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MODELING OF MULTIPHASE FLOWS IN FINITE-DEFORMED POROUS MEDIA

Y.V. PEREPECHKO*, E.I. ROMENSKI† AND G.V. RESHETOVA‡

* Institute of Geology & Mineralogy (IGM SB RAS)
3 ave Koptyuga, 630090 Novosibirsk, Russia
e-mail: perep@sibmail.ru, www.igm.nsc.ru

† Institute of Mathematics (IM SB RAS)
4 ave Koptyuga, 630090 Novosibirsk, Russia
e-mail: evrom@math.nsc.ru, www.math.nsc.ru

† Institute of Computational Mathematics & Mathematical Geophysics (ICMMG SB RAS)
6 ave Lavrentyeva, 630090 Novosibirsk, Russia
e-mail: kgv@nmsf.sscc.ru, www.sscc.ru

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Abstract. A computational model for filtration of a mixture of compressible fluids with an arbitrary number of phases in a elastic porous medium is developed based on the thermodynamically compatible system of conservation laws theory. High accuracy numerical method based on the Runge-Kutta-WENO algorithm is developed and some two-dimensional test problems are solved.

1 INTRODUCTION

This paper is devoted to mathematical and numerical modeling of multiphase filtration in elastic porous media with finite deformation. The importance of the problem is caused by the necessity of intensifying oil extraction, since intensive development of existing deposits has led to depletion of oil stock, the major part of which has become hard to recover. The problem has appeared of creating new technology for increasing oil recovery from low permeability reservoirs as well as previously exploited heavily water-flooded reservoirs. Using methods such as forced pumping-out of oil by creating high pressure gradients in the area around the well leads to liberation of the gas phase and massive deformation of rock, even up to its disintegration. As a result, there appears a multiphase flow of oil, water and gases through deforming rock with a complex rheology. At the nonlinear stage of filtration the role of tangential stress becomes significant. When modeling such systems, the reliability of numerical modeling results should be provided for by both the methods of derivation of the model's consistent equations and the methods of their solution. In the current paper mathematical modeling of filtration of a mixed fluid with an arbitrary number of phases through a porous medium is carried out in the approach developed in [1]. The equations of motion obtained on the basis of the method of thermodynamically compatible systems of conservation laws satisfy the fundamental laws of thermodynamics: law of conservation of energy and the increase of entropy; they are hyperbolic which ensures the existence and uniqueness of the solution. An important property of the obtained equations is the possibility to present them in the divergent form. The hyperbolicity and divergent form of all equations of the model allows one to apply effective numerical methods developed for solving hyperbolic system of conservation laws. This approach has been employed successfully in modeling dynamics of two-phase compressible flows and filtration in a saturated elastic porous medium [2-4].

2 THERMODYNAMICALLY COMPATIBLE MODEL OF MULTIPHASE FILTRATION IN A DEFORMABLE POROUS MEDIUM

Consider a model for the flow of a mixture of compressible fluids in a deformable porous medium. It can be shown that such a model allows a representation in the terms of the generating thermodynamic potential. As a consequence, the model equations are of divergence form and may be reduced to a symmetric hyperbolic form, i.e., they are hyperbolic if the generating potential is convex. Let's describe an element of the poroelastic medium saturated with a mixture of compressible fluids (or gases) with the following parameters: Volume concentrations: α_1 - the volume concentration of the elastic skeleton (phase 1), α_n - volume concentrations of the fluids (phase n, $n = 2,...,N$, where $N-1$ - the number of fluids constituting the fluid mixture filtering through the porous medium); note that $\alpha_1 + \sum_{n=2}^{N} \alpha_n = 1$ and the concentration of the fluid mixture $\sum_{n=2}^{N} \alpha_n = 1 - \alpha_1$ is called the porosity of the elastic skeleton. Mass concentrations: c_1 - mass concentration of the elastic skeleton (phase 1), c_n - mass concentrations of the filtered fluids (phase n); here $c_1 + \sum_{n=2}^{N} c_n = 1$. Densities: ρ_n - mass density of phase n, $\rho = \sum_{n=1}^{N} \alpha_n \rho_n$ - mass density of an N-phase medium. Velocities: u_i^n - velocity of phase n, $n = 1,...,N$; $w_i^n = u_i^1 - u_i^n$ - relative velocity of fluid phase α with respect to the porous medium; u_i - velocity of the N-phase medium; $u_i = \sum_{n=1}^{N} c_n u_i^n$, where $c_n = \dfrac{\alpha_n \rho_n}{\rho}$. Denote the tensor of deformation gradient of the whole N-phase medium by F_{ij}. Denote the mass density of entropy of the whole N-phase medium by s. Hereinafter the index «n» denotes the phase number «n». For highlighting summands connected with the presence of liquid phases, the sum over phase number indices will be written out explicitly, for example $\sum_{n=2}^{N} \alpha_n$.

