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USE OF MULTIBODY SYSTEM APPROACH FOR TORQUE AND DRAG ANALYSIS OF LONG DRILL STRINGS

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Abstract. The paper focuses on application of multibody system approach for the torque and drag analysis of long drill strings. The approach is used to get improved results as compared with those obtained by soft-string models. The analysis is carried out by integration of equations of motion of the drill strings till reaching equilibrium under externally applied loads corresponding to conditions of drilling operations. A small value of kinetic energy serves as a criterion for finishing the simulation. To increase the convergence, high damping is applied to the drill string. Such damping is absent in the real well bores. It leads to speedy fall of the kinetic energy in a few seconds after start of the integration. However, low frequency oscillations with very small value of the energy appear if length of the drill string is several kilometers. As the result, the equilibrium position is reached very slowly. In order to overcome this effect, the three-step algorithm is proposed. It is based on the solution of static equations of the drill string for calculation of initial conditions that is used for dynamic simulation. Using the algorithm, torque and drag analysis of the drill string up to ten kilometers in length takes several minutes on modern computers. Some simulation aspects, such as efficiency of parallel computations are also considered in the paper.

1 INTRODUCTION

Computer simulation is a widespread approach for torque and drag analysis of drill strings. It is the important phase of planning of wells and selection of proper parameters of drilling operations. Dynamic simulation of long drill strings is related to certain difficulties. One of those is large number of degrees of freedom of the models. The second one is stiff equations of motion that require special integration methods. For calculations of very long drill strings, so called soft-string models that ignore bending stiffness are commonly used [1, 2, 3]. It is assumed that a drill string contacts with a well bore in each point along the full length. Such models produce quite accurate results for axial loads and torques if the drill string does not buckle. However, it cannot compute real contact forces between the drill string and wellbore, cannot simulate buckling and real drill string rotation.

In this paper, a multibody system approach to dynamical simulation of long drill strings is suggested. The aproach is described in details in paper [4]. A drill string is simulated as a set of uniform flexible beams connected via viscous-elastic force elements. Each beam can undergo arbitrary large displacements as a rigid body but its flexible displacements due to elastic deformations are assumed to be small. The methods of floating frame of reference for flexible bodies and component mode synthesis are used for modelling of dynamics of the beams. Parameters of the coupling force elements are calculated automatically based on stiffness and inertia characteristics of the connected beams. The approach allows simulating the dynamics of drill strings including such processes as vibrations, rock cutting, friction, hydraulics as well as buckling and post-buckling behaviour. Its use for the torque and drag analysis is considered below.

2 MATHEMATICAL MODEL

2.1 Equations of motion

Equations of motion of a flexible beam can be written in the following general form:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{k} = \mathbf{f}_g + \mathbf{f}_a + \mathbf{f}_c - \mathbf{C}\mathbf{q} - \mathbf{D}\dot{\mathbf{q}},$$

where $\mathbf{q}$ is the column vector of generalized coordinates, $\mathbf{k}$ is the column vector of generalized inertia forces; $\mathbf{f}_g$, $\mathbf{f}_a$, $\mathbf{f}_c$ are the generalized gravity, applied and reaction forces; **M**, **C** and **D** are the mass, stiffness and damping matrices of the beam.

Flexible displacements of the beam are described using the modal approach. Its application to dynamic simulation of drill strings is considered in [4]. In this section, the main ideas of the method are briefly recalled for completeness.

The local coordinate system (CS1) is linked to the body (Figure 1). Position $\mathbf{r}_k$ of an arbitrary point *K* in the global coordinate system (CS0) can be presented as the sum of the radius-vector $\mathbf{r}_1$ of the origin CS1 relative to CS0 and the radius-vector $\mathbf{r}_{k1}$ of point *K* in CS1.

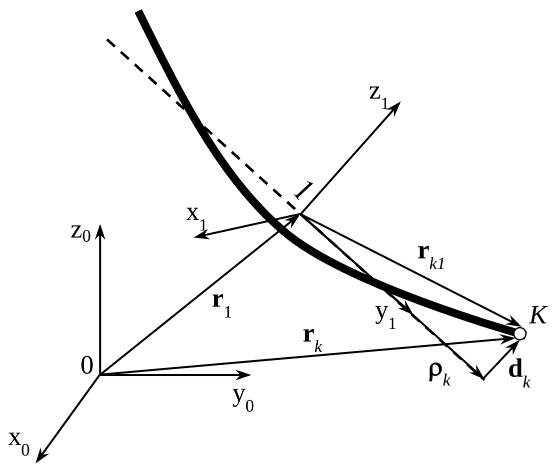


Figure 1: Position of an arbitrary point *K* of a flexible body

Vector $\mathbf{r}_{k1}$ is as a sum of vector $\boldsymbol{\rho}_k$ of the point coordinates of the undeformed body, constant in CS1, and vector $\mathbf{d}_k$ of the flexible displacement.

