rho_1 c_1^2)}\dfrac{\partial v}{\partial x} + \eta Q + \dfrac{\rho}{\rho_I}\dot{Y} \\[2mm] \dfrac{\partial \alpha_1\rho_1}{\partial t} + \dfrac{\partial(\alpha_1\rho_1 v)}{\partial x} = \rho\dot{Y} \\[2mm] \dfrac{\partial \alpha_2\rho_2}{\partial t} + \dfrac{\partial(\alpha_2\rho_2 v)}{\partial x} = -\rho\dot{Y} \\[2mm] \dfrac{\partial \rho v}{\partial t} + \dfrac{\partial(\rho v \otimes v + P)}{\partial x} = 0 \\[2mm] \dfrac{\partial \rho E}{\partial t} + \dfrac{\partial(\rho E + P)v}{\partial x} = 0, \end{cases} \tag{3}$$

where $E = e + \frac{1}{2}v^2$ is the mixture total energy and $e = (\alpha_1\rho_1 e_1 + \alpha_2\rho_2 e_2)/\rho$ is the mixture internal energy.

In order to find the solution of system (3), two steps should be solved:

- *Step 1*: find the numerical solution of the hyperbolic part of system (3) without heat and mass transfer term source.

- *Step 2*: update the solution of temporal ODEs system considering heat and mass transfer terms.

The splitting method is physically acceptable, since we assume that the mechanical relaxation characteristic time $1/\mu$ is much smaller than the characteristic time scales $1/\theta$ and $1/\nu$ of heat and mass transfer and that thermal and chemical relaxation occur in conditions of pressure equilibrium. On the contrary of Saurel, we apply the procedure to a complete system (1). In Step 1, the discrete equation method (DEM) [?] is applied to the system (1) for finding its numerical discretization. Then, an asymptotic development is applied, at a discrete level, in order to impose to the semi-discrete system a mechanical relaxation, finally, obtaining a final semi-discrete system that is equivalent to discrete the five equation model (3). For further details please see [4]. Step 2 is explained in the next section.

2.2 Step 2: Numerical solution of the temporal ODEs with heat and mass transfer terms

Let us consider now the numerical treatment of heat and mass transfer terms, solving the ODEs equation system, that allows to evolve the solution, considering the new sources term:

$$\begin{cases} \dfrac{\partial \alpha_1}{\partial t} = \dfrac{\alpha_1\alpha_2(\rho_2 c_2^2 - \rho_1 c_1^2)}{(\alpha_1\rho_2 c_2^2 + \alpha_2\rho_1 c_1^2)}\dfrac{\partial v}{\partial x} + \eta Q + \dfrac{\rho}{\rho_I}\dot{Y} = S_{\alpha_1} \\[2ex] \dfrac{\partial \alpha_1\rho_1}{\partial t} = \rho\dot{Y} = S_{Y_1} \\[2ex] \dfrac{\partial \alpha_2\rho_2}{\partial t} = -\rho\dot{Y} = -S_{Y_1} \\[2ex] \dfrac{\partial \rho v}{\partial t} = 0 \\[2ex] \dfrac{\partial \rho E}{\partial t} = 0. \end{cases} \tag{4}$$

where:

$$Q = \theta(T_2 - T_1)$$
$$\dot{Y} = \nu(g_2 - g_1)$$

with

$$g_k(P, T_k) = (\gamma_k C v_k - q')T_k - C v_k T_k \log\left(\dfrac{T_k^{(\gamma_k)}}{(P + P_{\infty,k})^{\gamma_k - 1}}\right).$$

The interface density been determined supposing an isentropic behavior of acoustic waves:

$$\rho_I = \frac{\frac{\rho_1 c_1^2}{\alpha_1} + \frac{\rho_2 c_2^2}{\alpha_2}}{\frac{c_1^2}{\alpha_1} + \frac{c_2^2}{\alpha_2}}$$