Let's determine the generalized internal energy of such a N-phase medium E:

$$E = E(\rho, \alpha_n, c_n, w_i^n, F_{ij}, s)\,,\ n = 2,...,N \tag{1}$$

The generalized internal energy depends upon the joint entropy s of the N-phase system by virtue of the assumption of phase equilibrium with respect to temperature.

Using the formalism suggested in [1] as a basis, the thermodynamically-consistent system of conservation laws for the flow of compressible fluid through a deformable matrix without taking dissipative processes into account may be presented in the following form:

$$\frac{\partial \rho}{\partial t} + \partial_k\left(\rho u_k\right) = 0 \tag{2}$$

$$\frac{\partial \rho c_n}{\partial t} + \partial_k\left(\rho c_n u_k - \rho E_{w_k^n}\right) = 0\,,\ n = 2,...,N \tag{3}$$

$$\frac{\partial \rho \alpha_n}{\partial t} + \partial_k\left(\rho \alpha_n u_k\right) = 0\,,\ n = 2,...,N \tag{4}$$

$$\frac{\partial w_k^n}{\partial t} + \partial_k\left(u_i w_i^n - E_{c_n}\right) = 0\,,\ n = 2,...,N \tag{5}$$

$$\frac{\partial \rho u_i}{\partial t} + \partial_k\left(\rho u_i u_k + \sum_{n=2}^{N} \rho w_i^n E_{w_k^n} + \rho^2 E_\rho \delta_{ik} - \rho F_{kl} E_{F_{il}}\right) = 0 \tag{6}$$

$$\frac{\partial \rho F_{ij}}{\partial t} + \partial_k\left(\rho F_{ij} u_k - \rho F_{kj} u_i\right) = 0 \tag{7}$$

$$\frac{\partial \rho s}{\partial t} + \partial_k\left(\rho s\, u_k\right) = 0 \tag{8}$$

The above equations represent conservation laws for total density of a mixture, mass and volume concentrations of liquid phases, relative velocities of liquid phases with respect to the porous matrix, total momentum, elastic deformation gradient and mass density of entropy. Solutions of this system satisfy additional consistency conditions

$$\partial_i w_k^n - \partial_k w_i^n = 0\,, \qquad \partial_k\left(\rho F_{kj}\right) = 0 \tag{9}$$

The energy conservation law of conservation of energy is also true for the system (2)-(8)

$$\frac{\partial}{\partial t}\left(\rho\left(E + \frac{1}{2}u_i u_i\right)\right) + \partial_k\left(\rho u_k\left(E + \frac{1}{2}u_i u_i + \frac{1}{\rho}p\right) + \sum_{n=2}^{N} \rho\left(u_i w_i^n - E_{c_n}\right)E_{w_k^n} - \rho u_i F_{kj} E_{F_{ij}}\right) \tag{10}$$

Here the pressure in the system is defined as: $p = \rho^2 E_\rho$.

Taking into account the dissipative interaction between phases in the form of relaxation of volume concentrations of phases towards an equilibrium value of phase pressures as well as interphase friction leads to the appearance of right-hand sides in the equations for α_n and for relative velocities w_k^n

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_k}{\partial x_k} = 0 \tag{11}$$

$$\frac{\partial \rho \alpha_n}{\partial t} + \partial_k \left(\rho \alpha_n u_k \right) = -\sum_{m=2}^{N} \lambda_{nm} \rho E_{\alpha_m} \ , \ \ n = 2,...,N \tag{12}$$

$$\frac{\partial \rho c_n}{\partial t} + \partial_k \left(\rho c_n u_k - \rho E_{w_k^n} \right) = 0 \ , \ \ n = 2,...,N \tag{13}$$

$$\frac{\partial w_k^n}{\partial t} + \partial_k \left(u_n w_n^n - E_{c_n} \right) = e_{knj} u_n \omega_j^n - \sum_{m=2}^{N} \chi_{nm} E_{w_k^m} \ , \ \ n = 2,...,N \tag{14}$$

$$\frac{\partial \rho u_i}{\partial t} + \partial_k \left(\rho u_i u_k + \sum_{n=2}^{N} \rho w_i^n E_{w_k^n} + \rho^2 E_\rho \delta_{ik} - \rho F_{kl} E_{F_{il}} \right) = 0 \tag{15}$$

$$\frac{\partial \rho F_{ij}}{\partial t} + \partial_k \left(\rho F_{ij} u_k - \rho F_{kj} u_i \right) = 0 \tag{16}$$

$$\frac{\partial \rho s}{\partial t} + \partial_k \left(\rho s \, u_k \right) = \frac{1}{T} R \tag{17}$$

