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ESTIMATION OF THE GLOBAL OPTIMALITY FOR MULTIPLE TUNED MASS DAMPER SYSTEMS USING ORDER STATISTICS

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Abstract. In the design of passive tuned mass damper (TMD) on the vibration control performance of spatial structures, we have to decide the both of a spatial arrangement and characteristics. To find an optimum design of the system is formulated as Mixed-Integer Programming (MIP) problem. It is difficult to obtain the exact global optimum solution in general form. We present a method which can guarantee to find a near-optimal solution with pre-assigned accuracy. The key concept of the method is random search based on prediction by order statistics. Through a numerical example, we investigate the applicability and effectiveness of the method to the design of TMD systems.

1. INTRODUCTION

Spatial structures generally possess little inherent damping. These structures tend to vibrate strongly in the normal direction of the curved roof. It has been reported that non-structural components; the plumbing, the ceiling systems and etc., were damaged by such vibrations, which caused the collapse and fall of the components, and injured and killed many people during the 2011 off the Pacific coast of Tohoku Earthquake [1]. It is known that passive tuned mass damper (TMD) can be effectively used for the vibration control. In particular, multiple TMD (MTMD) is robust when the system is excited by a wideband random disturbance [2,3]. This means that the effective use of TMDs enables to prevent or reduce such damages. The concept of the spatially distributed MTMD system is illustrated in Fig. 1 [3].

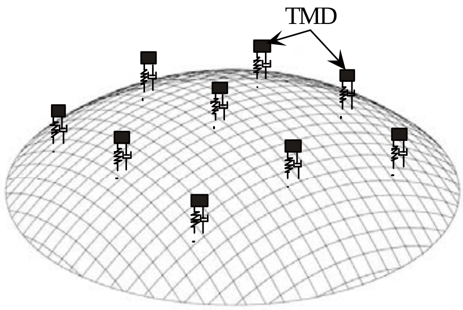

Figure 1: The spatially distributed TMD system for spatial structures

In this paper, we study an influence of placement of TMDs, i.e., MTMD systems, on the vibration control performance. The first explicit formulation of optimal TMD parameters to undamped structure have been derived by Den Hartog [4]. Many studies treated optimization of TMD system [2-7]. We have to decide the both of a spatial arrangement and parameters of the TMDs, which are frequency and damping characteristics. The possible locations for TMDs can be modelled as discrete variables, and the characteristics of the ones are treated as continuous variables. Thus, to find an optimum design of the MTMD system on the spatial structures is formulated as Mixed-Integer Programming (MIP) problem [5].

For solving this problems, some techniques are available. The ones are standard combinatorial optimization methods. In general, this type of the methods has rigorous mathematical backgrounds and can find exact global optimum solution; however, the methods need extremely high computational cost depending on the size of the problem. Heuristics are other choices; genetic algorithms and simulated annealing are representative ones in this category. It has been often reported that heuristics quickly find several near-optimal solutions and hence have become popular recently, e.g., [5]. This must be useful approach but we do not fully agree with the positive opinion because some of them lack mathematical backgrounds. How accurate and/or efficient the solutions are?

We attempt to mix two methods to design of the spatially distributed MTMD system. The one is enumeration method with deterministic procedures, which is used for decision on a spatial arrangement of TMDs. The other is random search method (RS) as a probabilistic approach [8,9], which is used for decision on characteristics of TMDs. The key concept of RS is prediction by order statistics. The method can guarantee the accuracy of a solution in terms of global optimality with a pre-assigned probability. The theoretical result indicates use of relatively small samples is enough to predict the large number of future samples. Thus, the proposed method is useful to estimate how accurate we obtain the solution in terms of global optimality.