The parameters θ and ν are thermal and chemical relaxation parameters, respectively. In order to retain metastable states, the relaxation parameters θ and ν will be set to zero far from the interfaces and they will be taken infinite in order to fulfill equilibrium interface conditions with mass transfer:

$$\theta, \nu = \begin{cases} +\infty & \text{if } \epsilon \leq \alpha_1 \leq 1 - \epsilon \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

where ϵ will be a very low value. In order to define this value, we first remember that the present model is a "diffusive interface model", $i.e.$ for numerical reason in all cells the two phases exist always, but a cell is considered filled by a pure fluid when its volume and mass fraction is equal to $1 - \epsilon_1$, with typically $\epsilon_1 = 10^{-8}$. In other words, in all cells the phases must have a volume fraction minimum equal to ϵ_1 or maximum equal $1 - \epsilon_1$, in order to assure that also the other phase exists.

So, for identifying the interface, we will take $\epsilon = 10^{-6}$ and the interface corresponds to mixture cells when volume fractions range is between ϵ and $1 - \epsilon = 10^{-6}$. If ϵ and ϵ_1 are too close, evaporation may occur too early and not only in the interfacial zone.

Moreover, the mass transfer is allowed if one of the fluids in the mixture cell is metastable, $i.e.$ if $T_k > T_{sat}(P^*)$. Let us denote with superscript ** the final quantities computed by solving the Step 1. In table (1) are reported the quantities of input and output of the second step and the variables that are not influenced by the heat or mass transfer. In fact, from the system (5), it is evident that by means of mass equations, we have the mixture density is constant ($\rho^{**} = \rho^*$) and thus, also the velocity ($v^{**} = v^*$) and the mixture total and internal energy ($E^{**} = E^*$ and $e^{**} = e^*$).

2.2.1 Stiff thermo-chemical solver

In order to solve the system (4), we need closure relations for Q and $\dot{Y}$, so two other equations are necessary.

Anyway, we know that physically, the single phase in metastable state evolves in a new equilibrium liquid/vapor state, during which, a heat and mass exchange happens. Thus, the mass transfer can stop only when a new state on the saturation curve is reached, since the liquid and its vapor are in equilibrium. In order to be sure that the two phases are in perfect equilibrium on the saturation curve, we needs to assure, in all time steps, that the system achieves the temperature and Gibbs free energy equilibrium between the phases ($T_1 = T_2$ and $g_1 = g2$).

Step 2 (Only on the interface: if $\epsilon_1 < \alpha_1 < 1 - \epsilon_1$ and $T_k > T_{sat}(P^*)$)		
IN variables	OUT variables	Constant MIXTURE variables
α_1^* ($\alpha_2^* = 1 - \alpha_1^*$)	α_1^{**} ($\alpha_2^{**} = 1 - \alpha_1^{**}$)	
ρ_1^* and ρ_2^*	ρ_1^{**} and ρ_2^{**}	
v^*	v^{**}	$\rho^{**} = \rho^*$
P^*	P^{**}	$e^{**} = e^*$
e_1^* and e_2^*	e_1^{**} and e_2^{**}	$E^{**} = E^*$
T_1^* and T_2^*	T^{**}	
g_1^* and g_2^*	g^{**}	

Table 1: Variables used in the Step 2. $\rho = (\alpha_1\rho_1) + (\alpha_2\rho_2)$; $\rho e = (\alpha_1\rho_1 e_1) + (\alpha_2\rho_2 e_2)$; $E = e + \frac{1}{2}v^2$.