Then the position of point K in CS0 can be found as follows:

$$\mathbf{r}_k^{(0)} = \mathbf{r}_1^{(0)} + \mathbf{A}_{01}(\boldsymbol{\rho}_k^{(1)} + \mathbf{d}_k^{(1)}),$$

where the superscript is the index of coordinate systems in which vectors are presented, $\mathbf{A}_{01}$ is the rotation matrix.

Flexible displacements of the body are presented using the finite element method and the modal approach:

$$\mathbf{x} = \sum_{j=1}^{H} \mathbf{h}_j w_j = \mathbf{H}\mathbf{w},$$

where $\mathbf{x}$ is the N×1 column vector of nodal degrees of freedom (DOF), N is the number of DOF, $\mathbf{h}_j$ are the modes of the flexible body, w_j are the *modal coordinates* of a flexible body, H is the number of modes used, $\mathbf{H}$ is the N×H *modal matrix*.

Thus, the set of generalized coordinates of the beam includes six coordinates for the description of motion of the local frame and H modal coordinates related to flexible modal displacements.

Basic matrix $\mathbf{H}$ is created in accordance with component modes synthesis method and can be represented in the following block form:

$$\mathbf{H} = \begin{bmatrix} \mathbf{Y} & \mathbf{S} \\ \mathbf{0} & \mathbf{E} \end{bmatrix},$$

where columns of the matrix $\mathbf{Y}$ are the fixed-interface normal modes, columns of the matrix $\mathbf{S}$ are the constraint modes and $\mathbf{E}$ is the unity matrix. The number of the constraint modes is equal to $6\times n_i$, where n_i is the number of the interface nodes. Number of normal modes is selected by a researcher. Then, matrix $\mathbf{H}$ is transformed and six modes are removed to exclude motion of the beam as rigid body relative to the local frame of reference [4].

Note that in most cases of simulation of drill strings, use of constrained modes only is enough for achievement of acceptable results. Thus, the minimal number of the general coordinates of the beam is equal to 12.

2.2 Numerical method and parallel computations

In order to overcome limitations related to the large number of degrees of freedom and to increase effectiveness of the simulation, the algorithm for parallel generation and numerical solution of equations of motion are developed. Parallel computations for multi-core processors are implemented according to the fork-join method.

The equations of motion of drill strings are normally stiff. The main cause of the equation stiffness is stiff forces such as forces in the connections of the beams and contact forces between the drill strings and well bores. A force is considered to be stiff if its value significantly changes under small variations of relative positions and velocities of interacting bodies.

The analytic expressions for Jacobian matrices of stiff forces are derived and applied within implicit Park method to increase the integration step size [5].

2.3 Model of contact between well bore and drill string

Contact interactions are simulated by specialized *Circle-Cylinder* contact force elements (Figure 2).

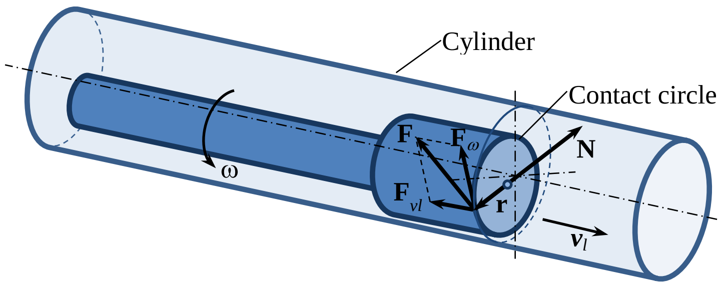
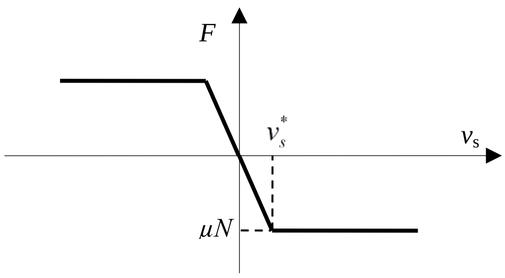


Figure 2: Circle-Cylinder contact model

The contact force element uses a compliant contact between a circle and a cylinder, which axis is set by a smooth curve. The diameter of the cylinder can vary to describe the dependency of cross section of a hole on its depth. The normal force **N** depends on the depth of penetration and damping rate in the contact. The model of the friction force can be described as follows.

$$\mathbf{F} = \begin{cases} -\mu N \dfrac{\mathbf{v}_s}{\|\mathbf{v}_s\|}, \|\mathbf{v}_s\| > v_s^* \\ -\mu N \dfrac{\mathbf{v}_s}{v_s^*}, \|\mathbf{v}_s\| \le v_s^* \end{cases}, \quad \mathbf{v}_s = \mathbf{v}_l + \boldsymbol{\omega} \times \mathbf{r} \tag{1}$$

where μ is the dynamic friction coefficient, N is the absolute value of the normal force, $\mathbf{v}_s$ is the vector of sliding velocity that can be presented as the sum of the longitudinal speed $\mathbf{v}_l$ of the drill string and the tangent velocity of the contact point that equal to $\boldsymbol{\omega} \times \mathbf{r}$, v_s^* is the small empirical value of sliding velocity. If the sliding velocity as not small, the classical model of friction is used, else the viscous damping is considered (see the figure below). In the simulation the v_s^* value is equal to $0.01 \times r$, where r is the radius of the contact circle.



