

MULTI-PHYSICS MODELLING OF THE CONSOLIDATION PROCESSES IN VARIABLY SATURATED ELASTO-PLASTIC SOILS DUE TO HIGH-TEMPERATURE

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Abstract. The aim of this paper is numerical analysis of the consolidation processes in soils due to high temperature.

A fully coupled finite element model for non-isothermal elasto-plastic multiphase materials at high temperature is therefore developed in the framework of Porous media mechanics for fully coupled multi-phase porous media.

The non-isothermal elasto-plastic consolidation processes of a Boom clay column due to a temperature rise of the top surface from 30 to 150 °C is studied in detail, in order to analyze the coupled thermo-hydro-mechanical behaviour of the material when the temperature is above the boiling value and water phase change develops inducing variably saturated conditions.

The numerical results reveal a temperature increase over time in the column; desaturation starts from the top surface, triggered by a water vapor mass exchange with the ambient at a RH=95%. When the temperature reaches the boiling value, phase change for the liquid water occurs and desaturation proceed much faster, developing high capillary pressures and inducing plastic strains, swelling and a final settlement when reaching the steady state conditions.

1 INTRODUCTION

In recent years, interest in coupled hydro-thermo-mechanical (HTM) analysis of variably saturated porous materials has increased, due to the wide spectrum of their engineering

applications. One area of particular interest is Environmental Geomechanics (see e.g. Schrefler [24]; Vulliet et al. [28]), in which some non-isothermal problems are challenging. In these cases, soils and rocks behave as deformable multiphase porous media, composed of a solid skeleton with voids containing one or more fluids, in which interactions among all the components of the porous material and thermal effects cannot be neglected.

To enable significant predictive simulations and analysis, suitable physical and mathematical coupled HTM models are needed.

Numerical models for HTM simulations of water saturated elasto-plastic materials or unsaturated elastic materials (with two fluid phases), are now well developed. Other models for unsaturated materials consider isothermal elasto-plastic solid skeletons. HTM models for unsaturated and non-isothermal elasto-plastic soils are scarce (Wenhua et al. [29]; Gens [7]; Dupray et al. [1]). A model describing the HTM behavior of elastic soils heated up to 150°C is presented in Thomas et al. [26], where phase change for liquid water is taken into account and an experimental test is simulated.

This paper is a contribution to the analysis of HTM physical processes, examining fluid phase changes inside pores in thermo-elasto-plastic Boom clay. In the following, the mathematical model for non-isothermal multiphase porous materials is summarized first. Small strains and quasi-static loading conditions are assumed. Then, the results of the numerical simulations are presented.

2 MATHEMATICAL MODEL

The full mathematical model necessary to simulate the HTM transient behavior of variably saturated porous media was developed within the Hybrid Mixture Theory in Lewis and Schrefler [13], Gawin and Schrefler [4] and Schrefler [24] with averaging theories following Hassanizadeh and Gray [9,10] and Gray and Hassanizadeh [8]. The model is briefly summarized in the present section (the full model can be found in Sanavia and Laloui [17]).

The partially saturated porous medium is treated as a multiphase system composed of a solid skeleton (s) with open pores filled with water (w) and gas (g). The latter is assumed to behave as an ideal mixture of dry air (noncondensable gas, ga) and water vapor (condensable gas, gw).

In the model, the solid is deformable and non-polar, and the fluids, solid and thermal fields are coupled. All fluids are in contact with the solid phase. The constituents are assumed to be isotropic, homogeneous, immiscible except for the dry air and water vapor, and chemically non-reactive. Local thermal equilibrium between the solid, gas and liquid phases is assumed. Heat conduction and heat convection, vapor diffusion, liquid water flow due to pressure gradients or capillary effects and water phase changes (evaporation and condensation) inside the pores are taken into account.

In the partially saturated zones the liquid water is separated from its vapor by a meniscus concave toward gas (capillary water). Due to the curvature of this meniscus the sorption equilibrium equation (Gray and Hassanizadeh [8]) gives the relationship $p^c = p^g - p^w$, between the capillary pressure $p^c(\mathbf{x}, t)$, also known as matrix suction $s(\mathbf{x}, t)$, gas pressure $p^g(\mathbf{x}, t)$ and water pressure $p^w(\mathbf{x}, t)$. \mathbf{x} is the vector of the spatial coordinates and t the current time.

