11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
E. Oñate, J. Oliver and A. Huerta (Eds)

A MODEL FOR NON-ISOTHERMAL VARIABLY SATURATED POROUS MEDIA IN DYNAMICS

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Key Words: *Unsaturated porous materials, Hydro-Thermo-Mechanical processes, Finite element method, Dynamics.*

Abstract. This work presents the development of a fully coupled mathematical and numerical model for the analysis of the thermo-hydro-mechanical behaviour of non-isothermal multiphase porous materials in dynamics. The model is developed following Lewis and Schrefler within the Hybrid Mixture theory [1]. The porous medium is treated as a multiphase system composed of a solid skeleton with open pores, filled with liquid water and gas. The solid is considered as deformable and non-polar. All the fluids are in contact with the solid phase. The constituents are assumed to be isotropic, homogeneous, immiscible, except for dry air and water vapour and chemically non-reacting. Local thermal equilibrium between the solid matrix, gas and liquid phases is assumed. Heat conduction, vapour diffusion, heat convection, liquid water flow due to pressure gradients or capillary effects and water phase change (evaporation and condensation) inside pores are all taken into account. In the partially saturated zones, liquid water is separated from its vapour by a meniscus concave toward gas (capillary water). In order to analyse the thermo-hydro-mechanical behaviour of a soil structure in the low frequency domain, e.g. under earthquake excitation, the u-p formulation is advocated for the finite element discretization.

1 INTRODUCTION

The analysis of the dynamic response of multiphase porous media has many applications in civil engineering. Onset of landslides due to earthquakes or rainfall and the seismic behaviour of dams are examples where inertial forces cannot be neglected. Moreover, there are situations where it is important to consider also the effect of temperature variation. It is the case of catastrophic landslides, where the mechanical energy dissipated in heat inside the slip zone may lead to vaporization of the pore water creating a cushion of zero friction, which may accelerate the movement of the landslides [3]. Another interesting case is the seismic analysis of deep nuclear waste disposal.

Many authors have developed models for the analysis of the dynamic behaviour of multiphase porous media in isothermal conditions. A state of art can be found in Zienkiewicz et al. [2] and Schanz [4]. Recently, Nenning and Schanz [5] presented an infinite element for wave propagation problems; Heider et al. [6] analyzed a numerical solution of dynamic wave propagation problems in infinite half spaces with incompressible constituents and Albers [7] analysed wave propagation problems in saturated and partially saturated porous media.

This work presents, as a novel contribution, a formulation of a fully coupled model for deformable multiphase geomaterials in dynamics including thermal effects.

The multiphase model is developed following Lewis and Schrefler [1]. The u-p-T formulation is obtained by neglecting the relative fluids acceleration and their convective terms, which is valid for low frequency problems as in earthquake engineering [2]. In the model devolvement, the dynamic seepage forcing terms in the mass balance equations and in the enthalpy balance equation and the compressibility of the grain at the microscopic level are neglected. The standard Galerkin method is applied to the governing equations for the spatial discretization, while the generalized Newmark scheme is used for the time discretization. The final non-linear set of equations is solved by the Newton method with a monolithic approach. The model has been implemented in the finite element code COMES-GEO, [1], [8], [13], [16], [19], [20], [21], [22].

The implemented model is validated through the comparison with analytical or finite element quasi-static or dynamic solutions.

2 MACROSCOPIC BALANCE EQUATIONS

The full mathematical model necessary to simulate the thermo-hydro-mechanical behaviour of partially saturated porous media was developed within the Hybrid Mixture Theory by Lewis and Schrefler [7], using averaging theories according to Hassanizadeh and Gray [9], [10]. After neglecting the acceleration of the fluids in the governing equations of Lewis and Schrefler [5], a set of balance equations for the whole multiphase medium is obtained and presented in this paper.