The contact force element is added to each of the points of beam connections. The diameter of the contact circle is equal to the greater of two outer diameter of adjacent beams.

The dynamic simulation for torque and drag analysis can be carried out by two ways. The first of them is modelling of real rotation of the drill string. Then the penetration and angular speed of the drill string segments will be different due to transient processes appear and stationary motion is observed after long simulation time. The second one is use of the values of v_l and ω as parameters of a drilling operation for direct calculation of the forces $\mathbf{F}_v$ and $\mathbf{F}_\omega$. The results presented in this paper are obtained by the second way.

3 PROCEDURE OF TORQUE AND DRAG ANALYSIS

3.1 Integration of equations of motion of a drill string

Torque and drag analysis of the drill string is reduced to the search of equilibrium conditions under given loads. A small enough value of the kinetic energy of the drill string is the criterion to finish the integration of the equations of motion. To increase the convergence of the calculations, unrealistic high (additional, not present in reality) internal and external damping forces are applied to the drill string. It can lead to the problems of simulation of very long drill strings when the processes with very low frequencies are observed. For example, torsional vibrations of the drill string of six kilometers in length are not completely damped when the threshold value of the kinetic energy equal to 1 Joule is reached. Therefore, values of axial torque are wrong to the end of simulation. Further decrease of the boundary value of the kinetic energy leads to multiple increase of the simulation time when the calculations can take few hours.

3.2 Algorithm of multistep torque and drag analysis

To speed-up the convergence to the equilibrium position, a three-step algorithm has been developed. The basic idea for the algorithm is a fast calculation of the initial position of a drill string that is near to equilibrium one.

At the first step, initial values of displacements and rotation angles of drill string parts are calculated using static equations. The drill string is considered as a chain of rigid segments connected via joints at their ends. Cross-sections, lengths and material properties of the segments correspond to the parameters of the drill string sections. The forces and moments in the interconnections of the segments are obtained from equilibrium conditions of each segment sequentially segment-by-segment. The boundary conditions for the first segment are defined by parameters of the drilling operation.

Flexible displacements and rotation angles of segments of the drill string expressed in the modal coordinates are computed using the results of the static solution. At the second step, the calculated longitudinal displacements and forces are applied to the full dynamic model of the drill string and equilibrium conditions are computed by integration of equations of motion. Finally, rotation angles and axial torques are applied and then values of all parameters corresponding to the operation conditions are obtained by integration of equations of motion.

Let us consider the steps of the algorithm in details.

3.2.1 Static equations

In this section, the procedure for calculation of all forces acting on the segments of the drill string under given boundary conditions and operation parameters is considered. It is based on the static equations.

A well bore centerline is defined by points in the global coordinate system (CS0). The cubic spline is used for approximation of the axis. The natural coordinate system in the point *i* on the centerline is specified as follows (Figure 3):

- $\mathbf{Z}_i$ axis is directed along the tangent τ to the well bore centerline against the direction of increase of the depth;
- $\mathbf{X}_i$ axis is normalized vector product of $\mathbf{Z}_0$ and $\mathbf{Z}_i$; $\mathbf{X}_i = \mathbf{Z}_0 \times \mathbf{Z}_i$. If the product is zero (angle between $\mathbf{Z}_0$ and $\mathbf{Z}_i$ equal to 0), $\mathbf{X}_i$ is parallel to $\mathbf{X}_0$;
- $\mathbf{Y}_i$ is normalized vector product of $\mathbf{Z}_i$ and $\mathbf{X}_i$; $\mathbf{Y}_i = \mathbf{Z}_i \times \mathbf{X}_i$.

For the torque and drag analysis of a drilling operation, end points of the rigid segments are placed on the well bore centerline (Figure 3). Forces acting on segments of the drill string are calculated segment-by-segment from the end with defined boundary condition to another one. For example, if axial force and torque that act on the bit are set, calculations are carried out from the bit to the hook.