The dissipative function $R \geq 0$ is of the form $R = \sum_{n,m=2}^{N} \chi_{nm} E_{w_k^n} E_{w_k^m} + \sum_{n,m=2}^{N} \lambda_{nm} E_{\alpha_m}^n$. The right-hand side in the equation for α_n provides for relaxation of phase pressures to an equilibrium value, and in the equation for w_k^n - the relaxation of phase velocities. The variables ω_j^n are connected to the curls of relative velocities $e_{ikj} \omega_j^n = \partial_i w_k^n - \partial_k w_i^n$, and are introduced to present the equations in divergence form (e_{ikj} - Levi-Civita symbol). They satisfy additional conservation laws

$$\frac{\partial \omega_k^n}{\partial t} + \partial_j \left(u_j \omega_k^n - u_k \omega_j^n + \sum_{m=2}^{N} e_{kjl} \chi_{nm} E_{w_l^m} \right) = 0 \tag{18}$$

which are consistency conditions for the system (11)-(17). This fact allows treating the summands $e_{klj} u_l \omega_j^n$ as source terms in the equations for w_k^n.

The form of the law of conservation of energy (10) does not change after the introduction of dissipative terms. To close the equations let's define generalized energy E as internal phase energy averaged over mass concentration, supplemented by the kinetic energy of relative phase movement. Let's define state equations for the elastic frame and liquid phases in the form of dependencies of internal energies on the parameters of phase states: $e^1 = e^1\left(\rho_1, F_{ij}, s\right)$, $e^n = e^n\left(\rho_n, s\right)$, ($n = 2,..,N$). We assume here, that the dependency e^1 on F_{ij} is given by means of a deviator of tensor g_{ij} (the Finger tensor $G = \{g_{ij}\} = f^T f$ [1], where $f = F^{-1}$).

Let's define the generalized internal energy of a three-phase medium by the relation

$$E\left(\rho, \alpha_n, c_n, w_k^n, F_{ij}, s\right) = e^1\left(\rho_1, F_{ij}, s\right) + \sum_{n=2}^{N} c_n \left(e^n\left(\rho_n, s\right) - e^1\left(\rho_1, F_{ij}, s\right)\right) + \tag{19}$$

$$+\frac{1}{2}\sum_{n=2}^{N}\left(1-c_n\right)c_n w_k^n w_k^n-\frac{1}{2}\sum_{\substack{n,m=2\\ n\neq m}}^{N}c_n c_m w_k^n w_k^m$$

A thermodynamic consequence of this definition of E

$$dE=\sum_{n=1}^{N}c_n e_{\rho_n}^n d\rho_n+c_1 e_{F_{ij}}^1 dF_{ij}+\sum_{n=2}^{N}(e^n-e^1)dc_n+\sum_{n=2}^{N}\left(\frac{1}{2}\left(1-2c_n\right)w_i^n w_i^n-\sum_{m=2}^{N}c_m w_k^n w_k^m\right)dc \tag{20}$$

$$+\sum_{n=2}^{N}\left(c_n w_i^n-\sum_{m=2}^{N}c_n c_m w_i^m\right)dw_i^n+\sum_{n=1}^{N}c_n e_s^n ds$$

and the use of the equalities

$$d\rho_n=\frac{c_n}{\alpha_n}d\rho+\frac{\rho}{\alpha_n}dc_n-\frac{\rho c_n}{\alpha_n^2}d\alpha_n,\; n=1,...,N \tag{21}$$

$$dE=E_\rho d\rho+\sum_{n=2}^{N}E_{\alpha_n}d\alpha_n+\sum_{n=2}^{N}E_{c_n}dc_n+E_{F_{ij}}dF_{ij}+\sum_{n=2}^{N}E_{w_i^n}dw_i^n+E_s ds$$

allows obtaining expressions for total energy derivatives through phase energy derivatives

$$E_\rho=\frac{1}{\rho^2}\sum_{n=1}^{N}\alpha_n\rho_n^2 e_{\rho_n}^n=\sum_{n=1}^{N}\alpha_n p_n=\frac{1}{\rho^2}p \tag{22}$$

$$E_{\alpha_n}=\frac{1}{\rho}\rho_1^2 e_{\rho_1}^1-\frac{1}{\rho}\rho_n^2 e_{\rho_n}^n=\frac{1}{\rho}\left(p_1-p_n\right),\; n=2,...,N \tag{23}$$

