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MODELING NON-EQUILIBRIUM TWO-PHASE FLOW IN ELASTIC-PLASTIC POROUS SOLIDS

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Abstract. The present paper is concerned with a numerical model that is developing to simulate dynamical processes in a heterogeneous two-phase medium consisting of two components – elastic-plastic porous solid and gas that occupies the domain in between of the solid. The scope of our interest is regimes of large deformations and intense loading-unloading processes when the solid and the gas have different velocities and temperatures, i.e., are in dynamical and thermal non-equilibrium. Such a model needs to describe, for example, combustion and detonation in condensed porous explosives that are manufactured by pressing granular propellants.

1 INTRODUCTION

The model to be considered is an extension to the well-known model of Baer and Nunziato [1] for description of detonation in granular explosives. The medium in this model is treated as a two-phase continuum that consist of the solid granular skeleton of unreacted explosive and the gaseous product of combustion. Each phase of the mixture is characterized by its own vector of state parameters governed by the compressible Euler equations.

The model we develop describes the behaviour of an elastic-plastic porous material (the solid phase) filled in by a gaseous component (the gas phase). The gas can flow through the porosity of the solid skeleton. The phases exchange mass, momentum, and energy due to combustion, interphase drag, and heat conduction.

Porosity of the solid phase is defined by the gas volume fraction. Moreover, the solid phase is instantaneously characterized by the field of density, velocity, and stress tensor, the gas phase is done by own density, velocity, and pressure. We use the model of Prandtl and Reuss with the plastic flow rule given by the isotropic Von Mises yield condition to describe dynamics of the solid phase. Only minor modifications are done in this model to account for porosity. The gas phase is described by the compressible Euler equations.

The system of governing equations is closed by the kinetic equation for porosity which takes into account change in time of the gas volume fraction due to deformation and combustion of the solid skeleton.

The resulting system of equations have no conservative form. Non-conservative terms appear because of gradient of the porosity, and are named in literature as nozzling terms by analogy with variable-area quasy-1D gas dynamics. A careful treatment is required for these terms when developing numerical methods [2].

Many investigations have been undertaken to analyze mathematical properties of the Bayer-Nunziato equations, in particular with the aim of solving the Riemann problem and extending the Godunov method to two-phase hydrodynamics [2-5]. The model of elastic-plastic two-phase flow is more complicated. For solving this model we implement the method of splitting that allows us to reduce the problem to more simple sub-problems.

2 MATHEMATICAL MODEL AND NUMERICAL METHOD

We consider heterogeneous two-phase medium consisting of two components – elastic-plastic porous solid and gas that occupies the domain in between of the solid. We use the continual approach where the medium is treated as a non-equilibrium two-velocity continuum. The gas phase is determined by time-depending spatial distributions of the volume fraction (the porosity) α, the density ρ_1^0, the pressure P_1, and the velocity vector $\vec{u}=(u_1,u_2,u_3)$. The solid phase is described by the solid volume fraction $\beta=1-\alpha$, the density ρ_2^0, the velocity vector $\vec{v}=(v_1,v_2,v_3)$, the stress tensor $\sigma=(\sigma_{ij})$, and the strain tensor $\varepsilon=(\varepsilon_{ij})$. Here we assume an orthogonal Cartesian coordinates.

The system of governing equations represents fundamental laws of conservation of mass, momentum, and energy. As a baseline model we use the model of Nigmatulin [14] in the form of the Baer-Nunziato [1] equations extended to take into account elasto-plastic properties of the solid phase:

$$\frac{\partial \vec{q}}{\partial t}+\frac{\partial \vec{f}_i}{\partial x_i}=\vec{H}_n+\vec{H}_e \tag{1}$$

where

$$\vec{q}=\left(\rho_1,\rho_1u_k,\rho_1E_1,\rho_2,\rho_2v_k,\rho_2E_2\right)^T$$

is the vector of conservative variables,

$$\vec{f}_i=\left(\rho_1u_i,\rho_1u_iu_k+\alpha P_1\delta_{ik},\rho_1u_iE_1+\alpha P_1u_i,\rho_2v_i,\rho_2v_iv_k-\beta\sigma_{ik},\rho_2v_iE_2-\beta\sigma_{ij}v_j\right)^T$$