The relations between $(\partial\Delta T/\partial t)$ and $(\partial\Delta g/\partial t)$ with Q and $\dot{Y}$ can be found using the mass equations and the energy mixture equation. So, we can write that:

$$\begin{cases} \dfrac{\partial\Delta T}{\partial t} = AQ + B\dot{Y} \\ \dfrac{\partial\Delta g}{\partial t} = A'Q + B'\dot{Y} \end{cases} \tag{6}$$

Let n and $n+1$ denote two successive time steps. By supposing that thermodynamic equilibrium is reached at the end of each time step, we can write the simplest numerical approximation of (6) :

$$\frac{\partial\Delta T}{\partial t} = \frac{\Delta T^{n+1} - \Delta T^n}{\Delta t} = \frac{0 - \Delta T^n}{\Delta t} = A^n Q^n + B^n \dot{Y}^n$$

$$\frac{\partial\Delta g}{\partial t} = \frac{\Delta g^{n+1} - \Delta g^n}{\Delta t} = \frac{0 - \Delta g^n}{\Delta t} = A'^n Q^n + B'^n \dot{Y}^n.$$

Thus, finally, we can determine the heat and mass transfer terms that are given by:

$$\begin{cases} Q = -\dfrac{B'}{AB' - A'B}\dfrac{(\Delta T)^n}{\Delta t} + \dfrac{B}{AB' - A'B}\dfrac{(\Delta g)^n}{\Delta t} \\ \dot{Y} = \dfrac{A'}{AB' - A'B}\dfrac{(\Delta T)^n}{\Delta t} + \dfrac{A}{AB' - A'B}\dfrac{(\Delta g)^n}{\Delta t} \end{cases} \tag{7}$$

2.2.2 Mass fraction and density positivity

These approximation of heat and mass transfer terms allow the calculation of source terms of volume fraction and mass conservative equations, but there is no guarantee that positivity of the solution be preserved. This means that, for example, supposing an evaporation process, the mass source terms estimated can be bigger than the liquid mass

fraction that can evaporate. A limitation is placed on the source terms, determining the maximum admissible values:

$$S_{max,\alpha_1} = \begin{cases} \frac{1-\alpha_1}{\Delta t} & \text{if } S_{max} > 0 \\ \frac{-\alpha_1}{\Delta t} & \text{otherwise,} \end{cases} \qquad S_{max,Y_1} = \begin{cases} \frac{(1-\alpha_1)\rho_2}{\Delta t} & \text{if } S_{max} > 0 \\ \frac{-\alpha_1\rho_1}{\Delta t} & \text{otherwise.} \end{cases}$$

Thus, if $|S_{max,\alpha_1}| > |S_{\alpha_1}|$ and $|S_{max,Y_1}| > |S_{Y_1}|$, the source terms is used otherwise, we consider that the mixture is composed nearly of the species k that has the highest entropy. In particular:

- if $s_g > s_l$, then the vapor volume fraction is fixed to $1 - \epsilon_1$,

- otherwise, if $s_g < s_l$, then the vapor volume fraction is fixed to ϵ_1,

where, as explained in section 2.2, $\epsilon_1 = 10^{-8}$.

Knowing α_1, we can find the new equilibrium pressure and temperature, P* and T*, that allow to find all other thermodynamic variables, using the mixture density equation and the mixture energy:

$$\begin{cases} (\alpha_1\rho_1)^{**} + (\alpha_2\rho_2)^{**} = \rho^* \\ (\alpha_1\rho_1e_1)^{**} + (\alpha_2\rho_2e_2)^{**} = (\rho e)^* \\ \rho_k(P^*, T^*) = \dfrac{P*+P_{\infty,k}}{Cv_k\Gamma_kT*} \\ \rho_ke_k(P^*, T^*) = \dfrac{P*+P_{\infty,k}\gamma_k}{\Gamma_k} + \rho_kq \end{cases} \tag{8}$$

where $\Gamma = \gamma - 1$.

