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COMPUTER MODELING OF OPERATION OF THE CONDUCTIVE MHD CENTRIFUGAL PUMP

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Abstract. The mathematical model of an MHD centrifugal conductive pump (CCP) is developed, which allows the flow of molten metal in the MHD channel and the output pressure to be calculated as a function of the basic determining major parameters: flow rate, current strength, and magnetic induction. The development and testing of the mathematical model were based on the experimental results obtained for the CCP working model. Determination of friction forces effect in scope of initial system of the MGD-equations is carried out on the basis of the known square-law of the drag for turbulent flow in a pipe, generalized on a case of two-dimensional flows in the channel. On base of the matching of computational and experimental values of pressure at exit of conductive centrifugal pump for different operating modes, dependences of coefficient of drag depending on the determining major parameters were got. The calculations showed that the dependences obtained have universal enough character. The implementation of this model in a computer code can be used to design pumps of this type and to control their operation in the metallurgical process technology.

1 INTRODUCTION

The casting process in metallurgy consists of numerous stages, including metal melting, its transportation, processing (degassing, alloying with various dopes, including nanodispersed, etc.), and finally pouring. In turn, each stage is a rather labor-consuming process with its own complicated technological chain. A common adverse factor, however, is the contact between the melt and the atmosphere. The problem could be solved by organizing a closed cycle. It should also be noted that organization of such a cycle would make this production more environmentally friendly. An important element of the cycle is the system of liquid metal supply, which should be sufficiently universal for the entire chain of out-of-furnace processing. It is commonly accepted now that a magnetohydrodynamic (MHD) system can serve as such a device and pumps based on conductive schemes, can be used at high working temperatures, in particular a centrifugal conductive pump (CCP).

Application of the CCP in the process flow of foundry production requires real-time control of its operation parameters (force being developed, pressure, flow rate, etc.) and optimization of structural concepts at the design stage, which requires an adequate and sufficiently simple mathematical model of the CCP to be developed.

2 MATHEMATICAL MODEL

The development and testing of the mathematical model were based on the experimental results obtained for the CCP model in [1, 2]. The disk channel of the MHD cell of the pump (Figure 1) is formed by two dielectric disks located at a distance h from each other. At the center of the upper disk, there is a branch tube of radius r_1, which is used to supply the metal into the MHD cell and serves as one of the electrodes. The second electrode is the side surface of the cell of radius r_2, which has orifices for working medium exhaustion. The voltage U is applied to the electrodes from an external current source; the magnetic field normal to the disk plane is generated by an external magnetic system.

