

# CONCURRENT TOLERANCE ALLOCATION IN MECHANICAL ASSEMBLIES BY DESIGN UNDER UNCERTAINTY METHODS

VICTOR E. RUIZ\*

\*†District University FJC(universidad Distrital Francisco José de Caldas),  
Grupo de investigación DisIng,  
Tv 70B73A35S Bogotá Colombia,  
veruizr@udistrital.edu.co

**Key Words:** *Tolerance allocation, Uncertainty, Aleatory and epistemic uncertainties.*

## 1 INTRODUCTION

The current conditions of the products development have advanced towards the integration and the interaction of multidisciplinary teams that contribute in each of the stages of the product life cycle, in such a way that they could reduce significantly the times for the placing on market, more innovations join and more complex problems are solved in a reduced time. [1] [2].

Equally the use of technologies that improve and promote a more adjusted way for the the different stages of the design process supported in the recent information and communication technologies, has allowed to advance in the multidisciplinary character of the product design and generated therefore appropriate interaction of the teams that intervene inside the life cycle, [2], all that to answer to the needs of a market with increased competitiveness that required shorter life cycles, increasing innovations, reduce manufacturing costs and obtain a high level of quality and satisfaction of the client. [1], [2]

In this context the dimensioning and the tolerances of the specification of the design play a fundamental role in obtaining a product of easy manufacture, repeatable in different productive environments and with the minor waste of resources of raw material and production times, avoiding the functionality conditions get lost, defining this both paradigms established inside the assignment of tolerances: minimal cost of manufacture and high quality dresses from the fulfilment of the function specified in the design. So that if very narrow margins of tolerance are assigned can cause an excess in the production costs, whereas very wide limits of tolerances increase the waste of raw material and cause problems of assembly and maintenance. [3 [4]]

In traditional design environments the process of tolerances allocation has the first stage, developed with the geometries definition of the pieces as well as the movement functions that establish the assemble. From this definition there is assigned the set of design tolerances, which guarantees the appropriate functioning of the parts and assemble, the interaction and interchange.

On the other hand the processes of manufacture reach in the generation of manufacture errors with regard to the dimensions and geometric nominal restrictions, the sources of variability can associate to multiple conditions, like precision of the manufacture, the montage of the piece in the machine, among others [5], [6], [7]. Bear all the previous aspects in mind together with the information of the set of design tolerances of, it conduces to determine the manufacture tolerances, as a second o complementary information set related with the manufacture and its associate cost.

Inside a concurrent design environment, where there is a permanent interaction of the specialized teams of every stage of the product life cycle, is therefore consistent with this approach, to obtain a only one set of design and manufacture tolerances that contributes in a suitable functional performance and in a guessed right selection of the systems of manufacture, the capacity of the machinery, the quality of the raw material(commodity), etc. [8]

The tolerances can relate directly to the uncertainty associated with the process of design and manufacture, taking into account that represent precisely the dimensional and geometric variations that can suffer the parts and assemble them associated to the decisions in the design and the manufacture. The way of representing and quantifying the uncertainty and to relate it to the sets of tolerances in assemblies is still a topic without a definitive definition and is affected by the discussion on the most appropriate way of classifying and representing the uncertainty [9], however the most common classification identifies two types, irreducible, aleatory or objective uncertainties and reducible epistemic or subjective uncertainties. The manufacture it is considered associated to the irreducible uncertainties [9] and are considered to be random, [10], and forms of representation are looked from the available information as the distributions of probability or random numbers, on the other hand the variations related to the design associate to lack of knowledge, inaccuracy of the models used for the calculation, and even to the precision used for the calculation of the dimensions, this uncertainty is considered epistemic and it is possible to reduce it from the increase of information. [9], [10], [11], nevertheless its representation still is in discussion. In the developed work there was in use the theory of the evidence [10], [11] for establishing the representation of the uncertainty so much random as epistemic.

A competing analysis of tolerances / uncertainties implies the mixing or simultaneous random consideration of the uncertainties and epistemic, which has not been completely studied, focus on the effects of the random uncertainty. [9].

The frame of work led to the approach into a strategy of competing assignment of based tolerances on uncertainties that took an initial model of multidisciplinary optimization in order to achieve an appropriate set of tolerances.

## **2 TOLERANCES / UNCERTAINTIES REPRESENTATION AND QUANTIFICATION**

### **2.1 Identification and classification of uncertainty sources**

To represent appropriately the associate uncertainties there were evaluated all the possible sources of variability assigned to one mechanic assembly and they qualified in agreement with the phase of the life cycle of design (design or manufacture) and later are classified by

their behavior (random or epistemic). With regard to the associated variability with the functionality (design) they were considered: precision of the models for dimensional quantification, conceptual development for the formal definition of the pieces, precision of the means used for the calculation (software), criteria of assignment of tolerances among assembled parts, and criteria of assemblies.