2. PROBLEM FORMULATION

This section summarizes the problem formulation. This formulation is not new. The reader may refer to related studies, e.g. [6], for further details. Consider a multi-degree-of-freedom (MDOF) structure with TMDs. Our interest focuses on design of the TMD parameters. The parameters are mass, damping, and stiffness of the ones, which are respectively denoted by

$$\tilde{m} = \left\{ m_j ; j = 1, \cdots, N_t \right\},\ \tilde{c} = \left\{ c_j ; j = 1, \cdots, N_t \right\},\ \tilde{k} = \left\{ k_j ; j = 1, \cdots, N_t \right\}, \tag{1}$$

where N_t is number of the TMDs. The differential equations governing the behavior of the system subjected to ground excitation can be written as

$$\left. \begin{aligned} \boldsymbol{M}(\tilde{m})\ddot{\boldsymbol{u}}(t) + \boldsymbol{C}(\tilde{c})\dot{\boldsymbol{u}}(t) + \boldsymbol{K}(\tilde{k})\boldsymbol{u}(t) = -\boldsymbol{M}(\tilde{m})\boldsymbol{r}a_g(t), \\ \boldsymbol{z}(t) = \boldsymbol{D}_1\dot{\boldsymbol{u}}(t) + \boldsymbol{D}_2\boldsymbol{u}(t) \end{aligned} \right\} \tag{2}$$

where $\boldsymbol{M}(\tilde{m}), \boldsymbol{C}(\tilde{c}), \boldsymbol{K}(\tilde{k}) \in \mathbb{R}^{N\times N}$ are mass, damping and stiffness matrices of the system, respectively; $\boldsymbol{r} \in \mathbb{R}^N$ is influence vector to the ground motion; and $\boldsymbol{D}_1 \in \mathbb{R}^{p\times N}$ and $\boldsymbol{D}_2^{p\times N}$ are velocity and displacement output matrices, respectively. The degree-of-freedom of the system is denoted by N, and the number of the outputs is denoted by p. Time-dependent vectors $\ddot{\boldsymbol{u}}(t), \dot{\boldsymbol{u}}(t), \boldsymbol{u}(t) \in \mathbb{R}^N$ are acceleration, velocity and displacement vectors relative to the ground, respectively; and $\boldsymbol{z}(t) \in \mathbb{R}^p$ is output vector; $a_g(t)$ is the ground acceleration. The mass, damping and stiffness matrices are assumed to be expressed as follows:

$$\boldsymbol{M}(\tilde{m}) = \boldsymbol{M}_0 + \sum_{j=1}^{N_t} m_j \boldsymbol{M}_j,\ \boldsymbol{C}(\tilde{c}) = \boldsymbol{C}_0 + \sum_{j=1}^{N_t} c_j \boldsymbol{C}_j,\ \boldsymbol{K}(\tilde{k}) = \boldsymbol{K}_0 + \sum_{j=1}^{N_t} k_j \boldsymbol{K}_j, \tag{3}$$

where $\boldsymbol{M}_0, \boldsymbol{C}_0, \boldsymbol{K}_0 \in \mathbb{R}^{N\times N}$ are mass, damping and stiffness matrices corresponding to the structure; and $\boldsymbol{M}_j, \boldsymbol{C}_j, \boldsymbol{K}_j \in \mathbb{R}^{N\times N}$ $(j = 1, \cdots, N_t)$ are ones corresponding to the TMDs. The equation (2) may be rewritten to a state space equation:

$$\left. \begin{aligned} \boldsymbol{E}(\tilde{m})\dot{\boldsymbol{x}}(t) = \boldsymbol{A}(\tilde{m}, \tilde{c}, \tilde{k})\boldsymbol{x}(t) + \boldsymbol{b}(\tilde{m})a(t), \\ \boldsymbol{z}(t) = \boldsymbol{D}\boldsymbol{x}(t) \end{aligned} \right\} \tag{4}$$

where $\boldsymbol{x}(t) = (\dot{\boldsymbol{u}}(t)^T\ \boldsymbol{u}(t)^T)^T \in \mathbb{R}^{2N}$ is state vector and

$$\boldsymbol{E}(\tilde{m}) = \begin{pmatrix} \boldsymbol{M}(\tilde{m}) & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{M}(\tilde{m}) \end{pmatrix} \in \mathbb{R}^{2N\times 2N},\ \boldsymbol{A}(\tilde{m}, \tilde{c}, \tilde{k}) = \begin{pmatrix} -\boldsymbol{C}(\tilde{c}) & -\boldsymbol{K}(\tilde{k}) \\ \boldsymbol{M}(\tilde{m}) & \boldsymbol{O} \end{pmatrix} \in \mathbb{R}^{2N\times 2N},$$

$$\boldsymbol{b}(\tilde{m}) = \begin{pmatrix} -\boldsymbol{M}(\tilde{m})\boldsymbol{r} \\ \boldsymbol{0} \end{pmatrix} \in \mathbb{R}^{2N\times 2N},\ \boldsymbol{D} = \begin{pmatrix} \boldsymbol{D}_1 & \boldsymbol{D}_2 \end{pmatrix} \in \mathbb{R}^{p\times 2N}.$$