$$E_{c_n}=e^n-e^1+\frac{p_n}{\rho_n}-\frac{p_1}{\rho_1}+\frac{1}{2}\left(1-2c_n\right)w_i^n w_i^n-\sum_{m=2}^{N}c_m w_k^n w_k^m,\; n=2,...,N \tag{24}$$

$$E_{w_i^n}=c_n\left(w_i^n-\sum_{m=2}^{N}c_m w_i^m\right)=c_n\left(u_i-u_i^n\right),\; n=2,...,N \tag{25}$$

$$E_{F_{ik}}=c_1 e_{F_{ik}}^1 \tag{26}$$

$$E_s=\sum_{n=1}^{N}c_n e_s^n=T \tag{27}$$

A equation of state allows to compute phase pressures $p_n=\rho_n^2 e_{\rho_n}^n$ ($p=\sum_{n=1}^{N}\alpha_n p_n$), shear stresses $\sigma_{ik}=\rho_1 F_{kj}e_{F_{ij}}^1$, as well as the temperature T.

When we substitute the abovementioned expressions for derivatives of generalized energy with respect to mixture state parameters in the system (11)-(17), we obtain a system of equations for the movement of the fluid mixture through the elastic medium in terms of phase state parameters:

$$\frac{\partial\rho}{\partial t}+\frac{\partial\rho u_k}{\partial x_k}=0 \tag{28}$$

$$\frac{\partial \rho \alpha_n}{\partial t} + \partial_k \left(\rho \alpha_n u_k \right) = -\sum_{m=2}^{N} \lambda_{nm} \left(p_1 - p_m \right), \; n = 2,...,N \tag{29}$$

$$\frac{\partial \alpha_n \rho_n}{\partial t} + \partial_k \left(\alpha_n \rho_n u_k^n \right) = 0, \; n = 2,...,N \tag{30}$$

$$\frac{\partial w_k^n}{\partial t} + \partial_k \left(\frac{1}{2} u_i^1 u_i^1 - \frac{1}{2} u_i^n u_i^n + e^1 + \frac{p_1}{\rho_1} - e^n - \frac{p_n}{\rho_n} \right) = e_{kij} u_i \omega_j^n - \sum_{m=2}^{N} \chi_{nm} c_m \left(u_k - u_k^m \right) \tag{31}$$

$$\frac{\partial \left(\rho u_i \right)}{\partial t} + \partial_k \left(\sum_{n=1}^{N} \alpha_n \rho_n u_i^n u_k^n + \sum_{n=1}^{N} \alpha_n p_n - \alpha_1 \sigma_{ik} \right) = 0 \tag{32}$$

$$\frac{\partial \rho F_{ij}}{\partial t} + \partial_k \left(\rho F_{ij} u_k - \rho F_{kj} u_i \right) = 0 \tag{33}$$

$$\frac{\partial \rho s}{\partial t} + \partial_k \left(\rho s \, u_k \right) = \frac{1}{T} R \tag{34}$$

where $R = \sum_{n,m=2}^{N} \chi_{nm} \left(u_i - u_i^n \right) \left(u_i - u_i^m \right) + \frac{1}{\rho^2} \sum_{n,m=2}^{N} \lambda_{nm} \left(p_1 - p_n \right) \left(p_1 - p_m \right)$.

This system should be supplemented by consistency conditions in the form of conservation laws:

$$e_{ikj} \omega_j^n = \partial_i w_k^n - \partial_k w_i^n, \; \frac{\partial \omega_k^n}{\partial t} + \partial_j \left(u_j \omega_k^n - u_k \omega_j^n + \sum_{m=2}^{N} e_{kji} \chi_{nm} E_{w_i^m} \right) = 0, \; \partial_k \left(\rho F_{kj} \right) = 0 \tag{35}$$

The law of conservation of energy will have the form

$$\frac{\partial}{\partial t} \left(\rho \left(E + \frac{1}{2} u_i u_i \right) \right) + \partial_k \left(\sum_{n=1}^{N} \alpha_n \rho_n u_k^n \left(e^n + \frac{1}{2} u_i^n u_i^n + \frac{1}{\rho_n} p_n \right) - u_i \alpha_1 \sigma_{ik} \right) = 0 \tag{36}$$

The equations formulated above describe nonlinear filtration of a mixture of compressible fluids through a deformable porous medium in the finite deformation case.