Replacing the density and energy in the first two equations of system 9, we obtain:

$$\begin{cases} T = \dfrac{\alpha_1Cv_2\Gamma_2(P^* + P_{\infty,1}) + \alpha_2Cv_1\Gamma_1(P^* + P_{\infty,2})}{\rho^*Cv_1\Gamma_1Cv_2\Gamma_2} \\ P^2 + bP + d = 0 \end{cases} \tag{9}$$

where:

$$b = \frac{F1}{F2} + \frac{q_1(\alpha_1Cv_2\Gamma_2) + q_2(\alpha_2Cv_1\Gamma_1)}{Z2} + F2\frac{P_{\infty,1}(\alpha_1Cv_2\Gamma_2) + P_{\infty,2}(\alpha_2Cv_1\Gamma_1)}{\rho^*\Gamma_1\Gamma_2}$$

$$d = \frac{F1}{Z2}\frac{P_{\infty,1}(\alpha_1Cv_2\Gamma_2) + P_{\infty,2}(\alpha_2Cv_1\Gamma_1)}{\rho^*\Gamma_1\Gamma_2} + \frac{P_{\infty,1}q_1(\alpha_1Cv_2\Gamma_2) + P_{\infty,2}q_2(\alpha_2Cv_1\Gamma_1)}{Z2}$$

$$F1 = P_{\infty,1}\gamma_1\frac{(\alpha_1Cv_2\Gamma_2)}{Cv_2} + P_{\infty,2}\gamma_2\frac{(\alpha_2Cv_1\Gamma_1)}{Cv_1} - (\rho e)^*\Gamma_1\Gamma_2$$

$$F2 = \alpha_1\Gamma_2 + \alpha_2\Gamma_1$$

$$Z2 = F2\frac{(\alpha_1 Cv_2\Gamma_2) + (\alpha_2 Cv_1\Gamma_1)}{\rho^*\Gamma_1\Gamma_2}.$$

So, solving the single quadratic equation of P^*, we select the physically admissible solution of the quadratic equation that maximizes the total entropy $s^* = Y_1^* s_1^* + Y_2^* s_2^*$.

Unlike other approaches in literature where a fractional time step is used for integration, instead, this procedure allows to solve the numerical integration with the hydrodynamic time step.

2.3 Thermodynamic closure

Concerning the thermodynamics closure, the EOS for the mixture can be easily obtained using the EOS of the single phase. This is not reported here for sake of brevity. For further details, please see [6].

3 RESULTS

All the numerical test-case have been performed by using Dodecane and Water. All the physical parameters are summarized in Tab. (2). Two classical test-cases are simulated for both showing the accuracy of the method and the possibility to take into account complex thermodynamics.

| FLUID	SG EOS					
	γ	P_∞ [Pa]	C_p [$\frac{J}{KgK}$]	C_v [$\frac{J}{KgK}$]	q	q'
Liq. Dodecane	2.35	4×10^8	2534	1077	-755×10^3	0
Vap. Dodecane	1.025	0	2005	1956	-237×10^3	-24.2×10^3
Liq. Water	2.35	10^9	4267	1816	-1167×10^3	0
Vap. Water	1.43	0	1487	1040	2030×10^3	-23×10^3

Table 2: EOS coefficients for liquid and gas phases.

3.1 TC1: Two-phase shock tube with mass transfer

The shock tube is filled out with liquid dodecane on the left and vapor dodecane on the right, but for numerical reasons, each chamber contains a weak volume of the other fluid ($\alpha_k = 10^{-8}$). The diaphragm is located at $x = 0.75$ m (the tube is 1 m long) and the results are shown at a time of $t = 473$ μs.