The state of the medium is chosen to be described by capillary pressure p^c , gas pressure p^g , absolute temperature T and displacements of the solid matrix \mathbf{u} . For a detailed discussion of

the primary variables chosen see Sanavia et al. [22].

The balance equations are now summarized; these equations were developed in a geometrically linear setting and written at the macroscopic level considering quasi-static loading conditions (see Sanavia et al. [20] for the corresponding model developed in a geometrically non-linear setting and Sanavia and Schrefler [18] for further improvements).

The equilibrium equation of the mixture in terms of the generalized effective Cauchy's stress $\boldsymbol{\sigma}'(\mathbf{x}, t)$ (Lewis and Schrefler [13]; Nuth and Laloui [14]) takes the form

$$\operatorname{div}\left(\boldsymbol{\sigma}' - [p^g - S_w p^c] \mathbf{1}\right) + \rho \mathbf{g} = \mathbf{0} \quad (1)$$

where $\rho = [1-n]\rho^s + nS_w\rho^w + nS_g\rho^g$ is the mass density of the overall medium, $n(\mathbf{x}, t)$ is the porosity, $S_w(\mathbf{x}, t)$ is the degree of saturation of water. \mathbf{g} is the gravity acceleration vector and $\mathbf{1}$ is the second order identity tensor. The mass balance equation for the liquid water and its vapor is (Sanavia and Laloui [17])

$$\begin{aligned} & n[\rho^w - \rho^{gw}] \frac{\partial S_w}{\partial t} + [\rho^w S_w + \rho^{gw} S_g] \operatorname{div} \mathbf{v}^s + nS_g \frac{\partial \rho^{gw}}{\partial t} \\ & - \operatorname{div} \left(\rho^s \frac{M_a M_w}{M_g^2} \mathbf{D}_g^{gw} \operatorname{grad} \left(\frac{p^{gw}}{p^c} \right) \right) - \operatorname{div} \left(\rho^w \frac{\mathbf{k}^w k^{rw}}{\mu^w} [\operatorname{grad}(p^g) - \operatorname{grad}(p^c) - \rho^w \mathbf{g}] \right) \\ & - \operatorname{div} \left(\rho^{gw} \frac{\mathbf{k}^g k^{rg}}{\mu^g} [\operatorname{grad}(p^g) - \rho^g \mathbf{g}] \right) - \beta_{sw} \frac{\partial T}{\partial t} = 0 \end{aligned} \quad (2)$$

$\mathbf{k}^\pi(\mathbf{x}, t) = k^\pi(\mathbf{x}, t) \mathbf{1}$ is the intrinsic permeability tensor of the porous matrix in π -fluid saturated condition [m^2], which is assumed to be isotropic, $k^{r\pi}(\mathbf{x}, t)$ is the fluid relative permeability parameter and $\mu^\pi(\mathbf{x}, t)$ is the dynamic viscosity of the fluid [$\text{Pa}\cdot\text{s}$], with $\pi = w, g$. $\beta_{sw} = [1-n]\beta_s[S_g\rho^{gw} + \rho^w S_w]$. $\mathbf{D}_g^{gw}(\mathbf{x})$ is the effective diffusivity tensor of water vapor in the gas phase contained within the pore space, and M_a, M_w and $M_g(\mathbf{x}, t)$ are the molar mass of dry air, liquid water and the gas mixture, respectively.

Similarly, the mass balance equation for the dry air is (Sanavia and Laloui [17]):

$$\begin{aligned} & -n\rho^{ga} \frac{\partial S_w}{\partial t} + \rho^{ga} S_g \operatorname{div} \mathbf{v}^s + nS_g \frac{\partial \rho^{ga}}{\partial t} - \operatorname{div} \left(\rho^g \frac{M_a M_w}{M_g^2} \mathbf{D}_g^{ga} \operatorname{grad} \left(\frac{p^{ga}}{p^g} \right) \right) \\ & + \operatorname{div} \left(\rho^{ga} \frac{\mathbf{k}^g k^{rg}}{\mu^g} [-\operatorname{grad}(p^g) + \rho^g \mathbf{g}] \right) - [1-n]\beta_s \rho^{ga} S_g \frac{\partial T}{\partial t} = 0 \end{aligned} \quad (3)$$