We formulate TMD's design problem to find TMD's parameter (m_t, c_t, k_t) which minimizes the real scalar γ, such that inputs and outputs of the system (4) satisfy

$$\begin{gathered}\int_{t=0}^{\infty} \|\boldsymbol{z}(t)\|^2 \, dt \le \gamma^2 \int_{t=0}^{\infty} \|a_g(t)\|^2 \, dt \\ \text{for all } a_g(t) \text{ such that} \int_{t=0}^{\infty} \|a_g(t)\|^2 \, dt < \infty.\end{gathered} \tag{5}$$

When a harmonic excitation $a_g(t) = a_0 \, \mathrm{e}^{i\omega t}$, where $a_0, \omega \in \mathbb{R}$ is the amplitude and the angular frequency, respectively, and $i = \sqrt{-1}$, the frequency response function matrix is given by

$$\boldsymbol{G}(i\omega) = \boldsymbol{D}\left(i\omega \boldsymbol{E}(\tilde{m}) - \boldsymbol{A}(\tilde{m}, \tilde{c}, \tilde{k})\right)^{-1} \boldsymbol{b}(\tilde{m}). \tag{6}$$

We can link the H_∞ norm of the system (4) to the scalar γ in inequality (5) as follows:

$$\|\boldsymbol{G}\|_\infty = \sup_{\omega \in \mathbb{R}} \sigma_{\max}\left(\boldsymbol{G}(i\omega)\right) \le \gamma, \tag{7}$$

where $\sigma_{\max}(\cdot)$ is the largest singular value of $(\cdot)$. When the steady-state response vector and ground acceleration are respectively given by $\boldsymbol{z}(t) = \boldsymbol{z}_0(\omega) e^{i\omega t}$ and $a(t) = e^{i\omega t}$, following condition is equivalent to condition (7):

$$\begin{gathered}\|\boldsymbol{z}_0(\omega)\| \le \gamma \quad \text{for all } \omega \in \mathbb{R}, \\ \left(\boldsymbol{z}_0(\omega) = \boldsymbol{D}\left(i\omega \boldsymbol{E}(\tilde{m}) - \boldsymbol{A}(\tilde{m}, \tilde{c}, \tilde{k})\right)^{-1} \boldsymbol{b}(\tilde{m})\right)\end{gathered} \tag{8}$$

Thus we formulate the TMD's design problem to the H_∞ optimal control problem, which can be written as

$$\left.\begin{array}{l}\text{Find} \quad \Theta := (\tilde{m}, \tilde{c}, \tilde{k}) \in \Omega \\ \text{which minimizes to} \quad g(\Theta) := \|\boldsymbol{G}\|_\infty\end{array}\right\} \tag{9}$$

where the parameters of TMD and its feasible set are denoted by $\Theta = (m_t, c_t, k_t)$ and $\Omega \subset \mathbb{R}^{N_t}$, respectively. Furthermore, we assume that $0 < \mathrm{vol}(\Omega) < \infty$, i.e., the parameters are continuous and bounded. Note that the design problem is a non-convex problem and its general form is known as NP-hard. It is extremely difficult to obtain the exact global optimum solution in general form. We may not need such exact solution in many practical situations. Thus various methods have been proposed to obtain reasonably a local solution in some sense. We present a new approach based on RS in next section, which can find approximately optimum solution. The method is also positioned to one of heuristics. However, our presenting method has advantages, which is easy implementation and enables to control the exactness of solutions with pre-assigned accuracy.

3. PURE RANDOM SEARCH AND ORDER STATISTICS

We apply RS to solve Problem (9) for obtaining an approximately optimum solution in some sense. In an algorithm of RS, a sequence of random points $\Theta_1, \Theta_2, \cdots, \Theta_n$ is generated. We will refer to this general scheme as Algorithm 1 (see, e.g., [10]).