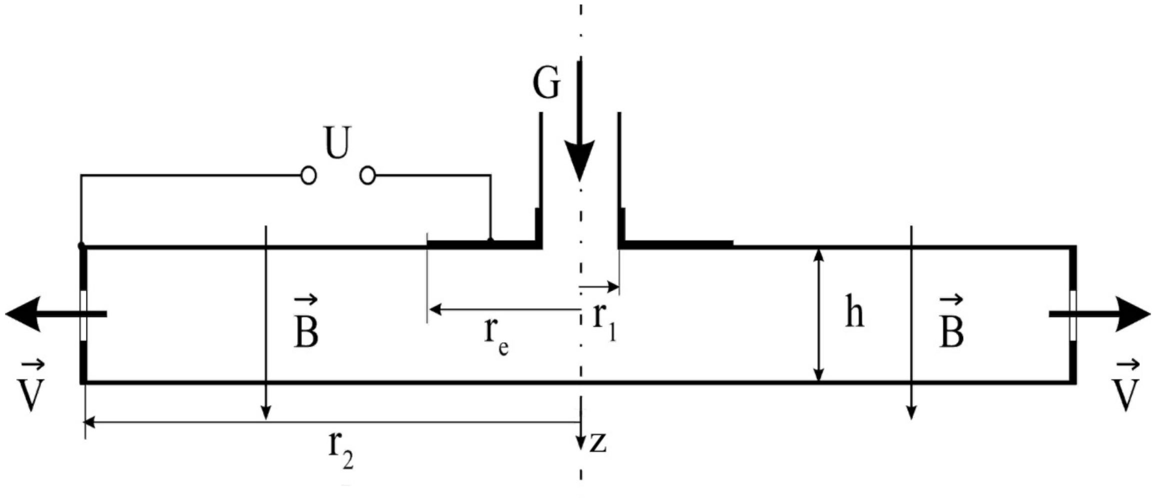

Figure 1: Scheme of the MHD cell

2.1 Remark 1

Calculation of the magnetic field. Let us write the magnetic field induction in the form $\vec{B}=\vec{B}_0+\vec{b}$, where $\vec{B}_0$ is the external magnetic field, $\vec{b}$ is the induced field determined by the Maxwell equation $\nabla\times\vec{b}=\mu_0\vec{j}$, and $\vec{j}$ is the current density in the flow. Thus, we obtain the estimate for the induced magnetic field $b_*/r_* \sim \mu_0 j_* \sim \mu_0 I_*/S_*$, where I_* is the characteristic total current; r_* and $S_*=2\pi r_* h$ are the characteristic radius and cross section at the current observation point. Then, the parameter characterizing the normalized induced field is

$$\Lambda=b_*/B_0 \sim \mu_0 I_* r_*/(S_* B_0)=4\pi 10^{-7} I_* r_*/(2\pi r_* h B_0)=2\cdot 10^{-7} I_*/(hB_0)\,.$$

For typical parameters of the experimental CCP, $I\approx 10^3 A;\;\; B=0,4T;\;\; h=10^{-2}m$, we have

$\Lambda = 2 \cdot 10^{-7} \cdot 10^{3} / (10^{-2} \cdot 4 \cdot 10^{-1}) = 0,05$.

The values of the parameters predicted by the estimates for a full-scale CCP with a flow rate of 10 … 20 kg/s and a pressure of 0.1 … 0.3 MPa are $I \approx 5 \cdot 10^{3} A$; $B = 2T$; $h = 2 \cdot 10^{-2} м$; therefore, we obtain

$\Lambda = 2 \cdot 10^{-7} 5 \cdot 10^{3} / (2 \cdot 10^{-2} 2) = 0,025$.

Thus, we can neglect the induced field in both cases and consider the flow of an electroconducting fluid in a given magnetic field, which is further assumed to be uniform along the channel.

2.2 Remark 2

Calculation of the current density. The determining major parameters of CCP operation are the flow rate, magnetic induction, and current strength. By virtue of the considerations discussed above, the magnetic induction is defined by an independent external magnetic system, whereas the current strength is a quantity that can be easily measured and controlled during the process. In this case, by setting the current, we can avoid considering the electric circuit of the pump; as the eddy currents in the MHD channel are closed, the total current passing through the pump is determined by the radial component of the current density; for a certain flow averaged over the channel width, we have $I = j_r 2\pi rh$.

2.3 Remark 3

Calculation of the friction force in the channel. The fluid in the CCP channel is set into rotational motion by the Ampere force; intense mixing and flow turbulization occur, which was directly observed in experiments. It is known that the results of the turbulent flow evolution in the channel are thinner boundary layers and more filled velocity profile; additional results in the case of MHD flows are thinner Hartmann layers and less pronounced adverse influence of the Hartmann effect on the character of the current flow in the channel.

In engineering calculations of steady viscous layered flows (with one component of the velocity), the friction force is taken in the form [3]:

$$f = (\lambda / \ell)\rho\bar{\upsilon}^2 / 2, \tag{1}$$

where λ is the dimensionless drag coefficient, ρ is the density, $\bar{\upsilon}$ is the average velocity, and ℓ is the characteristic size normal to the flow velocity direction. This expression for the friction force was obtained on the basis of the exact solution for a steady laminar flow of a viscous fluid in a circular tube. It should be noted that the laminar flow has a unique relation between the drag coefficient and the Reynolds number $\lambda = 32\mu / (\rho\bar{\upsilon}R) = 64 / \mathrm{Re}$ and a linear law for the friction force For turbulent flows, the proposed definition of the friction force is eventually an empirical presentation of the experimentally obtained quadratic law for the friction force, where the parameter λ is not constant; it depends on the channel geometry and flow regime and is determined on the basis of experimental results. In the case of plane turbulent flow in a channel the friction force (1) is defined as:

$$\vec{f} = (\lambda / \ell)(\rho V^2 / 2)\vec{V} / |V| \equiv (\lambda\rho / 2\ell)V\vec{V} \tag{2}$$