The enthalpy balance equation of the mixture has the following form (Lewis and Schrefler [13]; Gawin and Schrefler [4]),

$$\left(\rho C_p\right)_{\text{eff}} \frac{\partial T}{\partial t} + [nS_w \rho^w C_p^w \mathbf{v}^{ws} + nS_g \rho^g C_p^g \mathbf{v}^{gs}] \cdot \operatorname{grad} T - \operatorname{div}(\chi_{\text{eff}} \operatorname{grad} T) = -\dot{m}_{gw} \Delta H_{va} \quad (4)$$

where $\left(\rho C_p\right)_{\text{eff}}(\mathbf{x}, t)$ is the effective thermal capacity of the porous medium, $C_p^w(\mathbf{x}, t)$ and $C_p^g(\mathbf{x}, t)$ are the specific heat of water and gas, respectively, and $\chi_{\text{eff}}(\mathbf{x}, t)$ is the effective thermal conductivity of the porous medium. The RHS term of Equation (4) considers the contribution

of the evaporation and condensation. A recent development a similar model, which considers the air dissolved in the liquid water and its desorption at lower water pressures, is presented in Gawin and Sanavia [5].

The constitutive relationships for the fluids are described first. The gas phase is assumed to be a perfect mixture of two ideal gases and the state equation of a perfect gas (Clapeyron's equation) and Dalton's law are applied to dry air, water and moist air. In the partially saturated zones, water vapor pressure at equilibrium $p^{gw}(\mathbf{x},t)$ is obtained from the Kelvin-Laplace equation, where the water vapor saturation pressure, $p^{gws}(\mathbf{x},t)$, depends only upon the temperature. The bulk density of liquid water that is dependent on the temperature is modeled using the relationship proposed by Furbish (1997), which is valid for incompressible liquid water at the microscopic level. The liquid water viscosity, dry air and water vapor viscosity, and the latent heat of evaporation are also temperature dependent relationships. Water saturation $S_w(\mathbf{x},t)$ is an experimentally determined function of temperature and capillary pressure, while relative permeability $k^{tr}(\mathbf{x},t)$ for liquid water and gas are experimentally determined functions dependent on capillary pressure only, for the sake of temperature dependent experimental results.

The thermo-elasto-plastic behavior of the solid skeleton under variably saturated conditions is formulated using the general ACMEG-TS constitutive model for clayey soils (François and Laloui [2]; François [3]), which extends the model developed by Laloui and François [12] for water saturated clays to unsaturated conditions. This model, founded on the main physical mechanisms involved (see e.g. François and Laloui [3] or Gens [7]), is based on the critical state concept and developed within the rate-independent multi-surface plasticity theory and bounding surface theory for geometrically linear problems.

In this model, the strain tensor increment is decomposed into non-linear thermo-elastic and plastic components. The elastic component is modeled within the non-linear hypo-elasticity theory, while the plastic component is given by two coupled hardening processes, one isotropic and one deviatoric. Following Hujieux [11] and applying the multi-mechanism plasticity theory, both mechanisms may induce volumetric plastic strain. The yield functions of the two plastic mechanisms have the following expressions,

$$f_{iso} = p' - p'_c \quad r_{iso} = 0 \quad ; \quad f_{dev} = q - Mp' \left[1 - b \log \left(\frac{d p'}{p'_c} \right) \right] r_{dev} = 0 \quad (5)$$

where $q(\mathbf{x},t)$ is the deviatoric stress and $p'_c(\mathbf{x},t)$ is the preconsolidation pressure. $b(\mathbf{x})$, $d(\mathbf{x})$ and $M(\mathbf{x},t)$ are material parameters. M is the slope of the critical state line, dependent on temperature (François and Laloui [3]). The evolution of the yield loci, Equations (5), with capillary pressure and temperature, is illustrated in Figure 1, where both the capillary hardening and thermal softening can be observed.

The preconsolidation pressure depends on temperature, capillary pressure and the volumetric plastic strain, which introduces thermal plasticity and suction-induced plasticity, Salager et al. [15].