Algorithm 1 (general random sampling algorithm)

1. Generate a random point Θ_1 according to a probability distribution P_1 on Ω; evaluate the objective function at this point; set iteration counter $j = 1$.
2. Using the points $\Theta_1, \cdots, \Theta_j$ and the results of the objective function evaluation at these points, check whether $j = n$; that is, check an appropriate stopping condition. If this condition holds, terminate the algorithm.
3. Alternatively, generate a random point Θ_{j+1} according to some probability distribution P_{i+1} and evaluate the objective function at Θ_{j+1}.
4. Substitute $j+1$ for j and return to step 2.

The algorithm is called "Pure Random Search (PRS)" if all the distribution P_j are the same ($P_j = P$ for all j) [10]. As a result of the application of PRS we obtain independent samples $\{\Theta_1, \cdots, \Theta_n\}$ from a distribution P on Ω. Additionally, we obtain an independent sample $\{Y_1 = g(\Theta_1), \cdots, Y_n = g(\Theta_n)\}$ of the objective function values at the points. The samples Y_j are independent identically distributed random variables (iidrv) with a common continuous cumulative distribution function (cdf) of a random variable Y. The cdf is defined as

$$F(t) = \Pr\left\{Y \le t\right\} = \Pr\left\{\Theta \in \Omega : g(\Theta) \le t\right\}, \tag{10}$$

where the function f is formally defined as the probability density function (pdf). The cdf F is assumed to be continuous but unknown.

The iidrv $Y_1, \cdots, Y_n$ are arranged in increasing order of magnitude and the kth value is denoted by $Y_{k,n}$ such that

$$Y_{1,n} \le Y_{2,n} \le \cdots \le Y_{k,n} \le \cdots \le Y_{n,n}.$$

The $Y_{k,n}$ is referred to as the kth order statistics, see, e.g., [11,12]. We use the kth order statistics instead of the unknown exact global minimum value. The accuracy of this approach is linked with cindtion:

$$\Pr\left\{F(Y_{k,n}) \le \gamma\right\} \ge \beta \tag{11}$$

with parameters β, γ. Equation (11) means at least $100\beta\,\%$ confident that at most a proportion γ of the population is less than $Y_{k,n}$. This type of formulation is known as distribution-free tolerance interval [11-13].

Consider we select (k, n) such that following condition is satisfied:

$$I_{1-\gamma}\left(k, n-k+1\right) \le 1-\beta, \tag{12}$$

where $I_p(a,b)$ is incomplete beta function which is given by

$$I_p(a,b) = \frac{\int_0^p t^{a-1}(1-t)^{b-1}dt}{\int_0^1 t^{a-1}(1-t)^{b-1}dt}, \tag{13}$$

then condition (11) holds true [9]. Therefore, condition (11) is equivalent to condition (12), which corresponds to stopping rule of Algorithm 1.

4. NUMEIRICAL EXAMPLE

4.1. Main structure

To demonstrate the applicability of the proposed method, we consider an arch-frame model, which is studied by Tsuda et al. [7], shown in Fig. 2. The span and height of the model are 79m and 20.2m, respectively. The model without TMDs, which is referred to as the main structure, has 13 nodes and lumped mass of 6,000 kg is placed each node. The total mass of the main structure is 78,000 kg. The flexural rigidity of columns and members of arch roof are 2.42×10^9 N·m^2 and 2.77×10^9 N·m^2, respectively. We assume a Rayleigh damping for the main structure such that the damping ratio is 2% for both first and second modes.