is the vector of flux in the *i-th* coordinate, $i=1,2,3$,

$$\vec{H}_n=\left(0,-P_1\nabla_k\alpha,-P_1v_k\nabla_k\alpha,0,P_1\nabla_k\alpha,P_1v_k\nabla_k\alpha\right)^T$$

is the non-conservative term due to the gradient of the porosity, which is often referred to as the nozzling term since it is similar to that appears in the quasi-one-dimensional gas dynamics in channels with variable area,

$$\vec{H}_e=\left(\dot{m},-g_k+\dot{m}v_k,-g_jv_j-q+\dot{m}(E_2+\delta Q),-\dot{m},g_k-\dot{m}v_k,-\dot{m}E_2\right)^T$$

is the exchange term that describes mass, momentum, and energy transfer due to combustion of the solid phase, viscous friction, and heat transport between the phases.

In the above equations, $\rho_1 = \alpha\rho_1^0$, $\rho_2 = \beta\rho_2^0$ are bulk densities of the phases, $E_1 = e_1 + 0.5u_j u_j$, $E_2 = e_2 + 0.5v_j v_j$ are specific total energies, e_1 and e_2 are internal energies, $\dot{m}$ is the mass transport rate due to combustion, $\vec{g} = (g_k)$ is the viscous drag force, q is the heat transfer due to difference in temperatures of the phases, and δQ is the heat released in the gas phase because of the solid phase burning.

The material of the solid phase is assumed isotropic, so that the material relations between stress and strain are given in the form of the generalised Hooke's law

$$S_{ij} = 2\mu\left(\varepsilon_{ij} - \frac{1}{3}\varepsilon_{kk}\delta_{ij}\right) \tag{2}$$

where μ is the shear modulus, and $S = (S_{ij})$ is the deviatoric stress tensor:

$$S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij} = \sigma_{ij} + P_2\delta_{ij}$$

By taking material derivative of Eq.(2) and implementing the Jaumann derivative [7]

$$\frac{d^J S_{ij}}{dt} = \frac{dS_{ij}}{dt} - S_{ik}\omega_{jk} - \omega_{ik}S_{jk} \tag{3}$$

to satisfy frame-indifference requirement, the constitutive relations from Eqs.(2) and (3) can be written in the following form:

$$\frac{dS_{ij}}{dt} = S_{ik}\omega_{jk} + \omega_{ik}S_{jk} + 2\mu\varepsilon_{ij} - 2\mu\varepsilon_{kk}\delta_{ij}/3$$

Here $\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)$ is the spin tensor, $\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$ is the strain rate tensor.

An additional yield condition is required to model elastic-to-plastic flow transition. The yield condition determines when the flow became plastic. In the present paper, we implement the von Mises yield condition which can be written as a restriction Wilkin's radial return imposed on deviatoric stress tensor:

$$\text{if } Q = S_{ij}S_{ij} \geq \frac{2}{3}Y^2, \text{ then } S_{ij} \to S_{ij}\sqrt{\frac{2}{3}}\frac{Y}{Q} \tag{4}$$

where Y is the yield strength of the material in simple tension.

The yield condition (4) can be recast and introduced in the constitutive Eqs.(3). Let us introduce a parameter λ and modify Eqs. (3) as follows:

$$\frac{dS_{ij}}{dt} = S_{ik}\omega_{jk} + \omega_{ik}S_{jk} + 2\mu\varepsilon_{ij} - 2\mu\varepsilon_{kk}\delta_{ij}/3 - \lambda S_{ij} \tag{5}$$

Direct consequence of these equations is an equation for Q:

$$\frac{1}{2}\frac{dQ}{dt} = W - \lambda Q \qquad \text{with} \qquad W = 2\mu S_{ij}\varepsilon_{ij}$$

The condition (4) is satisfied provided that the parameter λ is assigned as

$$\lambda = 0, \quad \text{if} \quad \frac{Q}{2} \le \frac{1}{3}Y^2, \qquad \text{and} \qquad \lambda = \frac{3W}{2Y^2}, \text{ otherwise.}$$

Therefore, Eq. (5) will meet the yield condition (4) once λ is determined as

$$\lambda = \frac{3W}{2Y^2} H\left(\frac{Q}{2} - \frac{Y^2}{3}\right) \tag{6}$$

The Eqs. (3) and (4) are equivalent to Eqs. (5) and (6), but the latter is more convenient in coding. Besides, the model of elastic-plastic solids easy converts to the model of hydrodynamics (the model of Baer and Nunziato) by simply defining the yield strength $Y = 0$.