Linear momentum balance equation of the mixture

$$\operatorname{div}\boldsymbol{\sigma} + \rho\boldsymbol{g} - \rho\boldsymbol{a}^s = 0 \tag{1}$$

Dry air mass balance equation

$$\begin{aligned}
&\operatorname{div}\left\{\rho^{ga}\frac{k^{rg}\boldsymbol{k}}{\mu^g}\left[-\operatorname{grad}p^g + \rho^g\boldsymbol{g}\right]\right\} + \operatorname{div}\left[\rho^g\frac{M_a M_w}{M_g^2}\boldsymbol{D}_g^{ga}\operatorname{grad}\left(\frac{p^{gw}}{p^g}\right)\right] \\
&+\rho^{ga}\alpha S_g\operatorname{div}\boldsymbol{v}^s + nS_g\dot{\rho}^{ga} - \rho^{ga}n\dot{S}_w - \rho^{ga}\beta_s\left(\alpha-n\right)S_g\dot{T} = 0
\end{aligned} \tag{2}$$

Water species (liquid and vapour) mass balance equation

$$\begin{aligned}
&\operatorname{div}\left(\rho^w\frac{k^{rw}\boldsymbol{k}}{\mu^w}\left(-\operatorname{grad}p^w + \rho^w\boldsymbol{g}\right)\right) + \operatorname{div}\left(\rho^{gw}\frac{k^{rg}\boldsymbol{k}}{\mu^g}\left(-\operatorname{grad}p^{gw} + \rho^{gw}\boldsymbol{g}\right)\right) \\
&-\operatorname{div}\left[\rho^g\frac{M_a M_w}{M_g^2}\boldsymbol{D}_g^{ga}\operatorname{grad}\left(\frac{p^{gw}}{p^g}\right)\right] + \left(\rho^w S_w + \rho^{gw}S_g\right)\alpha\operatorname{div}\boldsymbol{v}^s \\
&-\left[\rho^w\beta_{sw} + \rho^{gw}\beta_s\left(\alpha-n\right)S_g\right]\dot{T} + \left[n\rho^w - n\rho^{gw}\right]\dot{S}_w \\
&+nS_g\dot{\rho}^{gw} + \left[\rho^w\frac{nS_w}{K_w}\right]\dot{p}^w = 0
\end{aligned} \tag{3}$$

Enthalpy balance equation for the multiphase medium

$$
\begin{aligned}
&\left[C_p^w \rho^w \frac{k^{rw}\boldsymbol{k}}{\mu^w}\left(-\operatorname{grad} p^w + \rho^w \boldsymbol{g}\right) + C_p^g \rho^g \frac{k^{rg}\boldsymbol{k}}{\mu^g}\left(-\operatorname{grad} p^g + \rho^g \boldsymbol{g}\right)\right] \cdot \operatorname{grad} T \\
&+\left(\rho C_p\right)_{eff} \dot{T} - \operatorname{div}\left(\chi_{eff} \operatorname{grad} T\right) \\
&-\rho^w \frac{nS_w}{K_w} \dot{p}^w \Delta H_{vap} - \rho^w S_w \alpha \operatorname{div} \boldsymbol{v}^s \Delta H_{vap} \\
&+\beta_{sw} \dot{T} \Delta H_{vap} - n\left(\rho^w - \rho^{gw}\right) \dot{S}_w \Delta H_{vap} - \operatorname{div}\left[\rho^w \frac{k^{rw}\boldsymbol{k}}{\mu^w}\left(-\operatorname{grad} p^w + \rho^w \boldsymbol{g}\right)\right] \Delta H_{vap} = 0
\end{aligned} \tag{4}
$$

The meaning of each variable of equations 1 to 4 is described in [8], [12], [13] or [20].

3 CONSTITUTIVE RELATIONSHIP

For the gaseous mixture of dry air and water vapour, the ideal gas law is introduced. The equation of state of perfect gas (Clapeyron's equation) and Dalton's law are applied to dry air (ga), water vapor (gw) and moist air (g).

In the partially saturated zones, the equilibrium water vapor pressure $p^{gw}(x,t)$ can be obtained from the Kelvin-Laplace equation, where the water vapor saturation pressure, $p^{gws}(x,t)$, depending only upon the temperature, can be calculated from the Clausius-Clapeyron equation or from an empirical correlation. The saturation degree $S_\pi(x,t)$ and the relative permeability $k^{r\pi}(x,t)$ are experimentally determined functions.

The solid skeleton is assumed elastic, homogeneous and isotropic in the numerical simulations described in Section 5.