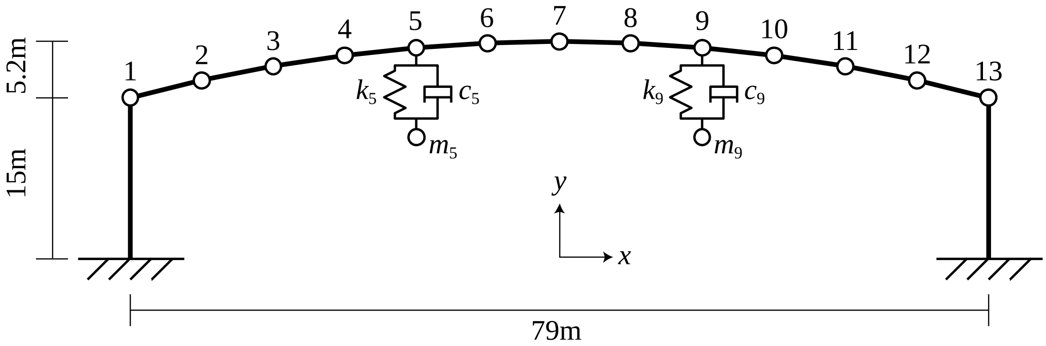

Figure 2: Arch-frame model with attached TMDs at the 5th and 9th nodes.

The natural periods of the main structure are shown in Tab. 1. In this problem, we are interested in reduction of vertical responses of the main structure under horizontal ground acceleration, and corresponding mode shapes in vertical y-directions are illustrated in Fig. 3. The mode shapes are normalized to a maximum value of 1.0. Thus influence vector $\boldsymbol{r}$ is given by a vector such that the element corresponding to a horizontal degree of freedom is one, otherwise zero; and velocity output matrix $\boldsymbol{D}_1$ is zero matrix; displacement output matrix $\boldsymbol{D}_2$ is given by a matrix such that the element corresponding to a vertical degree of freedom is one, otherwise zero. Under such condition, we define transmissibility as the ratio of root sum square of output displacements to input ground acceleration , i.e., $\| \boldsymbol{G} \|_\infty$ which is H_∞ norm of the frequency response function matrix. Transmissibility of the main structure is shown in Fig. 4, the peak value of which is 8.88 m/(m/s^2). From Tab. 1 and Fig. 4, we can see that the second mode is dominant in vertical vibration and we may need to take the forth mode into account to reduce the vertical vibration.

Table 1: Natural periods of the main structure

	Natural period [sec]
1st	0.934
2nd	0.789
3rd	0.364
4th	0.222

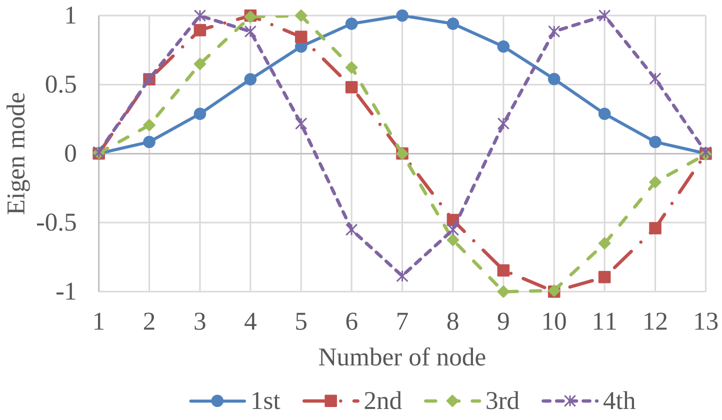

Figure 3: Mode shapes in vertical y-directions

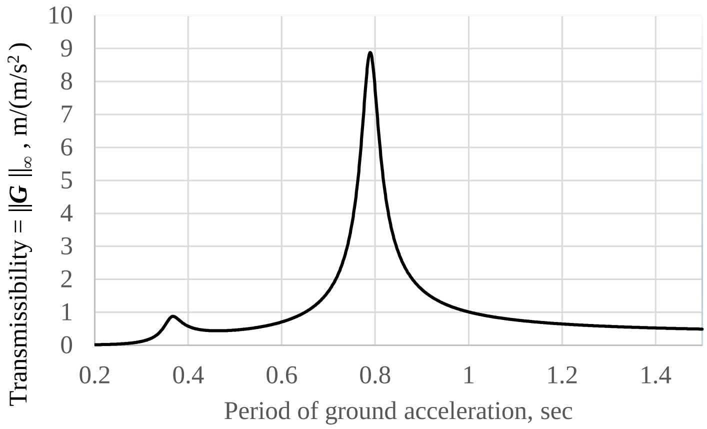

Figure 4: Frequency response function of the main structure

4.2. Design of TMDs

This model is symmetrical with respect to the y-axis, so that we also place TMDs symmetrically, e.g., we set same parameters to the TMDs at 5th and 9th nodes as shown in Fig. 2. Hence we only consider design of the left half side of the model as placement of attached TMDs; it is needless to say that TMDs of the other side is placed symmetrically. The placement of TMDs can be expressed by a multiset of number of nodes, which is denoted by $\mathcal{I}$. Note that members of multiset are allowed to appear more than once because we allow overlapping placement of TMDs. Such approach is known as MTMD, the effectiveness and robustness of which have been also known. The number of TMDs are constrained to less than or equal 3. The conditions can be summarized as follows:

$$\varnothing \subset \mathcal{I} \subseteq \left\{1,1,1,2,2,2,\cdots,7,7,7\right\}, \quad \left|\mathcal{I}\right| \le 3, \tag{14}$$

where $\varnothing$ is the empty multiset, e.g., $\mathcal{I} = \{5,5,5\}$ is admissible.

Total mass of TMDs are constrained to equal or less than 10% of the total mass of the main structure; and upper and lower bounds of the parameters are given, and then feasible set is given by

$$\Omega(\mathcal{I}) = \left\{ \Theta := \left(m_j, c_j, k_j; j \in \mathcal{I} \right) \left| \begin{array}{c} \sum_{j\in\mathcal{I}} m_j \le \dfrac{6000 \text{ kg} \times 13}{2} \times 10\% = 3900 \text{ kg}; \\ 1 \text{ kg} \le m_j \le 3{,}900 \text{ kg}, \;\; j \in \mathcal{I}; \\ 1 \text{ N/(m/s)} \le c_j \le 10^4 \text{ N/(m/s)}, \;\; j \in \mathcal{I}; \\ 1 \text{ N/m} \le k_j \le 10^6 \text{ N/m}, \;\; j \in \mathcal{I} \end{array} \right. \right\}. \quad (15)$$

We redefine our design problem as

$$\left. \begin{array}{ll} \text{Find} & \mathcal{I}, \Theta \\ \text{which minimizes to} & g(\Theta) := \left\| \boldsymbol{G} \right\|_\infty \\ \text{subjectto} & \varnothing \subset \mathcal{I} \subseteq \left\{1,1,1,2,2,2,\cdots,7,7,7\right\}, \;\; \left|\mathcal{I}\right| \le 3, \\ & \Theta \in \Omega(\mathcal{I}) \end{array} \right\}. \quad (9)'$$

By solving equation (12) for $\beta = \gamma = 0.99$, we obtain $k = 1$ and $n = 459$. Thus we need 459 samples to find a near-optimal solution for a given placement of TMDs. Under constraint (14) we can enumerate all possible $\mathcal{I}$, i.e., combinations of the placement of TMDs and the number of all combinations is 119. Hence, we select the best design among 119×459=54,621 samples, which means we evaluate equation (7) 54,621 times. We treat the design as a probabilistic optimum solution of problem (9)’ in a sense of (11). Furthermore, we try 10 cases for verification because RS is probabilistic method. The results of 10 cases are summarized in Tab. 2, in the row of which labeled by ‘RS’ the peak values of transmissibility of the model with TMDs are shown. We can reduce the peak value by averagely 77% from the one of model without TMDs.

Table 2: The peak values of transmissibility of the model with TMDs

Case	1	2	3	4	5	6	7
RS	1.98	2.06	2.01	2.01	2.00	1.99	2.02
DS	1.86	**1.74**	1.84	1.87	1.91	1.96	1.88

Case	8	9	10	best	average	SD
RS	2.02	**1.96**	2.12	1.96	2.01	0.043
DS	1.96	1.95	1.84	1.74	1.88	0.066

We expect there are better local solutions near the solutions in Tab. 2 and hence we solve problem (9)’ again by local search method from start point of the solutions in Tab. 2. This type of problem is known as nonsmooth problem, so that we use mesh adaptive direct search method (DS) [14], which is known as effective method for nonsmooth optimization [15]. The results are also summarized in the row labeled by 'DS' in Tab. 2. By the local search, we can obtain averagely 6.5% better solution in this example. The total best solution is

$$m_2 = 123.7 \text{ kg}, m_4 = 3767 \text{ kg},$$
$$c_2 = 80.6 \text{ N/(m/s)}, c_4 = 9138.8 \text{ N/(m/s)},$$
$$k_2 = 3739.2 \text{ N/m}, \ k_4 = 189588 \text{ N/m}.$$