2.4 The system of governing equations

In view of the remarks made above, the initial system of equations is taken as a system of one-dimensional equations of magnetic hydrodynamics, where all functions change only in the radial direction. We write the coordinate presentation of the vectors in a cylindrical coordinate system: $\vec{V}(u,v,0)$; $\vec{B}=(0,0,B)$; $\vec{j}(j_r, j_\theta, 0)$,
then, the system of equations becomes

$$d(r\rho u)/dr = 0\,; \tag{3}$$

$$\rho u du/dr - \rho v^2/r = -dp/dr - j_\theta B - (\lambda\rho/2h)(u^2+v^2)^{1/2}u\,, \tag{4}$$

$$\rho u dv/dr + \rho uv/r = j_r B - (\lambda\rho/2h)(u^2+v^2)^{1/2}v\,, \tag{5}$$

$$d(rj_r)/dr = 0\,, \tag{6}$$

$$j_\theta = \sigma u B\,. \tag{7}$$

Equation (3) is rigorously valid outside the region of flow turning ($r > r_1$), and Eq. (6) is rigorously valid outside the region of current turning ($r > r_e$); in these regions, Eqs. (3) and (6) are the laws of conservation of the flow rate and total current, respectively:

$$\rho u 2\pi r h = G,\ u = G/(2\pi h\rho r),\ r \ge r_1\,, \quad . \tag{8}$$

$$j_r 2\pi r h = I,\ j_r = I/(2\pi h r),\ \ r \ge r_e\,, \quad . \tag{9}$$

(G is the mass flow rate and I is the total current).

For Eqs. (4) and (5), we have to impose boundary conditions for the sought functions on $r = r_1$ and $r = r_e$, which are, generally speaking, unknown and are determined by solving two-dimensional problems in the regions of flow and current turning.

Let us obtain model distributions of the radial components of flow velocity and current density in the regions of turning, which satisfy the laws of conservation of the flow and total current. We consider the flow through a cylinder of radius r; the flow enters from the upper base of the cylinder and leaves through the orifice in the side surface. Then we have the following relations for the mass flow and current:

$$\rho v_z \pi r^2 = \rho u 2\pi r h,\ j_z \pi r^2 = j_r 2\pi r h\,. \tag{10}$$

Let us define v_z and j_z as

$$v_z = G/(\pi r_1^2 \rho),\ \ \mathrm{j}_z = I/(\pi r_e^2)\,, \tag{11}$$

Equations (8)-(11) yield the following expressions for the flow velocity and current density:

$$u = \begin{cases} Gr/(2\pi r_1^2 h\rho) & 0 \le r \le r_1, \\ G/(2\pi h\rho r) & r_1 < r \le r_2 \end{cases} \tag{12}$$

$$j_r = \begin{cases} Ir/(2\pi h r_e^2) & 0 \le r \le r_e, \\ I/(2\pi h r) & r_e < r \le r_2 \end{cases} \tag{13}$$

Finally, the system of the governing equations in the domain $0 \le r \le r_2$ can be presented as

$$dv/dr = j_r B/(\rho u) - v\left(1/r + (\lambda/2h)\sqrt{1+(v/u)^2}\right), \tag{14}$$

$$d\left(p + \rho u^2/2\right)/dr = \rho v^2/r - \sigma B^2 u - (\lambda/2h)\sqrt{1+(v/u)^2}\,\rho u^2, \tag{15}$$

where u and j_r are given by (12), (13).

Equation (14) was solved numerically under the boundary condition $v(0) = 0$ by the IVPRK subroutine of the FORTRAN library. The expression for the pressure generated by the pump is determined by integrating Eq. (15) from 0 to r_2 with allowance for Eq. (12) for the velocity u:

$$p = \int_0^{r_2} \rho\left(v^2/r - (u^2\lambda/2h)(\lambda/2h)\sqrt{1+(v/u)^2}\right)dr - \\ -g\left(\sigma B^2\left(1/2 + \ln(r_2/r_1)\right) + \rho g(2r_2^2)\right), \quad g = G/(2\pi h\rho) \tag{16}$$

System (3) – (7) has an exact solution in the case with a zero flow rate, at $u = 0$, Eqs. (5) and (13) yield

$$v = \sqrt{2hj_r B/(\lambda\rho)}, \quad j_r = \begin{cases} Ir/(2\pi h r_e^2) & 0 \le r \le r_e, \\ I/(2\pi h r) & r_e < r \le r_2 \end{cases} \tag{17}$$

and Eq. (15) yields

$$dp/dr = \rho v^2/r \tag{18}$$

Integrating Eq. (18) and taking into account Eq. (17), we obtain

$$p = IB/(\pi\lambda)\left(2/r_e - 1/r_2\right) \tag{19}$$

4 CALCULATION RESULTS

As the dimensionless turbulent drag coefficient is determined in this work from comparisons of the calculated pressure with the experimental value $p_{\exp}(IB,G) = P_{calc}(IB,G,\lambda)$, we can process an appropriate set of experimental results and obtain an approximating expression $\lambda = \lambda(IB, G)$, which will allow us to perform calculations in a unified manner within the entire range of admissible parameters of the facility. Note that the pressure at the pump exit in the case with $r_1/r_e \sim 1$, which is the optimal ratio, changes weakly with increasing channel length, and the results obtained for the model facility will be also valid for the full-scale CCP.