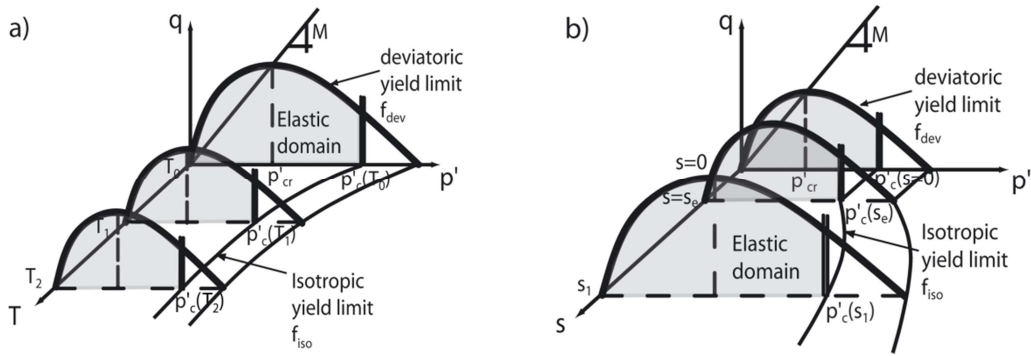


Figure 1: Effect of (a) temperature and (b) capillary pressure on the shape of the coupled mechanical yield limits

3 FINITE ELEMENT FORMULATION

The finite element model is derived by applying the standard Galerkin procedure for spatial integration and the Generalised Trapezoidal Method for time integration of the weak form of the balance equations of the previous section (e.g. Lewis and Schrefler [13]). After spatial discretisation within the isoparametric formulation, a non-symmetric, non-linear and coupled system of equations is obtained, where the solid displacements $\mathbf{u}(\mathbf{x},t)$, the capillary pressure $p^c(\mathbf{x},t)$, the gas pressure $p^g(\mathbf{x},t)$ and the temperature $T(\mathbf{x},t)$ are expressed in the whole domain by the global shape function matrices and nodal value vectors. The terms of the equation system are given in Sanavia et al. [22] and Gawin and Sanavia [5].

After time integration, the non-linear system of equations is linearized, thus yielding an equation system that can be solved numerically. Owing to the strong coupling between the mechanical, thermal and pore fluid fields, a monolithic solution of the linearized system of equations is preferred according to a Newton scheme.

4 NUMERICAL SIMULATIONS

The HTM consolidation processes in thermo-elasto-plastic Boom clay due to various mechanical, thermal and environmental loads are analysed in Sanavia and Laloui [17]. With respect to that work, two additional cases are simulated here:

Case A: analysis of the consolidation due to a compressive load of 10.0 MPa applied to the top surface of the column depicted in Figure 2a. The material is saturated with liquid water and temperature is of 303.15 K (30°C).

Case B: analysis of the HTM processes caused by heating the top surface of the column to a temperature of 423.15 K (150°C).

A soil column of Boom clay 7.0 m high and 2.0 m wide is studied (Figure 2). Plane strain conditions are assumed. Figures 2a and 2b show the position of the nodal points selected for plotting the time history of the relevant HTM variables.

The material parameters for Boom clay used in the computation are listed in Sanavia and Laloui [17]. Gravity load is taken into account. The soil water retention curve for Boom clay is used (Salager et al. [16]; see also Sanavia and Laloui [17]). The variation in the relative permeability of gas and liquid water with the capillary pressure is modeled following the experimental results of Volkaert et al. [27].

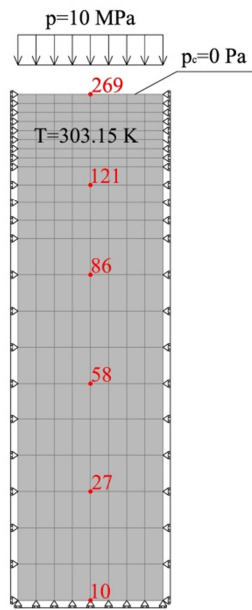


Figure 2a: Spatial discretization, b.c. and plotting nodes, case A.