To complete the governing equations, we need also equation of states (EOS) that relate thermodynamical parameters such as density, pressure, internal energy, temperature, etc. We assume mechanical and thermal EOS in the following form: $e_k = e_k(\rho_k^0, P_k)$ and $e_k = e_k(\rho_k^0, T_k)$, k=1,2 for the gas and solid phase, respectively, where T denotes the temperature.

To closure the system of equations (1), (5), and (6), one needs to add an equation for the evolution of the porosity. In this paper we assume that the porosity is transferred with the solid phase dynamics and is also changed in time due to combustion and deformation of the solid phase. The corresponding equation takes the following form:

$$\frac{\partial \alpha}{\partial t} + v_j \frac{\partial \alpha}{\partial x_j} = \dot{\alpha}_{def} + \frac{\dot{m}}{\rho_2^0}, \tag{7}$$

where $\dot{\alpha}_{def}$ is the rate of change of the porosity due to deformation of the solid phase. In this paper we assume that this change of porosity occurs isotropically and depends on the difference in pressures, $\dot{\alpha}_{def} \sim P_2 - P_1$ in accordance with the model proposed in [15]:

$$\dot{\alpha}_{def} = \alpha\left(\frac{P_1 - P_2 + \frac{2}{3}Y\ln\alpha}{4\eta} H\left(P_1 - P_2 + \frac{2}{3}Y\ln\alpha\right) + \frac{P_2 - P_1 - \frac{2}{3}Y\ln\alpha}{4\eta} H\left(P_2 - P_1 - \frac{2}{3}Y\ln\alpha\right)\right),$$

where η is the dynamical viscosity and H is the Heaviside function. The Eq. (7) can be recast in the conservative form as

$$\frac{\partial \rho_2^0}{\partial t} + \frac{\partial \rho_2^0 v_j}{\partial x_j} = \frac{\rho_2^0 \dot{\alpha}_{def}}{\beta}, \tag{8}$$

We also should modify the exchange term $\vec{H}_e$ in Eq. (1) to account for the work of change the gas volume due to deformation of the solid phase:

$$\vec{H}_e = \left(\dot{m}, -g_k + \dot{m}v_k, -g_j v_j - q + \dot{m}(E_2 + \delta Q) - P_1\dot{\alpha}_{def}, -\dot{m}, g_k - \dot{m}v_k, -\dot{m}E_2 + P_1\dot{\alpha}_{def}\right)^T$$

Thus, the system of equations to be solved consists of equations (1), (5), (6), and (8) with the above modified $\vec{H}_e$.

The basic idea of the numerical method we develop lies on a general concept of splitting the governing equations in accordance with main processes [8, 9]. By using this approach, solving the problem can be reduced to more simple sub-problems without loss of fidelity. To do so, let us combine Eqs. (1), (5), (6), and (8) in a common conservative form

$$\frac{\partial \vec{q}}{\partial t} + \frac{\partial \vec{f}_k}{\partial x_k} = \vec{H}_n + \vec{H}_e + \vec{H}_m \tag{9}$$

by extending the state vector $\vec{q}$, corresponding flux vectors $\vec{f}_k$, and the r.-h. s. vectors $\vec{H}_n$, $\vec{H}_e$:

$$\vec{q} = \left(\vec{q}, \rho_2^0, \rho_2 S\right)^T; \ \vec{f}_k = \left(\vec{f}_k, \rho_2^0 v_k, \rho_2 S v_k\right)^T; \tag{10}$$

$$\vec{H}_n = \left(\vec{H}_n, 0, 0\right)^T; \ \vec{H}_e = \left(\vec{H}_e, \frac{\rho_2^0 \dot{\alpha}_{def}}{\beta}, 0\right)^T$$