4 SPATIAL AND TIME DISCRETIZATION

The finite element model is derived by applying the Galerkin procedure for the spatial integration and the generalized Newmark method for the time integration of the weak form of the balance equations of the previous section [1], [2], [14]. In particular, after spatial discretization within the isoparametric formulation, the following non-symmetric, non-linear and coupled system of equations is obtained:

$$
\begin{cases}
\boldsymbol{C}_{gg}\dot{\bar{p}}^g + \boldsymbol{C}_{gc}\dot{\bar{p}}^c - \boldsymbol{C}_{gT}\dot{\bar{T}} + \boldsymbol{C}_{gu}\dot{\bar{u}} + \boldsymbol{K}_{gg}\bar{p}^g - \boldsymbol{K}_{gc}\bar{p}^c - \boldsymbol{K}_{gT}\bar{T} = \boldsymbol{f}_g \\
\boldsymbol{C}_{cg}\dot{\bar{p}}^g + \boldsymbol{C}_{cc}\dot{\bar{p}}^c + \boldsymbol{C}_{cT}\dot{\bar{T}} + \boldsymbol{C}_{cu}\dot{\bar{u}} - \boldsymbol{K}_{cg}\bar{p}^g + \boldsymbol{K}_{cc}\bar{p}^c + \boldsymbol{K}_{cT}\bar{T} = \boldsymbol{f}_c \\
-\boldsymbol{C}_{Tg}\dot{\bar{p}}^g - \boldsymbol{C}_{Tc}\dot{\bar{p}}^c + \boldsymbol{C}_{TT}\dot{\bar{T}} - \boldsymbol{C}_{Tu}\dot{\bar{u}} - \boldsymbol{K}_{Tg}\bar{p}^g + \boldsymbol{K}_{Tc}\bar{p}^c + \boldsymbol{K}_{TT}\bar{T} = \boldsymbol{f}_T \\
\boldsymbol{M}_{uu}\ddot{\bar{u}} + \int \boldsymbol{B}^T \boldsymbol{\sigma}' dW - \boldsymbol{K}_{ug}\bar{p}^g + \boldsymbol{K}_{uc}\bar{p}^c = \boldsymbol{f}_u
\end{cases} \tag{5}
$$

where the displacements of the solid skeleton $u(x,t)$, the capillary pressure $p^c(x,t)$, the gas pressure $p^g(x,t)$ and the temperature $T(x,t)$ are expressed in the whole domain by global shape function matrices $\boldsymbol{N}_u(x), \boldsymbol{N}_c(x), \boldsymbol{N}_g(x), \boldsymbol{N}_T(x)$ and the nodal value vectors $\bar{u}(t), \bar{p}^c(t), \bar{p}^g(t), \bar{T}(t)$.

In this study, the generalized Newmark time integration scheme is applied to the nonlinear system equations and a nonlinear system of algebraic equations is obtained, in which the unknowns are $\boldsymbol{x} = \left[\Delta\dot{\bar{p}}^g, \Delta\dot{\bar{p}}^c, \Delta\dot{\bar{T}}, \Delta\ddot{\bar{u}} \right]$. The nonlinear system is solved by Newton-Raphson method.

5 FINITE ELEMENT VALIDATION

5.1 Numerical validation of the non-isothermal water saturated model

This problem deals with a water saturated thermo-elastic consolidation [15], simulating a column, 7 m high and 2 m wide, of a linear elastic material subjected to an external surface load of 10000 Pa and to a surface temperature jump of 50 K above the initial temperature of 293.15 K (Figure 1). The material parameters used in the computation are summarised in [16]. The liquid water and the solid grain are assumed incompressible for the static analysis, whereas the compressibility of the liquid water is taken into account in the dynamic analysis. For the numerical calculation, the problem is solved as a two-dimensional problem in plane strain condition. The column is discretized with eight-node isoparametric elements (4 elements/meter); nine Gauss points are used.

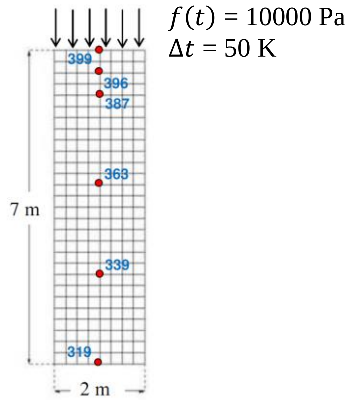


Figure 1: Description for model test

The initial and boundary conditions are described in Table 1.

Table 1: Initial and boundary conditions

Initial condition		**Boundary condition**	
P^g = Patm	fixed	P^g = Patm	fixed
P^c = idrostatic		P^c = 0.0	at the top
T = 293.15 K	fixed	T	not fixed
u_x = 0.0	on the lateral nodes	u_x = 0.0	on the lateral nodes
u_y = 0.0	on the bottom	u_y = 0.0	on the bottom

The solution of the finite element model presented in this work is compared with the quasi-static solution [16] and is plotted in Figures 2 - 4. The results show that the dynamic solution is faster than the quasi-static one at the beginning of the analysis, and that the dynamic solution reaches the quasi-static one at steady-state.