3 WENO-RUNGE-KUTTA METHOD FOR EQUATIONS OF MULTIPHASE FILTRATION IN DEFORMABLE POROUS MEDIA

An efficient method for solving the obtained equations is the finite volume method using WENO reconstruction to achieve a high order of accuracy of space approximation [5] in conjunction with the high-order Runge-Kutta method [6]. Runge-Kutta-WENO method is well recommended one in various applied problems and, in particular has been successfully applied to analyzing acoustic wave fields in a cylindrically symmetrical model of a well in a porous fluid-saturated medium in the paper [7].

Differential equations of the model in a two-dimensional coordinate system (x, y) may be written out in the following vector form:

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} + \frac{\partial G(U)}{\partial y} = S(U) \tag{37}$$

Here $U=(U_1,U_2,...,U_n)^T$ - the vector of unknown functions (conservative variables), $F(U)=(F_1(U),F_2(U),...,F_n(U))^T$, $G(U)=(G_1(U),G_2(U),...,G_n(U))^T$ - flux vectors along axes x and y correspondingly, $S(U)=(S_1(U),S_2(U),...,S_n(U))^T$ - right-hand side vector. We present the Runge-Kutta-WENO method for solving the system (37) in a rectangular area $(x,y)\in[X_1,X_2]\times[Y_1,Y_2]$ with cells $I_{ij}=[x_{i-1/2},x_{i+1/2}]\times[y_{j-1/2},y_{j+1/2}]$, where $x_{i\pm1/2}=x_i\pm h_x$, $h_x=(X_2-X_1)/N_x$, $x_i=X_1+(i-1/2)h_x$, $y_{j\pm1/2}=y_j\pm h_y$, $h_y=(Y_2-Y_1)/N_y$, $y_i=Y_1+(j-1/2)h_y$. N_x,N_y are the numbers of steps of the computational mesh along the axes x,y correspondingly. The difference approximation of I_{ij} the equation (37) may be written out in the form:

$$\frac{dU_{ij}}{dt}+\frac{F_{i+1/2,j}-F_{i-1/2,j}}{h_x}+\frac{G_{i,j+1/2}-G_{i,j-1/2}}{h_y}=S_{ij} \tag{38}$$

where U_{ij} are the values in the cell I_{ij}, $F_{i\pm1/2,j},G_{i,j\pm1/2}$ are the flux values on the lateral faces of the parallelepiped $(t-t_0)\times I_{ij}$, S_{ij} are the right-hand side vector values in the cell I_{ij}.

Thus, on the condition that the values of fluxes on lateral faces of a cell are known, we obtain the following differential equation for solution values in a cell

$$\frac{dU}{dt}=L(U)=-\frac{F_{i+1/2,j}-F_{i-1/2,j}}{h_x}-\frac{G_{i,j+1/2}-G_{i,j-1/2}}{h_y}+S_{ij} \tag{39}$$

which is solved by the Runge-Kutta method of the required order of accuracy. The time step Δt is chosen in accordance with the formula $\Delta t=C_{CFL}\times\min\left(\frac{h_x}{S_{ij}^x},\frac{h_y}{S_{ij}^y}\right)$ and is calculated in each time moment (the minimum is taken over all mesh points, S_{ij}^x and S_{ij}^y are the velocities of the fastest sound waves in the cell ij, C_{CFL} is the Courant number). For stability the condition $0<C_{CFL}<1/2$ is required.

Consider equation (39) and denote with U^n the value of the function in the time moment t_n, U^{n+1} - in the time moment t_{n+1} and $\Delta t=t_{n+1}-t_n$. The formulae for the TVD Runge-Kutta method of the third order of accuracy are of the form [8]:

$$U^{(1)}=U^n+\Delta tL\left(U^n\right) \tag{40}$$

$$U^{(2)}=U^n+\frac{1}{4}\Delta tL\left(U^n\right)+\frac{1}{4}\Delta tL\left(U^{(1)}\right) \tag{41}$$

$$U^{n+1}=U^n+\frac{1}{6}\Delta tL\left(U^n\right)+\frac{1}{6}\Delta tL\left(U^{(1)}\right)+\frac{2}{3}\Delta tL\left(U^{(2)}\right) \tag{42}$$

To close the numerical algorithm we have to specify the values of fluxes $F_{i\pm1/2,j},G_{i,j\pm1/2}$ on the faces of the computational cells and an approximation method for the right-hand side vector S_{ij}. Right-hand sides modeling friction forces are calculated in a standard way as S_{ij} = $S\left(U_{ij}\right)$. Right-hand sides modeling phase pressure relaxation are approximated on the assumption of instantaneous pressure relaxation. The fluxes $F\left(U\right), G\left(U\right)$ are nonlinear functions of conservative variables U, therefore for their calculation it is sufficient to know the values of $U_{i\pm 1/2,j}, U_{i,j\pm 1/2}$ on the cell faces. To calculate fluxes on cell faces the WENO approximation of the solution by polynomials (reconstruction) is implemented using the solution values in neighbor cells. A decomposition with respect to space variables is employed, i.e. for calculating F and G one-dimensional systems are used which describe the propagation of waves along the axes x and y, correspondingly:

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} = 0, \qquad \frac{\partial U}{\partial t} + \frac{\partial G(U)}{\partial y} = 0 \tag{43}$$

The Lax–Friedrichs method is employed to calculate fluxes in equations (43). For the equation in the direction of the x axis, assuming that in the initial data the values U_L and U_R on the left and right of point $x_{i+1/2}$ are known, calculate the flux in this point by the formula

$$F_{i+1/2}^{LF} = \frac{1}{2}\left(F\left(U_L\right) + F\left(U_R\right)\right) - \frac{1}{2}\frac{\Delta x}{\Delta t}\left(U_R - U_L\right) \tag{44}$$

In exactly the same way in the direction of the y axis, assuming that in the initial data the values of U_L and U_R on the left and right of the point $y_{j+1/2}$ calculate the flux in this point by the formula

$$G_{j+1/2}^{LF} = \frac{1}{2}\left(G\left(U_L\right) + G\left(U_R\right)\right) - \frac{1}{2}\frac{\Delta y}{\Delta t}\left(U_R - U_L\right) \tag{45}$$

To calculate the values U_L and U_R on the left and right of the cell border $x_{i+1/2}$ an algorithm of WENO-polynomial reconstruction is implemented, allowing the required order of approximation with respect to space. We present the formulae for the WENO-polynomial reconstruction of variables U_L and U_R for one scalar variable (denoted by w). All the other variables are processed in similar fashion.

We present an example of WENO reconstruction providing for the fifth order of accuracy. Inside the computational cell $I_i = [x_{i-1/2}, x_{i+1/2}]$ we define "right" and "left" values, which should be calculated with the use of reconstruction $w_{i-1/2}^R = w(x_{i-1/2} + 0)$, $w_{i+1/2}^L = w(x_{i+1/2} - 0)$:

$$w_{i-1/2}^R = \frac{\omega_0}{6}\left(11w_i - 7w_{i+1} + 2w_{i+2}\right) + \frac{\omega_1}{6}\left(2w_{i-1} + 5w_i - w_{i+1}\right) + \frac{\omega_2}{6}\left(2w_i + 5w_{i-1} - w_{i-2}\right) \tag{46}$$

$$w_{i+1/2}^L = \frac{\omega_0}{6}\left(2w_i + 5w_{i+1} - w_{i+2}\right) + \frac{\omega_1}{6}\left(-w_{i-1} + 5w_i + 2w_{i+1}\right) + \frac{\omega_2}{6}\left(2w_{i-2} - 7w_{i-1} + 11w_i \right. \tag{47}$$

Nonlinear weight coefficients are defined by the relations

$$\omega_k = \frac{\alpha_k}{\alpha_0 + \alpha_1 + \alpha_2}, \qquad \alpha_k = \frac{d_k}{(\varepsilon + \beta_k)^2}, \; k = 0,1,2. \tag{48}$$

with a smoothness indicator

$$\beta_0 = \frac{13}{12}(w_i - 2w_{i+1} + w_{i+2})^2 + \frac{1}{4}(3w_i - 4w_{i+1} + w_{i+2})^2 \tag{49}$$

$$\beta_1 = \frac{13}{12}(w_{i-1} - 2w_i + w_{i+1})^2 + \frac{1}{4}(w_{i-1} - w_{i+1})^2 \tag{50}$$

$$\beta_2 = \frac{13}{12}(w_{i-2} - 2w_{i-1} + w_i)^2 + \frac{1}{4}(w_{i-2} - 4w_{i-1} + 3w_i)^2 \tag{51}$$

The constant ε is introduced to avoid division by zero when $\beta_k = 0$ and is usually set to $\varepsilon = 10^{-6}$. Optimal weight coefficients d_0, d_1, d_2 for “right” values: $d_0 = 1/10$, $d_1 = 3/5$, $d_2 = 3/10$, for left values: $d_0 = 3/10$, $d_1 = 3/5$, $d_2 = 1/10$.

Thus, we get two reconstructed functions $w^L_{i+1/2}$ and $w^R_{i+1/2}$ correspondingly on the left and right on each face of a cell in each internal cell of the computation area. Performing reconstruction for all variables we obtain the values U_L and U_R on the left and right of the cell borders which are then used to calculate flows by the Lax–Friedrichs formula.