The vector $\vec{H}_m$ in Eq.(9) represents the r.-h.s. of the constitutive equation (5):

$$\vec{H}_m = \left[\vec{0}, 0, \rho\left(\omega S - S\omega\right) + 2\mu\rho Dev\left(\varepsilon\right) - \lambda\rho S\right]^T \tag{11}$$

The dynamical process in the two-phase elasto-plastic porous medium can be thought of as the combination of two sub-processes [6]. One is hydrodynamic where the medium behaves like a two-phase fluid with the stress tensor assumed to be frozen (with no change) in the solid phase related Lagrangian particle. It is also assumed that no exchange process between the phases occurs at this stage. Another sub-process is considered as Lagrangian one. It goes in the medium on the frozen in time velocity fields and accounts for the change in time of the deviatoric stress S due gradients in the solid velocity field and the change of the state vector $\vec{q}$ caused by the exchange term $\vec{H}_e$. This splitting is mathematically described by 2 systems of equations: for the hydrodynamic part

$$\frac{\partial \vec{q}}{\partial t} + \frac{\partial \vec{f}_k}{\partial x_k} = \vec{H}_n , \tag{11}$$

and for the Lagrangian part

$$\frac{\partial \vec{q}}{\partial t} = \vec{H}_e + \vec{H}_m . \tag{12}$$

At the first, hydrodynamical, stage, we calculate an intermediate solution by solving Eqs. (11). This solution represents changes in the state vector due to fluid dynamics factors providing that the deviatoric stress is carried by the media inalterably, simply as a Lagrangian characteristics.

In general, Eqs. (11) are solved with a moving grid that is adapted to the boundaries. We want to track, for example, contacts between different media, free boundaries, shock waves etc. At the second Lagrangian stage, this grid is frozen in time, and Eqs. (11) are integrated in each computational cell as a system of ODEs. Thus, updating the solution vector $\vec{q}$ from a discrete time level *n* to a new one *n+1* is accomplished in the following two steps:

$$\vec{q}^{\,*} = L_1\left(\Delta t, \vec{q}^{\,n}\right) ; \qquad \vec{q}^{\,n+1} = L_2\left(\Delta t, \vec{q}^{\,*}\right) \tag{13}$$

In the present paper, L_1 is the second-order accurate Godunov discrete operator with a MUSCL-type cell interpolation scheme [10] (extended to unstructured grids [11]).

Let $G = G(t)$ be a grid that discretizes the computational domain at the time t. The grid is in general unstructured, and consists of non-overlapping polyhedrals. We assume that the topology of the grid doesn't change in time: the set of control volumes and the structure of cell links remain invariant. Then, the cell-centered control volume method applied to Eqs. (11) leads to a semi-discrete equations:

$$\frac{dV_i \vec{q}_i}{dt} = -\sum_{\sigma}\left[\left(\vec{f}_k - \delta\vec{f}_k\right)n_k - \vec{q}U_n\right]_{\sigma} S_{\sigma} \tag{14}$$

where the subscripts i and σ denote a computational cell and its face, respectively, V_i =*Volume(*i*)* is the cell volume and S =Area(σ) is the face area , $n_k, k = 1,2,3$ are Cartesian components of the unit normal to the face directed outward the cell.

The additional flux $\delta\vec{f}_k$ in Eq. (14) takes into account the non-conservative nozzling term $\vec{H}_n$. The Godunov method assumes continuity of all flow parameters inside each computational cell. For the first order scheme, these are just constant. Therefore the porosity

at the beginning of the time step may have discontinuity only across the cell face. Next time moments, this discontinuity moves with the material velocity of the solid phase, and is found to be just inside the cell. To treat $\vec{H}_n$ in the finite volume method discretization, the integration over the whole cell is represented by the integrations over two sub-domains separated with the surface of porosity discontinuity. In the case of the first order scheme this results in Eq. (14), and the additional flux $\delta\vec{f}_k$ takes the following form:

$$\delta\vec{f}_k = \begin{cases} \vec{f}_k^{c,+} - \vec{f}_k^{c,-}, \ if \ v_n^c \le 0 \\ 0, \ otherwise \end{cases}, \tag{15}$$