3.1.1 TC2: modified dodecane test-cases

Now, we assume to have the same liquid initial conditions of the previous test case. We vary the vapor initial conditions, and we consider two conditions (1) and (2) where (2) is closer to the saturation curve. Note that the results are all shown at a time $t = 473$ μs. For these cases, strong differences are observed when using SG or PRSV, not only in terms
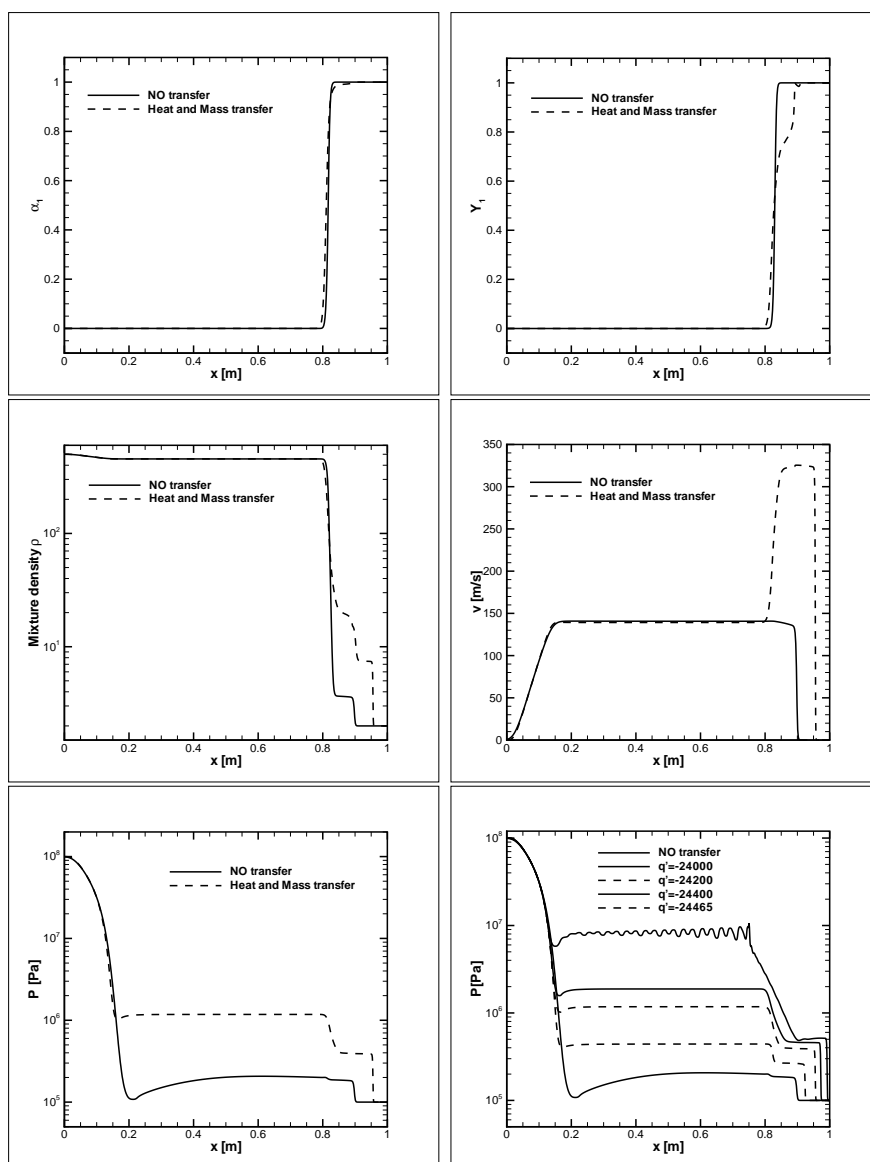
Figure 1: Vapor volume fraction, $Y = \alpha_1 \rho_1 / \rho$, mixture density, velocity, pressure and pressure in function of q' variation

of temperature, but also in term of density, pressure and velocity. Vapor volume fraction, vapor density, mixture density, velocity, pressure and gas temperature are reported in figures 2, 3, respectively.

First, note that the vapor volume fraction does not change significantly when we modify the EOS or the vapor initial conditions. Then, we focus on the SG solutions. Vapor density and mixture density vary according to the operating conditions, as expected. In particular, the vapor density in the state between the shock and the contact discontinuity

increases when approaching the saturation curve. The shock position changes according to the cases, *i.e.* getting closer to $x = 0.8$, when the vapor operating condition is nearer and nearer the saturation curve. This can be seen by looking at velocity and pressure. Mixture pressure is not very much influenced by the operating point. In particular, the mixture pressure after the passage of the compression shock is very similar: we have difference of about 3%. As expected, temperature changes according to the variation of the operating point. Generally, it lowers when vapor conditions is closer to the saturation curve.