We allow overlapping placement of TMDs; however the total best solution is non-overlapping placement. Furthermore, the number of TMDs is only 2, i.e., total 4 TMDs attached to the structure. The natural periods are shown in Tab.3. Real part of the mode shapes in vertical y-directions are illustrated in Fig. 5. The imaginary part of the modes are relatively smaller than the real part of the ones. In Fig. 6, the dashed line, red line and green line represent transmissibility without TMDs, the one with optimum TMDs and peak value with the optimum TMDs, respectively.

Table 3: Natural periods of the model with TMDs

	Natural period [sec]
1st	1.144
2nd	1.143
3rd	0.945
4th	0.934
5th	0.886
6th	0.740

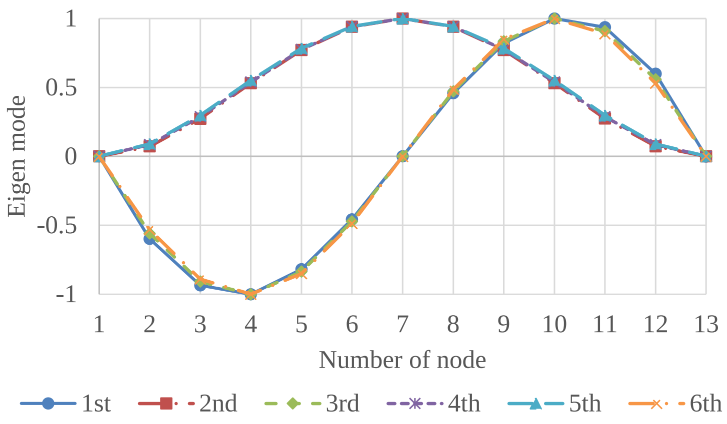

Figure 5: Real part of mode shapes in vertical y-directions

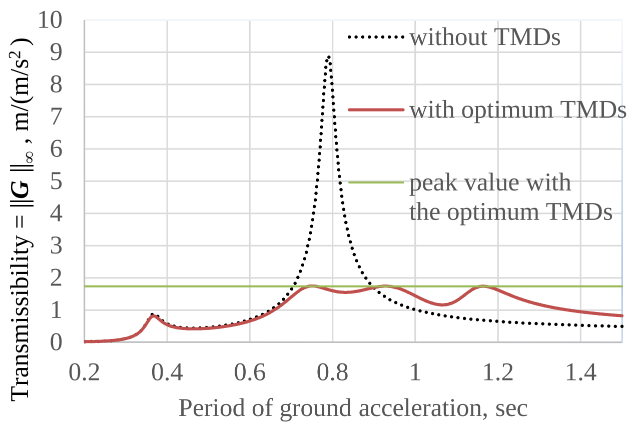

Figure 6: Comparison of frequency response function

From Tab.3, the 1st and 2nd natural periods, and the 3rd and 4th ones closely spaced. Furthermore, in Fig. 5, the 1st, 3rd and 6th mode shapes and the 2nd, 4th and 5th mode shapes are almost same. In Fig. 6, the three peaks have also almost same value. It is likely that closely spaced frequencies and correspondence of mode shapes cause small number of optimum TMDs because it needs to control a fewer number of modes.

5. CONCLUSION

To find an optimum design of TMDs for the vibration control of spatial structures, we proposed a probabilistic optimization method. We investigated the behavior of the method through a numerical example. The results are as follows:

- We propose a new stopping rule of random search method based on order statistics. This approach can guarantee to find a near-optimal solution with pre-assigned accuracy in a sense of probability.
- We consider an arch-frame model as a numerical example, in which the number of possible locations for TMDs is 13. By using the proposed methods, we can find design of TMDs which can reduce the peak response by averagely 77% from the one without TMDs.
- By combining the method with local search method, we can obtain further better solution by averagely 6.5% than the one without local search method.
- The best solution we found has closely spaced frequencies and correspondence of mode shapes, which cause small number of TMDs because it needs to control a fewer number of modes.

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