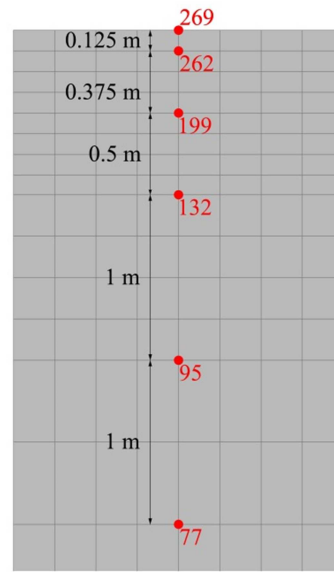


Figure 2b: Spatial discretization of the upper part of the column and plotting nodes, case B

Case A – This case simulates the mechanical consolidation of a column of soil at the initial temperature of 30°C (303.15K). The upper surface of the column is drained ($p^c = 0$ Pa, $p^g = p_{\text{atm}} = 101325$ Pa) and the lateral and bottom surfaces are sealed and insulated (Figure 2a). Horizontal displacements are constrained along the vertical boundaries, and the vertical displacements are constrained at the bottom surface. First, the material is consolidated and saturated with liquid water and the linear elastic behavior is assumed for the solid skeleton. Afterwards, the upper side of the column is subjected to an external compressive surface load of 10.0 MPa and the elasto-plastic consolidation is studied up to the development of the steady state condition (elasto-plasticity is modelled using AcmeG-TS).

The HM behavior computed for this case is illustrated with Figures 3a-b, showing the elasto-plastic consolidation of the soil column.

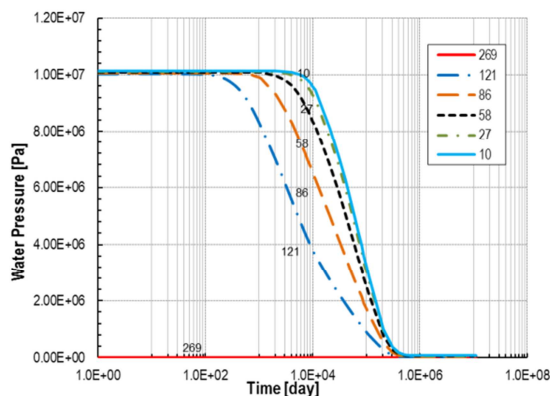


Figure 3a: Water pressure [Pa] versus time [day] at different nodes.

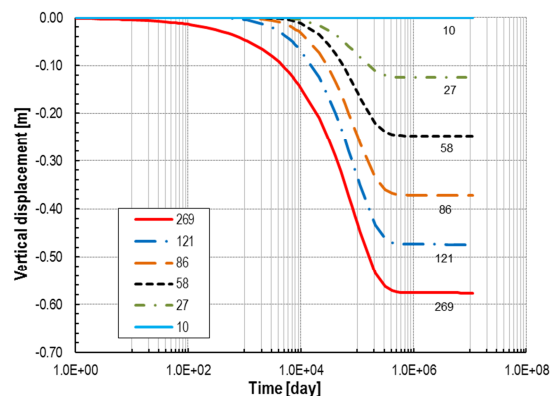


Figure 3b: Vertical displacement [m] versus time [day] at different nodes.

Case B: heating the top surface of the column up to a temperature of 150°C. This case studies the coupled HTM behavior of the column due to heating up to a temperature higher than the boiling temperature, inducing evaporation of the liquid water (preliminary results were shown in Sanavia et al. [19], where only the initial heating phase was analyzed).

In this case, the upper side of the column is heated by applying a temperature load increasing linearly from 30°C to 150°C (423.15K) with the rate of 12°C/day. The upper surface is in contact with an ambient at atmospheric pressure ($p^s=p_{atm}$) and a relative humidity (RH) of 95%; for this case, a mass exchange coefficient of $\beta_c=0.005 \text{ m s}^{-1}$ is used during the computation (see the boundary conditions of the third kind described in Gawin and Schrefler [4] or in Sanavia and Laloui [17]).