As shown in the previous section, using SG or PRSV when the vapor conditions are very distant from the saturation curve, has a non-negligible impact only in terms of temperature. For estimating whether the SG solutions are predictive or not, a comparison with respect to a more precise EOS, such as for example PRSV, is done. We compute differences of using SG with respect to PRSV when approaching the saturation curve.

It is important to observe that getting closer to the saturation curve, *i.e.* from TC2(1) to TC2(2), the shock wave propagation velocity in PRSV EOS get lower, producing a change in the shock position.

Finally, the whole set of solutions highlights how SG EOS solutions are not reliable when considering conditions close to the saturation curve: more complex EOS, as PRSV, for describing the gas conditions, are needed.

4 CONCLUSIONS

In this paper, a semi-discrete scheme for the resolution of interface problems, taking into account complex EOS for evaluating real gas effects, is presented. A reduced five equation model, under the hypothesis of pressure and velocity equilibrium, is used and discretized through the Discrete Equations Method (DEM), considering mass and heat transfer. Then, the system is coupled with a mixture equation of state, that has been formulated for a general EOS, thus permitting real EOS-SG based mixture. In this work, SG has been systematically used for the liquid phase, while different EOS have been considered for the gas phase, i.e. SG, PRSV and SW. Finally, the proposed method has been applied to the simulation of two-phase flows of some configurations well-known in literature.

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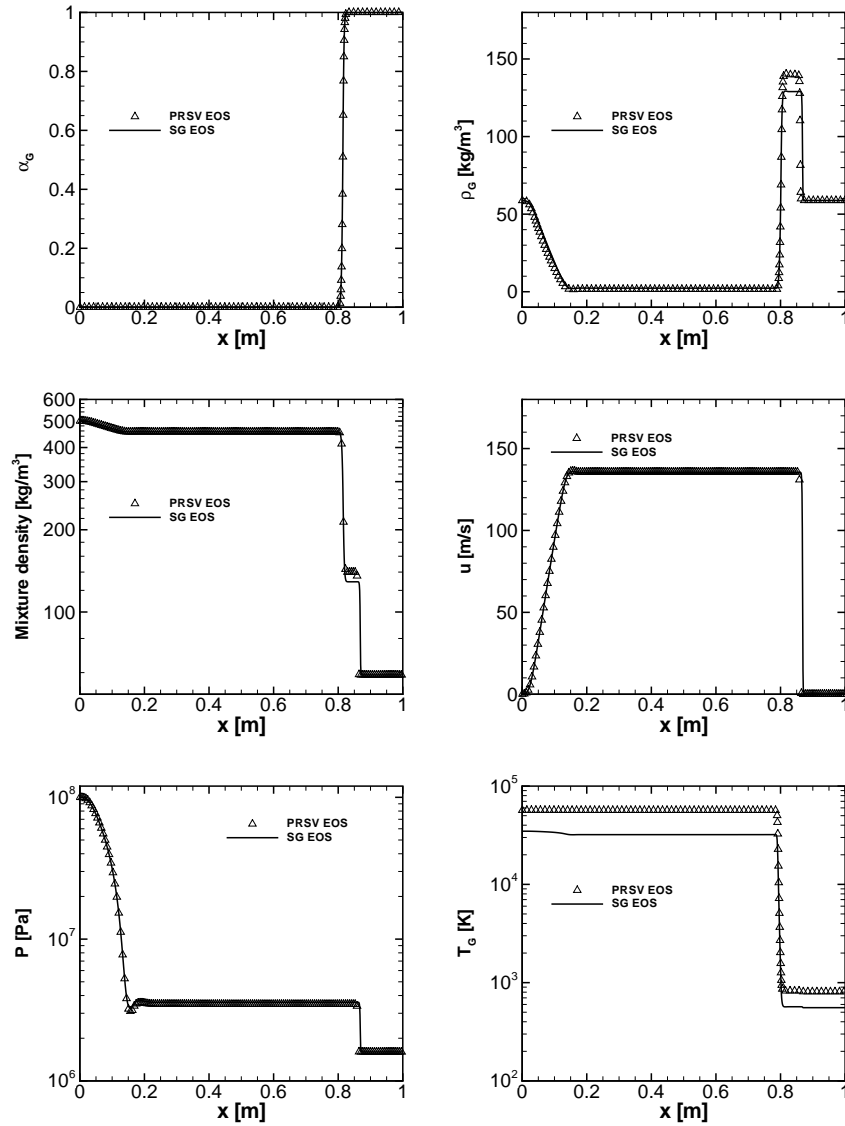
Figure 2: Vapor volume fraction, vapor density, mixture density, velocity, pressure and gas temperature profiles for TC2(1) test case.