In general case, the following loads can be applied to the segment *i* (Figure 4):

- $\mathbf{Q}_1$, $\mathbf{Q}_2$ are the lateral forces at the end points 1 and 2;
- $\mathbf{T}_1$, $\mathbf{T}_2$ are the axial forces;
- $\mathbf{M}_{1b}$, $\mathbf{M}_{2b}$ are the bending moments;
- $\mathbf{M}_{1t}$, $\mathbf{M}_{2t}$ are the torques;
- $\mathbf{G}$ is the weight of the segment in the fluid;
- $\mathbf{F}_c$ is the centrifugal force acting on the segment if the drilling operation with rotation is simulated; it is computed using the expressions presented in [6];
- $\mathbf{F}_b$ is the buckling normal force arising if drill string loses stability helically; the expressions for the force value are given in [7, 8];
- the friction force $\mathbf{F}_f$ and the friction moment $\mathbf{M}_f$. They act in the end points with contact circles. The radius of the contact circle is equal to the bigger radius of two adjacent sections (one uniform section can includes several segments).

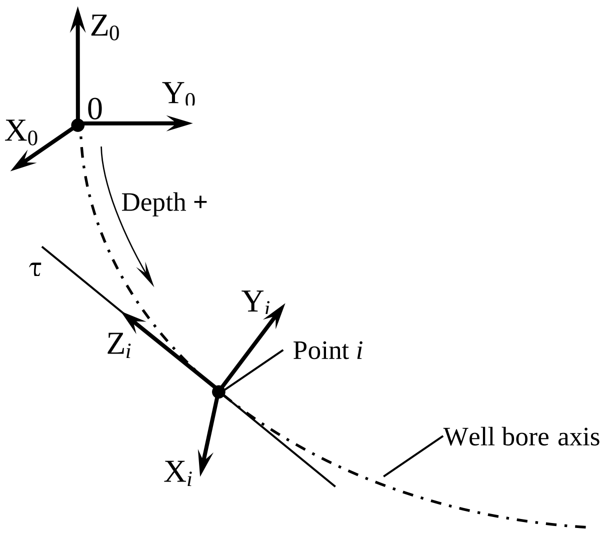


Figure 3: Natural coordinate system in the point *i* on the centerline of a well bore

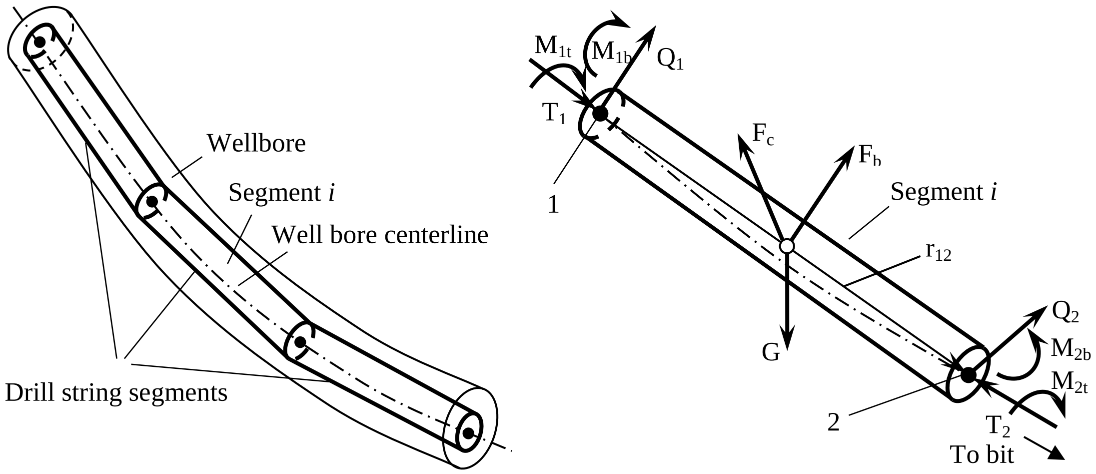


Figure 4: Arrangement of end points of the drill string on the wellbore centerline and the forces acting on segment *i*.

Let us consider some expressions and equations for calculation of the values mentioned above. The bending moments can be optionally computed. Their values in the end points of the segments are calculated using the relative bending angles α_i between axes of the segments *i-1* and *i*:

$$M_b = EI\alpha_i / L,$$

where *EI/L* is the smallest stiffness of two adjacent segments; *E, I, L* are correspondingly Young modulus, the bending inertia moment and the length of the segment with smallest stiffness.