Fragments calculation of the drag coefficient on the basis of the experimental data [1] presented in Figures (2) – (3). The experimental MHD cell had the following geometric parameters: r_1 = 0.4 cm, r_2= 4 cm, r_e = 0.4 cm and h = 0.8 cm.

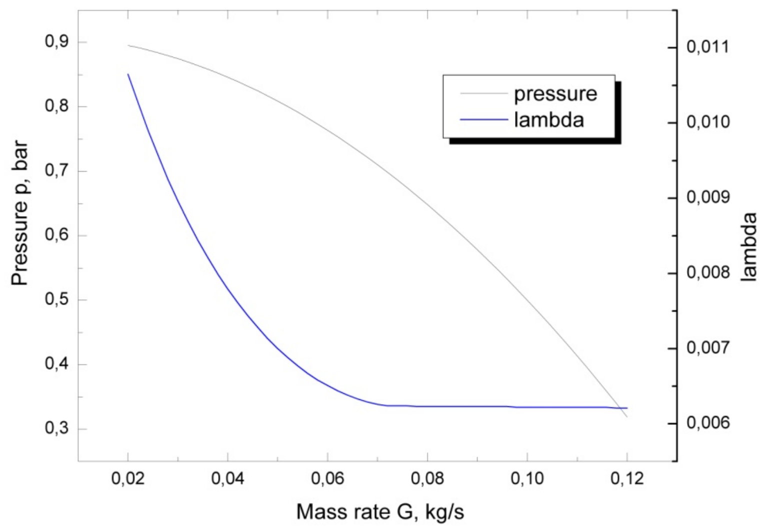

Figure 2: Drag coefficient versus the flow rate at *IB* = 0.15 kATl

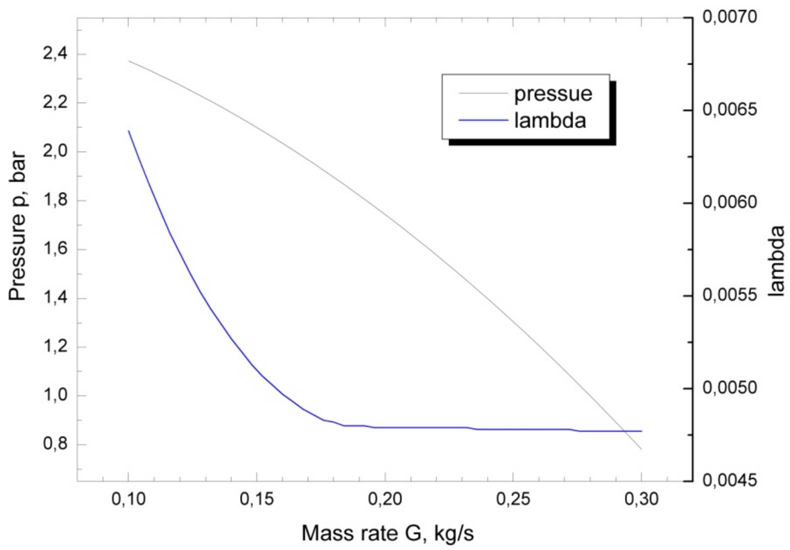

Figure 3: Drag coefficient versus the flow rate at *IB* = 04.5 kATl

The calculations were performed for the Rose's alloy. In calculating the error in the determination of the pressure is not exceeded 1%. It is seen that with increasing flow rate drag coefficient decreases and goes on the shelf. Its absolute value decreases with increase in the parameter IB. Figures 4, 5 shows the distributions of the azimuthally component of velocity along the channel for G = 0.1 kg/s and *IB* 0.15, 0.45 kA·Tl respectively.