The numerical results reveal the temperature increase in the column (Figure 4a) and the desaturation of the top surface (node 269, Figure 2b) triggered by a water vapor mass exchange with the ambient at a RH=95% (Figure 4b). Then, when the temperature reaches the boiling value (at about 7 days for node 269), phase change for the liquid water occurs and the desaturation proceed much faster because the relative humidity decreases rapidly (Figure 4b), developing high capillary pressures (Figure 4c) and inducing compressive plastic strains and a partial release of the of the initial settlement due to the mechanical load (Figure 4d). Finally, a further compaction is observed during the consolidation stage reaching the steady state conditions at about $6.0 \cdot 10^3$ days.

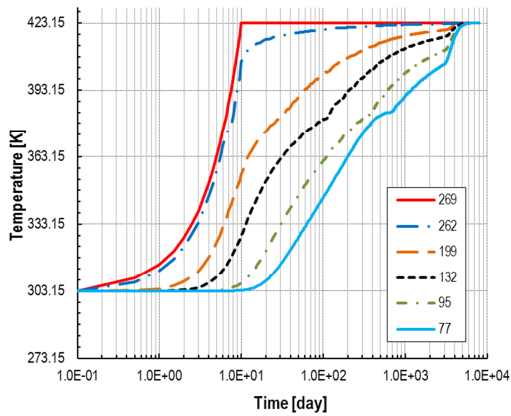


Figure 4a: Temperature [K] versus time [day] at different nodes.

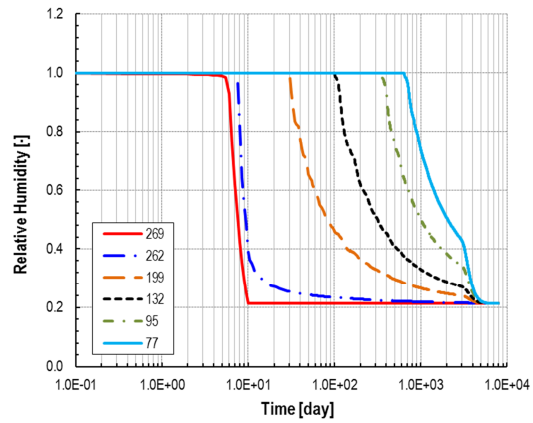


Figure 4b: Relative humidity [-] versus time [day] at different nodes.

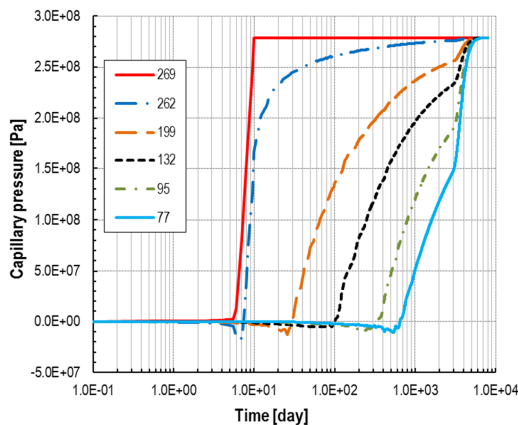


Figure 4c: Capillary pressure [Pa] versus time [day] at different nodes.

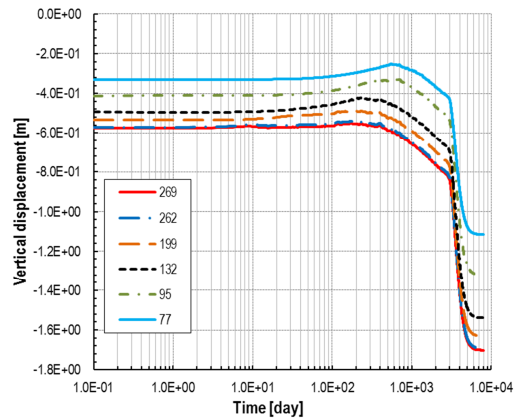


Figure 4d: Vertical displacement [m] versus time [day] at different nodes.

5 CONCLUSIONS

A fully coupled finite element formulation for thermo-elasto-plastic multiphase geomaterials based on Porous Media Mechanics was developed. A finite element study of the consolidation processes in a Boom clay column due to mechanical and thermal loads was presented, permitting a deeper understanding of the HTM behavior of the material. In particular, the heating case up to a temperature above the boiling value shows that the liquid water phase change develops, inducing desaturation of the column together with a first release of the compaction due to the mechanical load, followed with a further compaction during the consolidation process.

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