The lateral force in the bottom point 2 of segment *i* is calculated using the following equilibrium condition:

$$\mathbf{M}_1 = \sum_j \mathbf{m}_1(\mathbf{F}_j) = \underbrace{0.5 \cdot \mathbf{r}_{12} \times (\mathbf{F}_c + \mathbf{G} + \mathbf{F}_b) + \mathbf{M}_{b1} + \mathbf{M}_{b2}}_{\mathbf{M}_1^*} + \mathbf{r}_{12} \times \mathbf{Q}_2 = \mathbf{M}_1^* + \mathbf{r}_{12} \times \mathbf{Q}_2 = 0,$$

The lateral force $\mathbf{Q}_2$ is calculated as the sum of the projections on $\mathbf{X}_2$ and $\mathbf{Y}_2$:

$$\mathbf{Q}_2 = \mathbf{Q}_{2x} + \mathbf{Q}_{2y},$$

$$\mathbf{Q}_{2x} = \mathbf{e}_{2x} \cdot (-\mathbf{e}_{2y} \times \mathbf{M}_1^*) / r_{12x}^*, \ \mathbf{Q}_{2y} = \mathbf{e}_{2y} \cdot (\mathbf{e}_{2x} \times \mathbf{M}_1^*) / r_{12x}^*,$$

where $\mathbf{e}_{2x}, \mathbf{e}_{2y}$ is the unit vectors (orts) along the axes of the natural coordinate systems in point 2, r_{12x}^* is the length of the projection of the vector product $\mathbf{r}_{12}$ and $\mathbf{e}_{2y}$ on $\mathbf{X}_2$:

$$r_{12x}^* = \mathbf{e}_{2x} \cdot (\mathbf{r}_{12} \times \mathbf{e}_{2y}).$$

The friction forces and moments can be calculated by formula (1) using the lateral forces. All calculated loads are applied for the computations of initial displacements and rotation angles of the segments at the first step of the algorithm.

3.2.2 Calculation of initial position of a drill string

Flexible modes of a beam can be separated on the longitudinal, torsional and bending ones. Then the elongation of the drill string under acting loads can be written as the sum of the longitudinal displacements in the connections between the beams and elongations of the beams described by corresponding modal coordinates. Twisting of the drill string can be presented in the same manner. Let us consider the expressions for calculation of mentioned values.

In an initial position, curvilinear coordinates of the origins of the local frames and nodes of the segments correspond to their longitudinal coordinates *z* on the straight drill string where zero value of *z* is the upper point, maximal value *z* is the bit point. The calculations are carried out in accordance with the following expressions (Figure 5):

$$z_{0(i)} = z_{n2(i-1)} + \frac{l_{0(i)} + \Delta l_i}{2} + \Delta z_{c1(i)},$$

$$\Delta z_{c1(i)} = \frac{F_i}{c_{1z(i)}},$$

$$\Delta z_{n1(i)} = \Delta z_{n2(i)} = \frac{\Delta l_i}{2} = \frac{F_i l_{0(i)}}{2E_i A_i},$$

where $z_{0(i)}$ is coordinate of local frame of the segment *i*, $\Delta z_{c1(i)}$ is displacement of the end node 1 of the segment *i* relative to the end node 2 of the segment *i-1*, $\Delta z_{n1(i)}, \Delta z_{n2(i)}$ are the longitudinal flexible displacements of the end nodes of segment *i* reference to the local frame, $l_{0(i)}, \Delta l_i$ are the length of the undeformed segment *i* and its elongation, E_i is Young modulus and A_i is the area of the cross-section of the segment *i.*

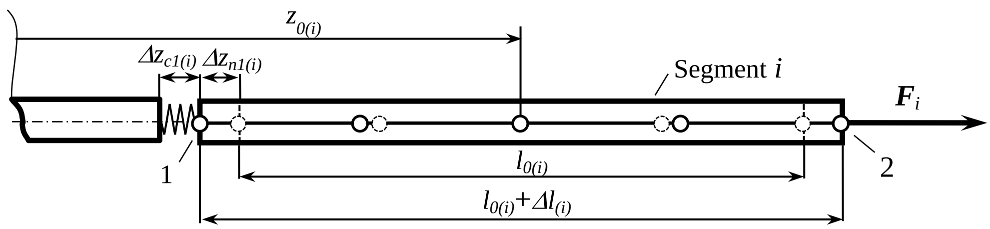


Figure 5: Calculation of the initial positions of the segment *i* with use of the static results

The following assumptions are used for the calculations:

1) the constant longitudinal force acts on each segment; its value is equal to the half-sum of the forces calculated in end nodes 1 and 2:

$$F_i = \frac{F_{i1} + F_{i2}}{2};$$

2) the drill string is elongated due to the first longitudinal mode only.

Note that only one longitudinal mode is usually used in the modal matrix.

Flexible displacements of the nodes are expressed in the modal coordinate. For example, formula for end node 1 can be presented as follows:

$$\Delta z_{n1(i)} = h_{l1(i)}(iz_{n1}) w_{l1(i)}, \text{ therefore } w_{l1(i)} = \frac{\Delta z_{n1(i)}}{h_{l1(i)}(iz_{n1})},$$

where $\mathbf{h}_{l1(i)}$ is the first longitudinal mode of the segment, $w_{l1(i)}$ is the modal coordinate corresponding to the first longitudinal mode, iz_{n1} is the index of z coordinate of the end node 1 in the list of nodal coordinates. Thus, the initial values for the generalized coordinates $z_{0(i)}$ and $w_{l1(i)}$ are calculated.