The sources related to the manufacture, among others were considered: the precision of the system of measurement and control of the manufacture, the assembly (montage), the thermo-mechanics conditions of piecework and the tools, the configuration of the machine, skills of the operator, development of the programming NC, suitable selection of the manufacture process, quantity of units produced by manufacture unit time.

Done one coincides the analysis with [9] as for which the majority of sources associated with the manufacture are irreducible, since in spite of the improvement of the conditions of the manufacture it is not possible to reach with absolute precision to the nominal value of a dimension, therefore they behave as random uncertainties. With regard to the related sources to the functionality, it is possible to find that the changes related to the capture of decisions and to the used means they can eliminate the variability of assembly, by what they can be considered to be reducible and therefore they behave like epistemic uncertainties.

For the random uncertainties one worked associating distributions of probability with every source [12] and the principal challenge consisted of representing properly the epistemic uncertainty in order were able to manage to use models who lead her to quantifying. Analyzing some methods of quantification raised it was decided to work with based models theoretically of the evidence. [10], [13]

## 2.2 Tolerances and uncertainty representation

In an assembly the dimensions are defined from the parts dimensions, so variations in the assembly depend on the parts variations, this relationship is defined in equation 1

$$Y = f(X) \quad (1)$$

Where Y=set of assembly dimensions and X=set of manufacturing dimensions

Functions that define the relationship between levels of assembly can be very complex however there are different representations, where the use of vector loops is one of the simplest ways to represent assembly variations from dimensional variations of pieces.

So assembly variations are defined from the first term of an expansion in Taylor series[15] according to equation 2

$$\Delta Y = \sum_{i=1}^n \frac{\partial f}{\partial x_i} \Delta x_i \quad (1)$$

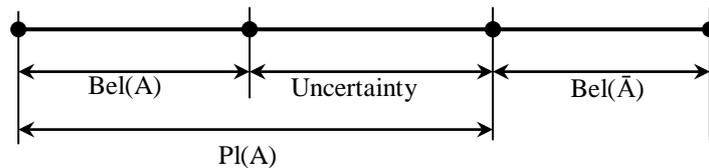
Where  $\Delta Y$  and  $\Delta x$  express the tolerances of dimensions by assembly and parts respectively.

The aleatory uncertainties representation was done from distributions of probability that were adjusting to an approximate behavior of the source of variation. For the present work was used normal distribution.

For the representation of the epistemic uncertainties there was used the theory of the evidence or Demspster-Schafer theory, which quantifies the uncertainty from two measures, belief (*Bel*) and plausibility (*Pl*) [10], [14]. These are defined from the known evidence without it is distributed, this is that the evidence can appear as a value inside an interval that experimental numerical parameters or from experts' judgments [10], on the other hand it is important to specify that given the existence of the uncertainty, the sum of the measure of evidential of an event and its complement is different to the unit (1), this is in agreement to the equation (1) a graphical way of representing is presented in the figure 1. [10]

$$Bel(A) + Bel(\bar{A}) \neq 1 \quad (1)$$

One of the parameters to measure the evidence on the most basic is the BBA or parameter of basic assignment of probability and corresponds(fits) when a mapped realizes m about an event To and the possible subsets of belief presents related to the lack(mistake) of knowledge inside a universal set named frame of discernment [14], the formulation for the BBA follows in agreement with the equation (2).



**Figure 1:** Plausibility (Pl) and Belief (BI) Representation. [10]

$$m(A) = \frac{\sum_{B \cap C = A} m_1(B)m_2(C)}{1 - \sum_{B \cap C = \emptyset} m_1(B)m_2(C)}, A \neq \emptyset \quad (2)$$

### 3 MODEL OF TOLERANCE ALLOCATION BASED ON UNCERTAINTIES

Once defined the uncertainties inside the space of design a method appears to calculate the uncertainties propagation and to realize the process of tolerance allocation. The challenge in this stage is to use aleatory and epistemic uncertainties at the same level to include in a simultaneous uncertainties propagation. For the present work a model appeared probabilistic for the managing of the random uncertainties and an algorithm of multidisciplinary optimization for the epistemic uncertainties in order to minimize the quality loss or in other words to reduce the quantity of rejected parts for not expiring functionally. Development strategy was based on the problem [15], in figure 2 solution methodology is presented.