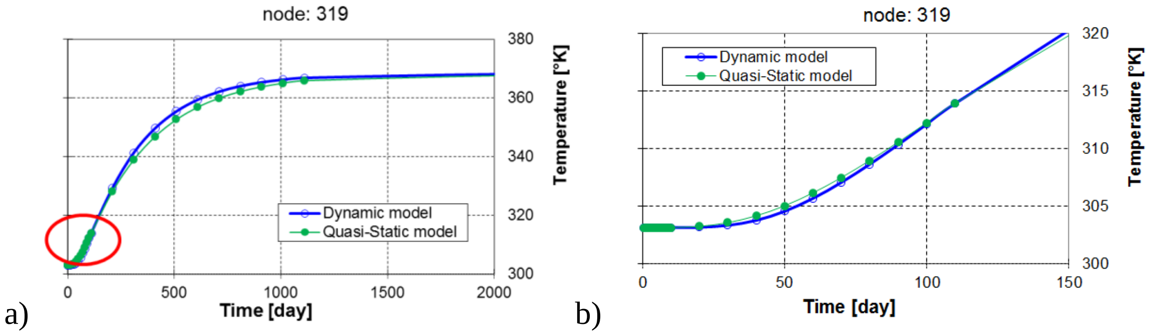


Figure 2: Temperature time history for node 319 up to the steady state solution (a) and in the first period (b) highlighted in a)

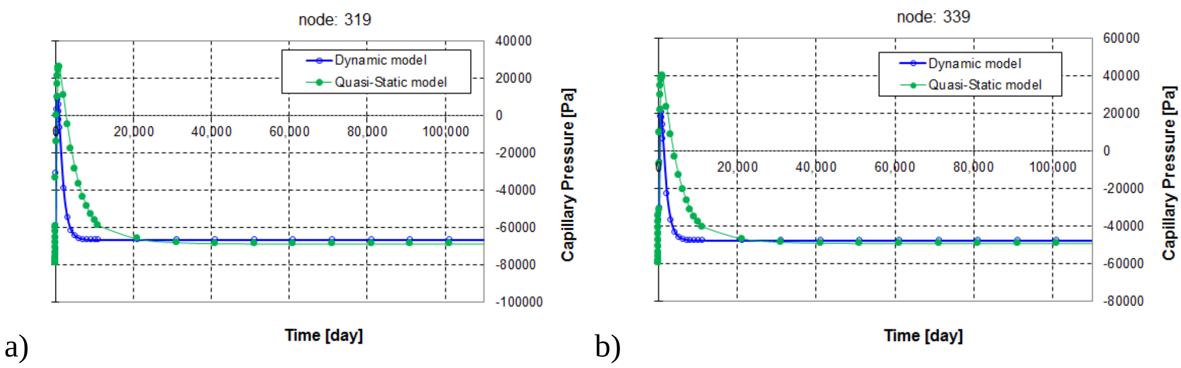
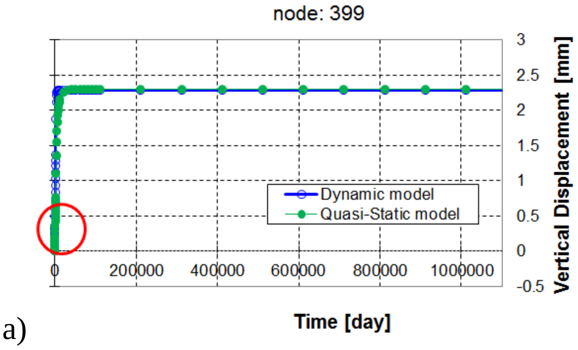


Figure 3: Capillary pressure time history for node 319 and node 339 up to the steady state solution (a) and (b)



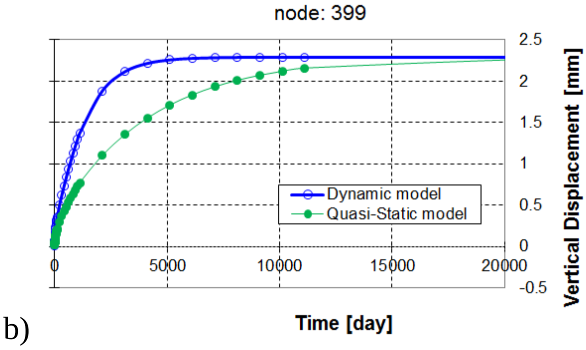


Figure 4: Vertical displacement time history for node 399 up to the steady state solution (a) and in the first period (b) highlighted in a)