The torsion initial conditions are calculated in the same way.

After the second step, equilibrium position of the drill string without taking into account the rotation angles is computed. The drill string bends depending on the shape of the well bore and acting loads. In the third step, each beam of the bended drill string is rotated along its longitudinal axis. In order to leave the drill string in the state near to equilibrium, the modal coordinates of the beams can be recalculated to turn back the bending plane. All bending modes of the uniform beams are pairwise. The pairwise modal coordinates with the indices *j* and *j+1* corresponding to the bending modes are transformed using the following expressions:

$$\begin{aligned} w_j &= w_j \cos(\varphi_z) + w_{j+1} \sin(\varphi_z) \\ w_{j+1} &= -w_j \sin(\varphi_z) + w_{j+1} \cos(\varphi_z) \end{aligned},$$

where φ_z is the rotation angle of the local coordinate system of the beam around local axis *z*.

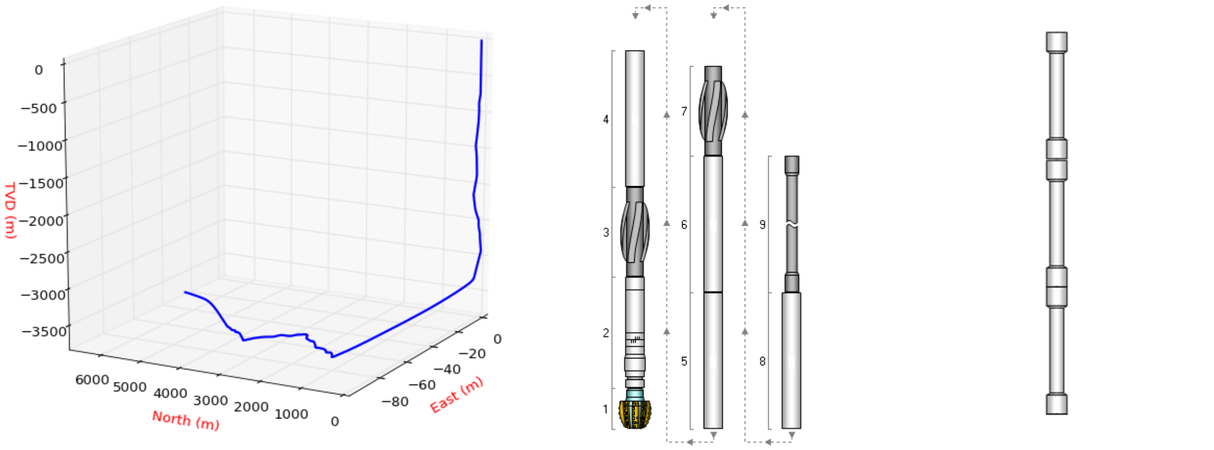
4 SIMULATION EXAMPLE

In this section, the efficiency of simulation of the field case with the long drill string using the proposed algorithm is considered. The shape of the well bore centerline and outline of the drill string are presented in Figure 6. The drill string consists of the bottom hole assembly and 1058 drill pipes. The operation parameters are given in table 1, the model parameters are in table 2. The numerical experiments are carried out using Intel® Core™ i7 processor 3.4GHz.

Dynamic simulation without setting initial conditions in accordance with the proposed algorithm leads to the following results. Kinetic energy of the drill string decreases from 315 kJ to 3 kJ during one and a half seconds after the simulation start. Then, decay of the kinetic energy slows down significantly. After 170 seconds of simulation time, the value of the kinetic energy is equal to 30 Joules (Figure 7). The low frequency oscillations are observed. To that moment, the bit is twisted around the longitudinal axis on 336 radians (more than 53 revolutions). Required time for the calculations exceeds three hours. The torques acting on the beams of the drill string does not achieve its true value.

If initial conditions calculated by static equations are set, the equilibrium position is found in 18 seconds of simulation time. It takes about 8 minutes of real time if 3 threads of multicore processor are used for the computations. There are no low frequency oscillations in the graph of kinetic energy (Figure 8). The subsequent increase of the thread number does not lead to speed-up of the simulation. The efficiency of the parallel calculations is shown in table 3.