The approximation of equations for volume concentrations of phases is obtained under the assumption of instantaneous phase pressure relaxation. In the case when the characteristic time of phase pressure relaxation towards an equilibrium value common for all phases is much smaller than the time intervals we consider here, it’s prudent to use the correction procedure for volume concentrations of phases under the assumption that pressure relaxation commences instantly. In real cases phase pressures are equalized by means of propagation and interaction of pressure waves in phases. Therefore, when characteristic sizes of irregularities are small compared to wavelengths of sound waves, applying the procedure of instant pressure relaxation is justified.

4 MODELING RESULTS

In this paper numerical computations were carried out for nonlinear non-isothermal filtration of a two-phase water-oil mixture through a deformable porous medium. The propagation of nonlinear acoustic oscillations near the boundary of the water-oil mixture and the fluid-saturated porous formation was examined on a model, the geometrical parameters of which were $5\,m$ by $4\,m$. The boundary of the area of water-oil mixture was located at a distance of $0.5\,m$ to the right of the lateral sides of the area. Initial velocity values were set to zero – the hydrodynamic flow was absent. Initial values of thermodynamic parameters corresponded to standard conditions. The values of the phases’ physical parameters were set in the model to the following: densities of formation $\rho_{10} = 2.5 \cdot 10^3\, kg/m^3$, water $\rho_{20} = 1.0 \cdot 10^3\, kg/m^3$, oil $\rho_{30} = 0.9 \cdot 10^3\, kg/m^3$; volume content of phases in the formation of water $\alpha_{20} = 0.1$, oil $\alpha_{30} = 0.1$; porosity $\alpha_{10} = 0.8$; shear modulus of the solid phase

$\mu = 8.9 \cdot 10^9\ Pa$; coefficients of volumetric expansion $K_1 = 3.7 \cdot 10^{10}\ kg/m\,s^2$, $K_2 = 2.25 \cdot 10^9\ kg/m\,s^2$, $K_3 = 1.3 \cdot 10^9\ kg/m\,s^2$.; coefficients of thermal expansion $\beta_1 = 1.2 \cdot 10^{-5} K^{-1}$, $\beta_2 = 1.1 \cdot 10^{-4} K^{-1}$, $\beta_3 = 9.2 \cdot 10^{-4} K^{-1}$.; thermal capacities $c_{p1} = 0.96 \cdot 10^3\ J/kg\,K$, $c_{p2} = 4.2 \cdot 10^3\ J/kg\,K$, $c_{p3} = 0.88 \cdot 10^3\ J/kg\,K$; kinetic coefficients λ_{ij} defining pressure relaxation in phases $\lambda_{11} = 0.11 \cdot 10^{-6}\ s/m^2$, $\lambda_{22} = 0.69 \cdot 10^{-6}\ s/m^2$; coefficients of interphase friction χ_{ik} are defined by Darcy's formulae, the dynamic viscosities and relative permeability of the phases being $\eta_2 = 0.93 \cdot 10^{-3}$ Pa s, $\eta_3 = 2.0 \cdot 10^{-3}$ Pa s and $k_{22} = 0.2 \cdot 10^{-12}\ \text{m}^2$, $k_{33} = 0.2 \cdot 10^{-12}\ \text{m}^2$.

In Figure 1,2 the fields of the following physical parameters are presented: the pressure in water/oil mixture and the xx-component of stress tensor of porous matrix for three moments of oscillation propagation from a Ricker type source: 0.0001, 0.0004 and 0.0012 s.

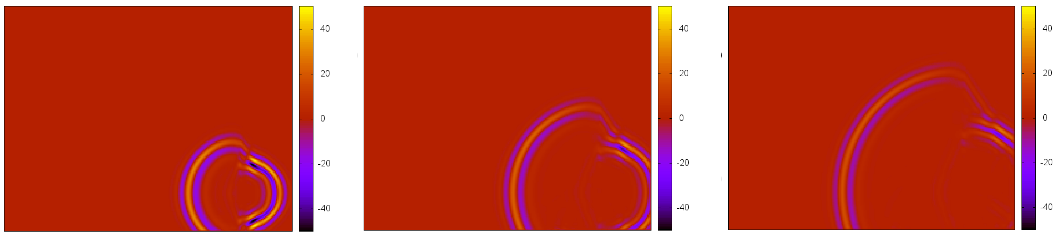

Figure 1: The pressure in water/oil mixture. Time =0.0001, 0.0006, 0.0012 s.