5.2 Drainage of liquid water from initially water saturated soil column

The proposed benchmark is based on an experiment performed by Liakopoulos [17] on a column 1 meter high of Del Monte sand and instrumented to measure the moisture tension at several points along the column during its desaturation due to gravitational effects. Before the start of the experiment, water was continuously added from the top and was allowed to drain freely at the bottom through a filter, until uniform flow conditions were established. Then the water supply was ceased and the tensiometer readings were recorded. The finite element simulation is performed with the two-phase flow model in isothermal conditions, with switching between saturated and unsaturated solution performed at $p^c = 2000$ Pa ($S_w = 0{,}998$), which corresponds to bubbling pressure of the analysed sand, and an additional lower limit for the gas relative permeability of 0,0001 [8]. For the numerical calculation, a two-dimensional problem in plane strain conditions is solved; the spatial domain of the column is divided into 20 eight-node isoparametric finite elements of equal size. Furthermore, nine Gauss integration points were used. The material parameters are listed in [8] or [20].

This problem has been solved considering single or two-phase flow mainly in quasi-static condition; a finite element solution in dynamics was presented in [18]. The initial hydro-mechanical equilibrium state is obtained via a preliminary quasi-static solution. The initial and boundary conditions for the dynamic analysis are summarized in Table 2.

Table 2: Initial and boundary conditions

Initial condition		Boundary condition	
P^g = Patm	on the top	P^g = Patm	on the top, on the bottom
P^c = idrostatic		P^c = 0.0	on the bottom
T = 293.15 K	fixed	T = 293.15 K	fixed
u_x = 0.0	on the lateral nodes	u_x = 0.0	on the lateral nodes
u_y, V_y = 0.0	on the bottom	u_y = 0.0	on the bottom

The comparison between the dynamic and the quasi-static solution is plotted in Figures 5 to 7, where the profiles for water pressure, water saturation and vertical displacement along the column are plotted. Since the inertial loads are negligible in the experiment, the finite element solution in dynamics gives almost the same results of the quasi-static model [8], [20].

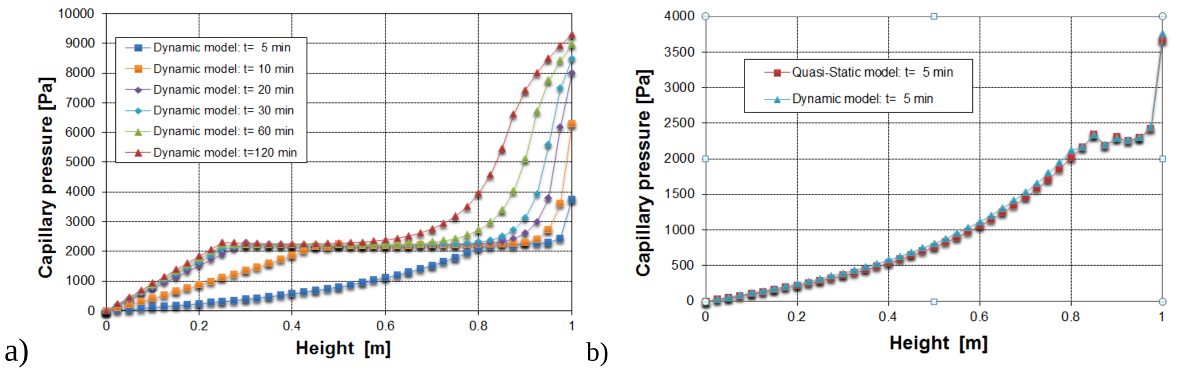


Figure 5: Profiles of capillary pressure versus height (a-dynamic solution) and comparison between the quasi-static and the dynamic solution