The specialized software including soft-string and multibody models has been developed for torque and drag analysis of the drill strings. In the presented case, the results obtained by both of approaches are very close (Figure 9). The graphs of the axial load are practically the same.



a) well survey 3D view b) bottom hole assembly c) 1058 x 4", drill pipe, H90, E75, IU, 11.85 ppf

Figure 6: Three-dimensional view of the well bore centerline and outline of the drill string in the example

Table 1: Operation parameters for rotary drilling

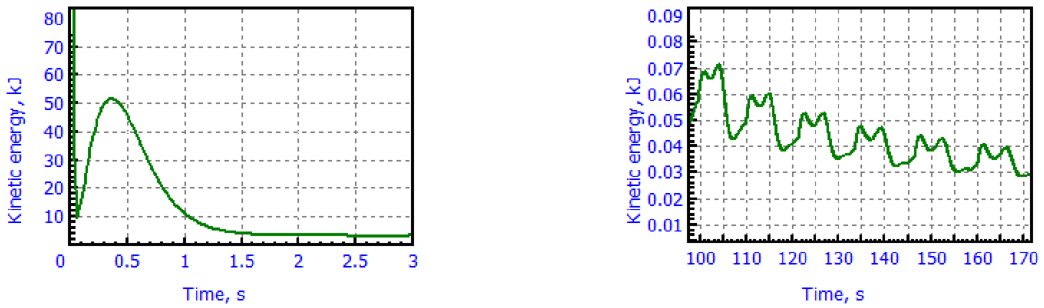
Parameter	Value in Imperial units	Imperial unit	Value in SI units	SI unit
Bit depth	32 991	ft	10 055.7	m
Weight on bit	2.25	kip	10	kN
Torque on bit	3.69	kip×ft	5	kN×m
Rate of penetration	98.43	ft/hr	30	m/hr
Rotating speed	50	rpm	50	rpm

Table 2: Model parameters

Parameter	Value
Number of beams	2 189
Number of degrees of freedom	26 304
Total mass of the drill string, kg	191 761
Maximal dogleg of the well bore, degrees/100ft	14.46
Maximal curvature of the well bore, 1/m	0.0077
Fluid density, kg/m^3	1138.35
Fluid dynamic viscosity, Pa·s	0.07

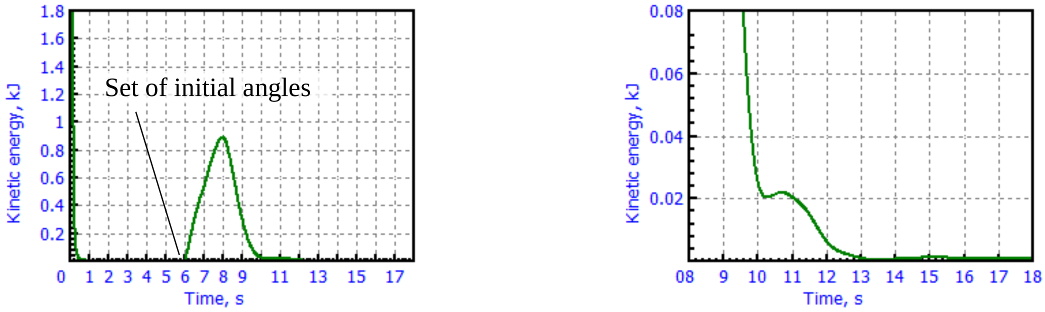
Table 3: Efficiency of parallel computing

Number of threads	CPU time, min:s	Speeding up factor
1	14:05	1.00
2	9:56	1.41
3	8:03	1.74



a) start period of the integration b) finish period of the integration

Figure 7: Kinetic energy of the drill string without set of initial conditions from static analysis



a) start period of the integration b) finish period of the integration

Figure 8: Kinetic energy of the drill string with set of initial conditions from static analysis