The optimization model used is based on robust optimization, looking for verify system sensitivity to uncertainty and minimizing the loss of the quality function.

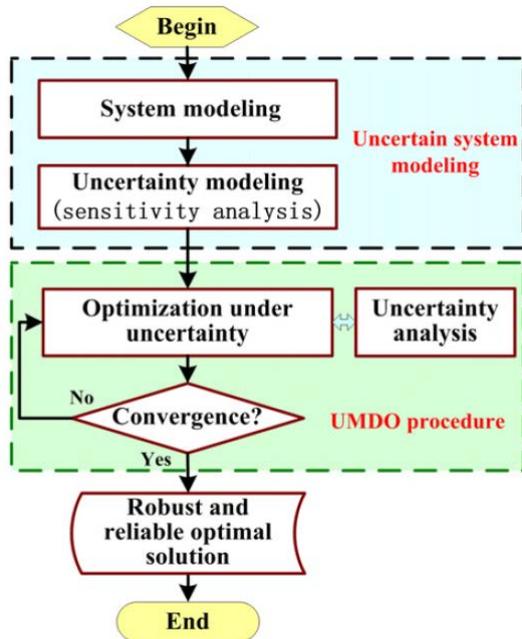


Figure 2: Structure of the model of tolerance allocation . [15]

#### 4 MODEL APPLICATION

The strategy was used in a five parts assembly as shown in figure 3, with different relations among parts. There was just one assembly dimension to be controlled from the parts dimensions, the variations for the parts include the epistemic uncertainties expressed by experts intervals, for the aleatory uncertainties it was assumed a normally distributed performance associated to the manufacturing.

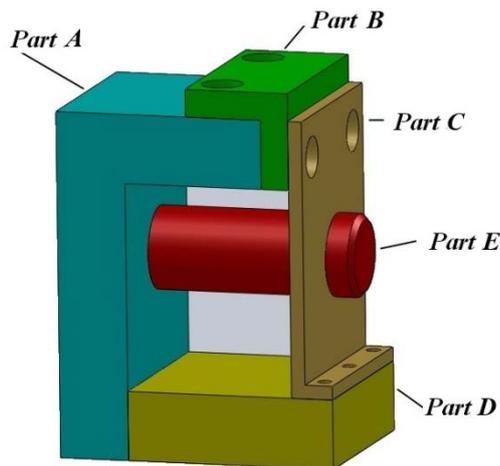
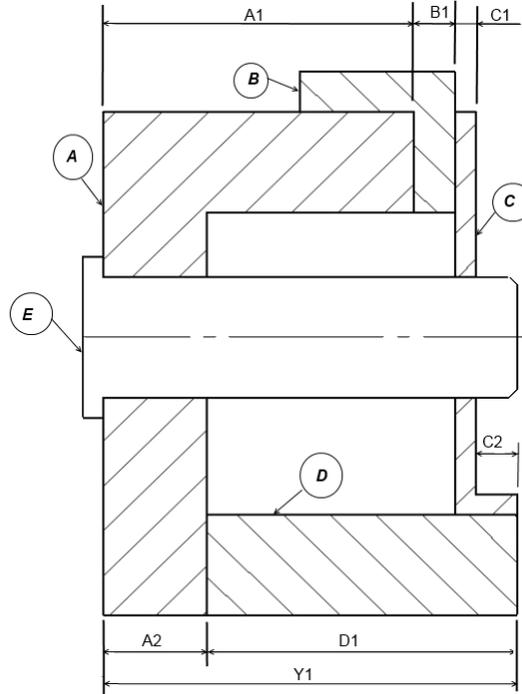


Figure 3: Assembly

The assembly dimension Y1 was assumed along the coincident axe with part E, for the model the geometrical tolerances were not considered, just dimensional tolerances. In figure 4 is presented the considered dimensions.



The obtained results in Table 1 are presented in a lower ( $x_l$ ) and upper ( $x_u$ ) limits for each dimension, a stack up analysis for y1 based on RSS method was implemented in order to establish the influence of each part dimension in the assembly.

**Table 1:** Results

Dimension	Nominal Value mm	$x_l$ mm	$x_u$ mm
A1	95	0,2134	0,5125
A2	50	0,0922	0,2138
B1	20	0,1481	0,4123
C1	10	0,2217	0,1754
C2	20	0,1954	0,0532
D1	150	0,0956	0,3327
Y1	200	RSS Min	RSS Max
		0,1041	0,0974

## 5 CONCLUSIONS

- To relate the dimensional tolerances to the uncertainties establishes a method that allows to unify the variations associated with the manufacture (aleatory) with the

- function design (epistemic).
- There tried to be in use a model who propagates simultaneously the aleatory and the epistemic uncertainties.
- The results showed one suitable behavior of the assembly variables and comparably with traditional methods of tolerances allocation.
- Even they exist bounding as for the high computational cost of the solutions of this type of problems. Some alternative methods for the solution of multidisciplinary optimization problems can be checked to adjust to the conditions for tolerance allocation in assemblies with multiple parts and dimensions.