where $\vec{f}_k^c$ is the flux at the surface of porosity discontinuity, the superscript "+" and "-" indicates the outer and inner side of this surface with respect to the outward normal $\vec{n}$, and v_n^c is the normal component of its velocity. Note, that for the second order scheme we have to write in the left-hand side of Eq. (14) an approximation of the cell integral of $\vec{H}_n$. Taking into account that $\vec{H}_n = \vec{h}_k \nabla_k \alpha$ this may be done as

$$\int_i \vec{H}_n dV = \sum_\sigma \left(\vec{h}_k\right)_i \alpha_i^\sigma \left(n_k S\right)_\sigma, \tag{16}$$

where α_i^σ is the MUSCL-interpolated σ-face value of the porosity in the *i-th* cell.

In Eq.(14), $U_n = \left(\vec{U}, \vec{n}\right)$ is the normal component of the face velocity. In practice, movement of the grid is determined by velocities of node points (vertices) which are calculated based on the data how the tracked surfaces is displaced. The face velocity is then defined so that the geometrical conservation law is satisfied:

$$\frac{dV_i}{dt} = \sum_\sigma U_{n,\sigma} S_\sigma \tag{17}$$

This can be realized in different ways. In particular, we can choose this velocity as

$$U_{n,\sigma} = \frac{\Delta V_\sigma}{\hat{S}_\sigma \Delta t} \tag{18}$$

where $\Delta V_\sigma = Volume\left(\sigma(t), \sigma(t+\Delta t)\right)$ is the volume that the face σ sweeps as the cell $i(t)$ goes to the position $i(t+\Delta t)$, and $\hat{S}_\sigma = Aver\left(\sigma(t), \sigma(t+\Delta t)\right)$ is an average position of the face

(e.g., arithmetic averaging). Such a choice guaranties the validity of geometrical conservation law at the discrete level:

$$V_i(t+\Delta t)-V_i(t)=\Delta t\sum_{\sigma}U_{n,\sigma}\hat{S}_{\sigma} \tag{19}$$

Letting $\vec{k}=(k_1,k_2,k_3)$ and $\vec{l}=(l_1,l_2,l_3)$ be unit vectors tangential to the face σ such that $\vec{n},\vec{k}$, and $\vec{l}$ form an orthonormal basis in R^3, the r.-h.s. of Eq. (14) can be written using one flux vector $\vec{F}$

$$\left(\vec{f}_k-\delta\vec{f}_k\right)n_k-\vec{q}U_n=T^{-1}\left(\vec{F}-\delta\vec{F}-\vec{Q}U_n\right);\ \ \vec{Q}=T\vec{q} \tag{20}$$

where T is the matrix of transformation from the absolute Cartesian coordinates to localcoordinates in the basis $\vec{n},\vec{k},\vec{l}$.

The flux vector $\vec{F}=\vec{F}(\vec{Q})$ is given by

$$\vec{F}=\left(\rho u_n;\rho u_n{}^2-\sigma_{nn};\rho u_n u_k-\sigma_{nk};\rho u_n u_l-\sigma_{nl};\rho u_n E-\sigma_{nn}u_n-\sigma_{nk}u_k-\sigma_{nl}u_l;\rho u_n S\right)$$

where the subscript n means normal, and k, l tangential coordinates, respectively: $u_a=u_i a_i$; $\sigma_{ab}=\sigma_{ij}a_i b_j$ with a and b taking n,k,l. Therefore, $\vec{F}$ can be thought of as a local one-dimensional flux of mass, momentum, energy and deviatoric stress in the normal to the face direction.

Thus, the semi-discrete form of the hydrodynamic part is written as

$$\frac{dV_i\vec{q}_i}{dt}=-\sum_{\sigma}T_{\sigma}{}^{-1}\vec{\Phi}_{\sigma}S_{\sigma} \tag{21}$$

where $\vec{\Phi}=\vec{F}-\delta\vec{F}-U_n\vec{Q}$.