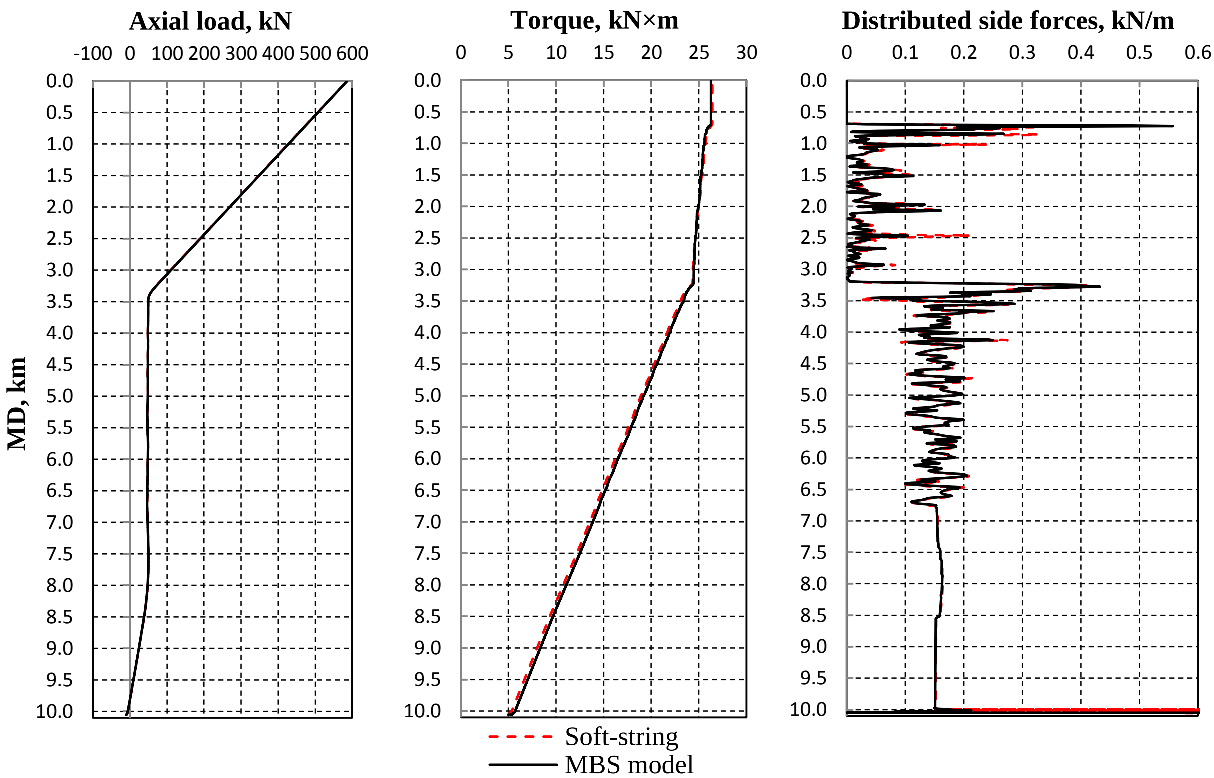


Figure 9: Comparision of results obtained by using soft-string and multibody system approach

5 CONCLUSIONS

- Use of multibody system approach for torque and drag analysis of long drill string is encountered with the problem of very slow damping of kinetic energy during the integration of the equations of motion of a drill string. The calculations can take several hours without success because of low frequency oscillations with small energy are observed.
- The three-step algorithm is suggested for the use within the scope of multibody system approach for torque and drag analysis of the long drill strings. In accordance with the algorithm, an initial position of the drill string near to the equilibrium is calculated based on solution of static equations. Longitudinal displacements of the beams and its rotation angles are set sequentially with the search of the intermediate equilibrium position. Using this algorithm allows decreasing the calculation time to several minutes on modern computers.
- The application of parallel calculations increases the efficiency of simulation. Speedup with the use of three threads is 1.73 as compared with the calculating in a single thread. Note, that it is less than acceleration which is usually reached by using parallel calculations on multi-thread computers. The typical ratio of the computational efficiency is about 2.5. The additional analysis of the calculating process is required.

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REFERENCES

[1] C.A. Johancsik, D.B. Freisen, and R. Dawson, Torque and Drag in Directional Wells – Prediction and Measurement. *Journal of Petroleum Technology* (1984) June: 987-992.

[2] S.A. Mirhaj, E. Kaarstad, B.S. Aadnoy, Improvement of Torque and Drag Modeling in Long-Reach Wells. *Modern Applied Science* (2011) Vol. **5**, No. 5, October: 10-28.

[3] V.S. Tikhonov, A.R. Nurgaliev, A.I. Safronov, L. Ring, Development of new program for torque-and-drag analysis of drill strings in the well bores with arbitrary profiles. *Proceedings of the VIII International Conference «Ashirov's Reading»* (2011), Vol *2*: 4-10 (in Russian).

[4] D. Pogorelov, G. Mikheev, N. Lysikov, L. Ring, R. Gandikota and N. Abedrabbo, A Multibody System Approach to Drill String Dynamics Modeling. *2012 Proceedings of the ASME 11th Biennial Conference on Engineering Systems Design and Analysis (ESDA2012)* (2012), Vol. **4**: 53-62.

[5] D. Yu. Pogorelov. Jacobian matrices of the motion equations of a system of bodies. *Journal of Computer and Systems Sciences International* (2007), **46**, Nr. 4: 563–577.

[6] G. Heisig, M. Neubert Lateral Drillstring Vibrations in Extended-Reach Wells. *Proceedings of the 2000 IADC/SPE Drilling Conference* (2000), New Orleans, paper No. IADC/SPE 59235.

[7] J. Wu. H.C. Juvkam-Wold. Coiled Tubing Implication in Drilling and Completing Horizontal Wells. *SPE Drilling & Completion* (1995), March: 16-21.

[8] J. Wu, H.C. Juvkam-Wold, The Effect of Wellbore Curvature on Tubular Buckling and Lockup, *Transactions of the ASME. Journal of Energy Resources Technology* (1995), Vol. 117, September: 214-218.