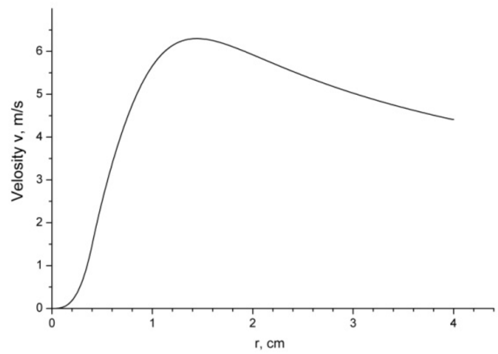

Figure 4: Distribution of the azimuthal velocity along the channel for G = 0.1 kg/s *IB* = 0.15 kATl

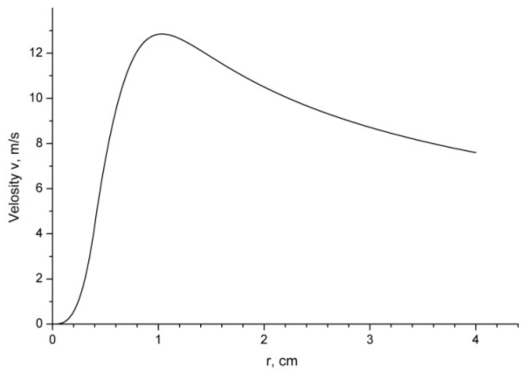

Figure 5: Distribution of the azimuthal velocity along the channel for G = 0.1 kg/s *IB* = 0.45 kATl

Maximum velocity is in the first half of the channel, increases with *IB* and moves to the top channel. Figure 6 shows the pressure as a function of the parameter *IB* in the regime with a zero flow rate, the point on the curve corresponds to the experimental value.

In accordance with the formula (19) this the relationship is linear. For the same regime in Figure 7 presents calculations, based on the experimental results of [2]. The experimental MHD cell had the following geometric parameters: r_1 = 0.5 cm, r_e = 0.8 cm, r_2 = 6 cm, and h = 1 cm.

4 CONCLUSIONS

- A simple mathematical model is developed, which ensures an adequate description of operation of an MHD centrifugal conductive pump as a function of the set of the governing parameters: current, magnetic induction, and flow rate.
- The implementation of this model in a computer code can be used to design pumps of

this type and to control their operation in the technological process.

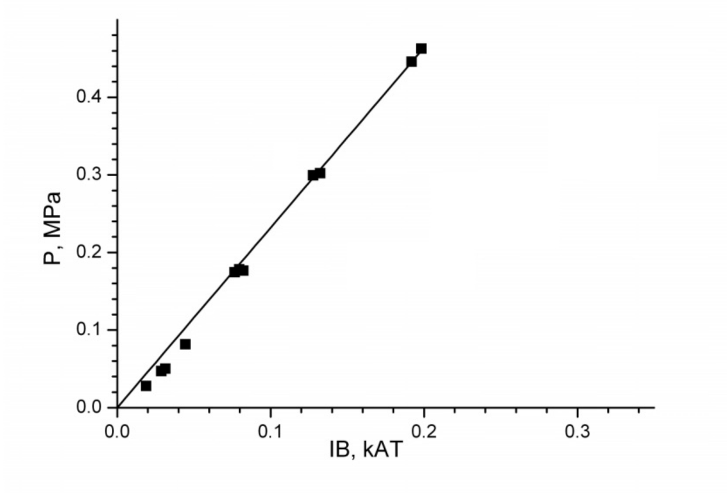

Figure 6: Comparative characteristics of the calculated and experimental data [1] in the regime with the zero flow rate. The solid curves show the calculated results; the squares are the experimental data

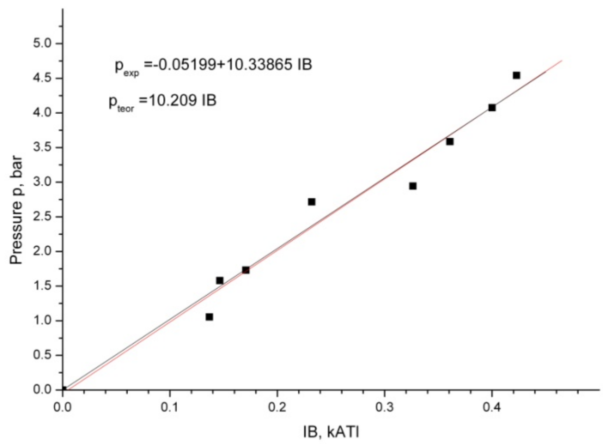

Figure 7: Comparative characteristics of the calculated and experimental data [2] in the regime with the zero flow rate. The solid curves show the calculated results; the squares are the experimental data

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