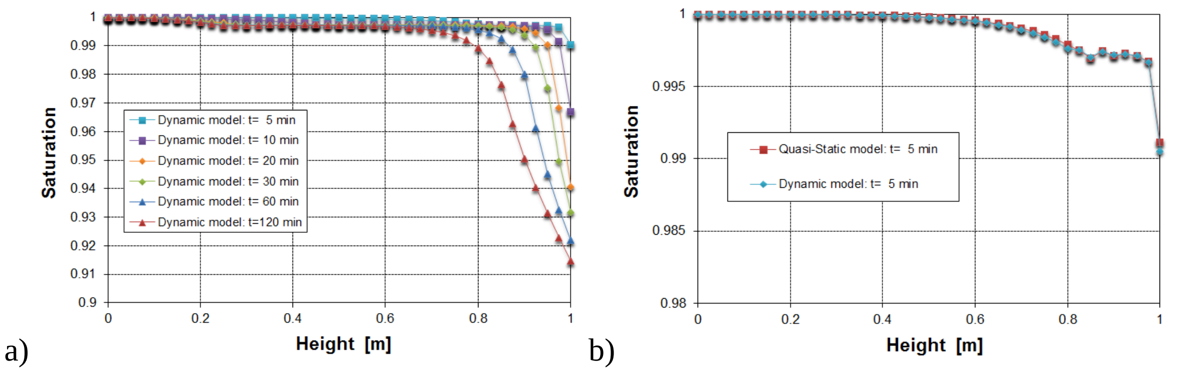


Figure 6: Profiles of saturation degree versus height (a-dynamic solution) and comparison between the quasi-static and the dynamic solution

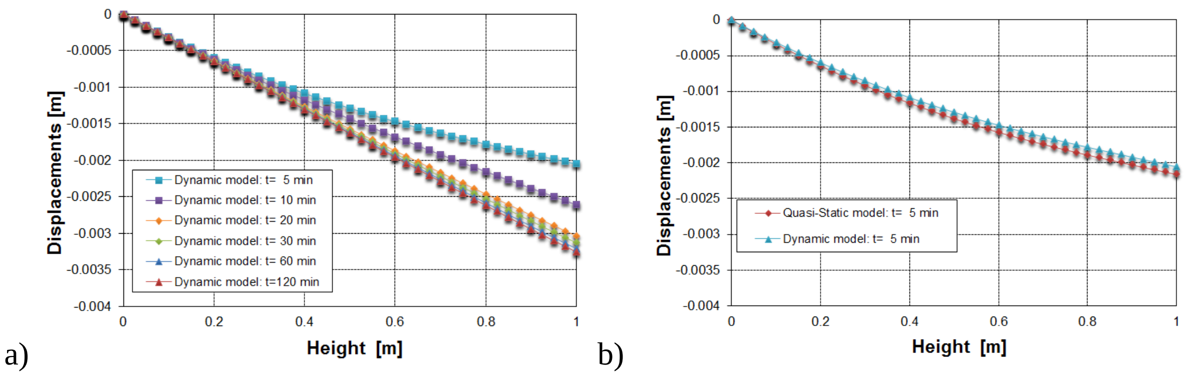


Figure 7: Profiles of vertical displacement versus height (a-dynamic solution) and comparison between the quasi-static and the dynamic solution

6 CONCLUSIONS

A model for the analysis of the thermo-hydro-mechanical behaviour of porous media in dynamics was developed. Starting from the generalized mathematical model developed by Lewis and Schrefler [1] for deforming porous media in non-isothermal conditions, the u-p-T formulation was derived following [2]. The validity of such an approximation is limited to low frequencies problems [2], as in earthquake engineering. In this formulation the relative accelerations of the fluids and the convective terms related to these accelerations are neglected. Moreover, in the model development, the compressibility of the solid grain at microscopic level and the dynamic seepage forcing terms were neglected.

The numerical model was derived within the finite element method: the standard Bubnov-Galerkin procedure [14] was adopted for the discretization in space, while the implicit and unconditionally stable generalized Newmark procedure was applied for the discretization in time [14]. The independent variables chosen are: the displacement of the solid skeleton u, the capillary pressure p^c, the gas pressure p^g and the temperature T.

The model was implemented in the finite element code Comes-Geo [1], [8], [13], [16], [19], [20], [21], [22]. The formulation and the implemented solution procedure were validated through the comparison with literature benchmarks, finite element solutions or analytical solutions. In this paper, comparison between the finite element solution in dynamics and the corresponding quasi-static solution is presented by studying the non-isothermal consolidation in a water saturated column and the drainage of liquid water in an initially water saturated soil column.

This work extends the model developed in [18] to non-isothermal conditions and removes the passive air phase assumption of the multiphase porous media model in dynamics developed in [2], [23], [24], [25].

Acknowledgement. The authors would like to thank the 7th Framework Programme of the European Union (ITN MuMoLaDe project 289911) for the financial support.

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