## REFERENCES

- [1] A. N. Bramley, Ed., *Advances in integrated design and manufacturing in mechanical engineering*. Dordrecht: Springer, 2005.
- [2] A. K. Kamrani and E. A. Nasr, *Collaborative engineering: theory and practice*. New York ; London: Springer, 2008.
- [3] K. W. Chase and W. H. Greenwood, "Design issues in mechanical tolerance analysis," *Manuf. Rev.*, vol. 1, no. 1, pp. 50–59, 1988.
- [4] H. P. Peng, X. Q. Jiang, and X. J. Liu, "Concurrent optimal allocation of design and process tolerances for mechanical assemblies with interrelated dimension chains," *Int. J. Prod. Res.*, vol. 46, no. 24, pp. 6963–6979, Dec. 2008.
- [5] J. J. Shah, Y. Yan, and B. C. Zhang, "Dimension and tolerance modeling and transformations in feature based design and manufacturing," *J. Intell. Manuf.*, vol. 9, no. 5, pp. 475–488, 1998.
- [6] A. Skander, L. Roucoules, and J. S. Klein Meyer, "Design and manufacturing interface modelling for manufacturing processes selection and knowledge synthesis in design," *Int. J. Adv. Manuf. Technol.*, vol. 37, no. 5–6, pp. 443–454, Jun. 2007.
- [7] M. S. Kumar and S. Kannan, "Optimum manufacturing tolerance to selective assembly technique for different assembly specifications by using genetic algorithm," *Int. J. Adv. Manuf. Technol.*, vol. 32, no. 5–6, pp. 591–598, Apr. 2006.
- [8] M. Siva Kumar and B. Stalin, "Optimum tolerance synthesis for complex assembly with alternative process selection using Lagrange multiplier method," *Int. J. Adv. Manuf. Technol.*, vol. 44, no. 3–4, pp. 405–411, Jan. 2009.
- [9] J. Y. Dantan, N. Gayton, A. J. Qureshi, M. Lemaire, and A. Etienne, "Tolerance Analysis Approach based on the Classification of Uncertainty (Aleatory/Epistemic)," *Procedia CIRP*, vol. 10, pp. 287–293, Jan. 2013.
- [10] H. Agarwal, J. E. Renaud, E. L. Preston, and D. Padmanabhan, "Uncertainty quantification using evidence theory in multidisciplinary design optimization," *Reliab. Eng. Syst. Saf.*, vol. 85, no. 1, pp. 281–294, 2004.
- [11] W. L. Oberkampf, S. M. DeLand, B. M. Rutherford, K. V. Diegert, and K. F. Alvin, "Error and uncertainty in modeling and simulation," *Reliab. Eng. Syst. Saf.*, vol. 75, no. 3, pp. 333–357, 2002.
- [12] S. H. Lee and W. Chen, "A comparative study of uncertainty propagation methods for

- black-box-type problems,” *Struct. Multidiscip. Optim.*, vol. 37, no. 3, pp. 239–253, May 2008.
- [13] P. Limbourg and E. De Rocquigny, “Uncertainty analysis using evidence theory-confronting level-1 and level-2 approaches with data availability and computational constraints,” *Reliab. Eng. Syst. Saf.*, vol. 95, no. 5, pp. 550–564, 2010.
- [14] V. Ruiz, “Index Numbers Calculation to Determine the Dimensional Tolerance Sensitivity in Linked Assemblies,” *Tecnura*, vol. 11, no. 22, pp. 53–62, 2008.
- [15] K. W. Chase, “BASIC TOOLS FOR TOLERANCE ANALYSIS OF MECHANICAL ASSEMBLIES,” in *Manufacturing Engineering Handbook*, McGraw-Hill, 2004, p. 13.
- [16] H.-R. Bae, R. V. Grandhi, and R. A. Canfield, “An approximation approach for uncertainty quantification using evidence theory,” *Reliab. Eng. Syst. Saf.*, vol. 86, no. 3, pp. 215–225, Dec. 2004.
- [17] W. Yao, X. Chen, W. Luo, M. van Tooren, and J. Guo, “Review of uncertainty-based multidisciplinary design optimization methods for aerospace vehicles,” *Prog. Aerosp. Sci.*, 2011.