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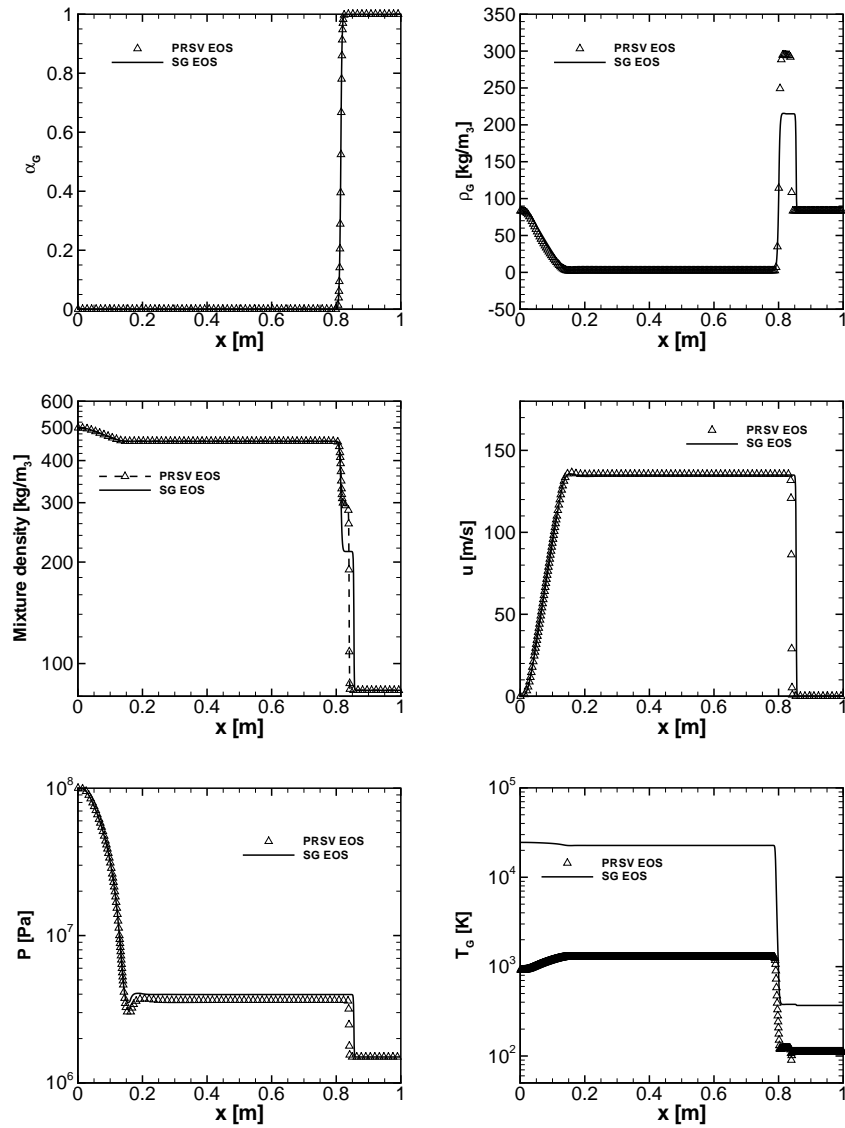
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