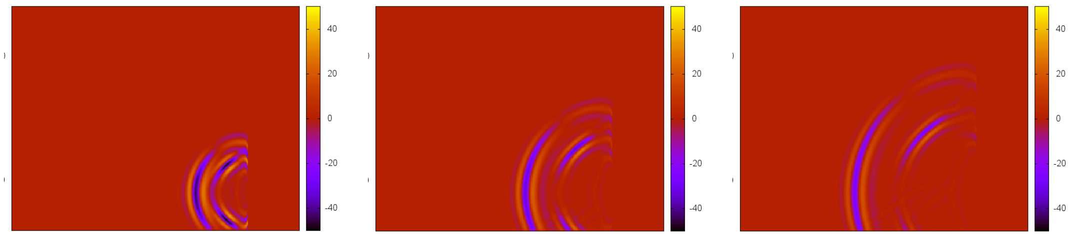

Figure 2: The stress field in porous matrix. Time =0.0001, 0.0006, 0.0012 s.

The problem of water-oil mixture filtration in the space beyond the well when the mixture is injected into the well was considered in the following statement. The geometrical parameters of the computation area were $500\,m$ by $500\,m$. On well is located in the center of the area, if the geometry with several wells was given, they were distributed evenly over the computation area. The physical parameter values were chosen the same way as in the previous problem. In Figures 3,4 we present patterns, illustrating the propagation of waves in the well system for the geometry with a single well and a Ricker type source. The effect of an impulse excitation of great magnitude on the character of multiphase filtration is demonstrated. On Figure 2 the results are presented for water injection into a model well in the absence of acoustic influence. A regular flow may be observed with water-oil displacement. Introducing a pressure source of great intensity on the hydrodynamic background of the water-oil mixture's filtration leads to a qualitative change in the filtration mode (see Figure 3).

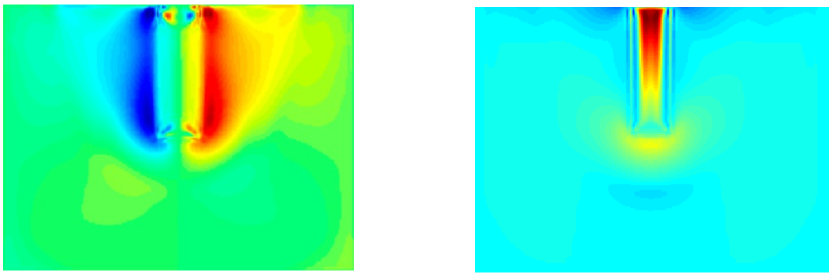

Figure 3: Flow velocity fields of water (horizontal (left) and vertical (right) components). A pressure source is absent.

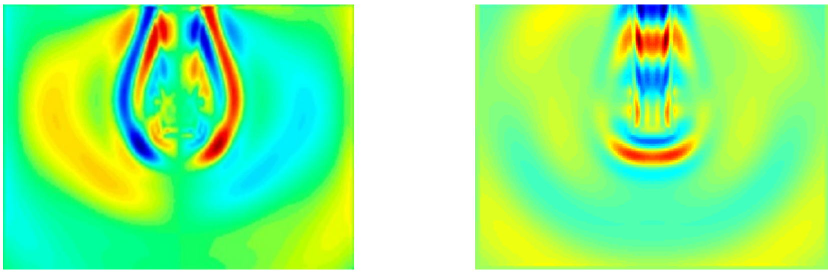

Figure 4: Flow velocity fields of water (horizontal (left) and vertical (right) components). A pressure source of the Ricker type is located in the center of the lower part of the well.

Thus, as a result of the calculations of test problems of multiphase filtration in deformable porous media, we demonstrate the possibility of modeling the propagation of nonlinear acoustic oscillations on the boundary of the water-oil mixture and porous formation saturated with the water-oil mixture as well as the problems of filtration of water/water-oil mixture in the formation, including the research of its influence upon the propagation of waves in a multiphase medium. Testing of the two-dimensional numerical method has shown its stability and applicability to complex wave problems of porous media acoustics.

5 CONCLUSIONS

- A mathematical model of multiphase filtration with an arbitrary number of phases in elastic porous media is developed. The employed finite-difference high precision algorithm based on the WENO-Runge-Kutta method allows modeling of processes of multiphase filtration in deformable porous media in oil-bearing beds in a broad spectrum of model parameters.
- The model allows studying the propagation of seismoacoustic waves in well systems accompanied by dissipative effects and temperature variations; the influence of acoustic excitation upon the flow of fluids (water, oil, their mixture) in such media; nonstationary, non-isothermal multiphase filtration of mixtures of compressible fluids in porous media under the conditions of large elastoplastic deformations for problems of developing oil and gas-condensate fields.

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