The vector-function $\vec{\Phi}$ is referred to as numerical flux. Known well in numerical methods for hydrodynamics, it is considered as a function that depends on state parameters at the cell interface

$$\vec{\Phi}=\vec{\Phi}\left(\vec{Q}_i^{\sigma},\vec{Q}_{\sigma(i)}^{\sigma}\right) \tag{22}$$

where $\sigma(i)$ denotes the cell adjoined the i-cell by the face σ, and the superscript σ indicates the cell interface value. In this paper we employ the Godunov method that treats the numerical flux by means of the solution to the cell interface Riemann problem. Details of this approach we discuss in the next section.

The option of choosing the σ-values in Eq. (22) is related to the accuracy of the scheme. For the first order scheme we just set $\vec{Q}_i^{\sigma}=\vec{Q}_i$. To increase the accuracy, we follow the MUSCL approach [12], which defines the σ - value as

$$\vec{Q}^{\sigma} = \vec{Q}^{n} + T\left(r_k^{\sigma} I - 0.5\Delta t A_k\right)\nabla_k \vec{q}^{\,n} \tag{23}$$

where $\vec{r}^{\,\sigma}$ is the radius-vector from the cell center to the face center, $A_k = \partial \vec{\mathrm{f}}_k \big/ \partial \vec{q}$ is the flux Jacobian, ∇_k is the operator of limited derivative, which satisfies $r_k^{\sigma}\nabla_k \varphi_i = C\left(\varphi_{\sigma(i)}, -\varphi_i\right)$ with $0 \le c \le 0.5$ [13].

The explicit time marching scheme for Eq. (14) results in a first or second order accurate discretization depending on the choice of σ-values,

$$\vec{q}_i^{\,n+1} = \frac{1}{V_i^{n+1}}\left[V_i^n q_i^n - \Delta t \sum_{\sigma} T_{\sigma}^{-1} \Phi_{\sigma} S_{\sigma}\right] \tag{24}$$

providing that the time step is calculated in accordance with the CFL stability condition $\lambda\left(\vec{q}_i^{\,n}\right) = \dfrac{V_i}{\sum\limits_{\sigma} \rho_{\sigma}^{\Phi} S_{\sigma}}$, $\Delta t = \min\limits_i \left[\lambda\left(\vec{q}_i^{\,n}\right)\right]$, where ρ^{Φ} is the spectral radius of the Jacobian $\partial \vec{\Phi} \big/ \partial \vec{Q}$.

Once an intermediate solution $\vec{q}^{\,*}$ of the hydrodynamic sub-problem is obtained, we then use it as initial data to integrate the Lagrangian stage given by Eq.(12).

In these equations, the vector $\vec{q}$ is prescribed to cell centers of the grid at the upper time level t^{n+1}; the grid itself is assumed to be frozen in the curse of time. The derivatives in $\vec{H}_m$ (Eq. (11)) are approximated by the least square method.

To integrate the resulting system of ODE we use a 2 stage Runge-Kutta explicit scheme:

$$\tilde{\vec{q}}^{\,*} = \vec{q}^{\,*} + \Delta t\left[H_e\left(\vec{q}^{\,*}\right) + H_m\left(\vec{q}^{\,*}\right)\right]; \qquad \vec{q}^{\,n+1} = 0.5\left(\vec{q}^{\,*} + \tilde{\vec{q}}^{\,*}\right) + \Delta t\left[H_e\left(\tilde{\vec{q}}^{\,*}\right) + H_m\left(\tilde{\vec{q}}^{\,*}\right)\right] \tag{25}$$

A common value of Δt for the hydrodynamic and Lagrangian stages is chosen in accordance with the CFL stability condition (24). Some restriction on Δt is also imposed by of the Lagrangian stage. At each time step we check Δt to satisfy the following inequality: $\Delta t \left\| H_m\left(\vec{q}^{\,*}\right)\right\| \le \varepsilon \left\|\vec{q}^{\,*}\right\|$ with $\varepsilon = 0.01 \sim 0.1$.

3 NUMERICAL FLUX APPROXIMATION

The calculation of the numerical flux $\vec{\Phi}$ introduced in the previous section is carried out in the framework of the Godunov method with implementing the solution to the Riemann problem. With this aim, we first reduce the system of equations (11) to the 1D case for the direction *x* determined by the outward normal to the cell face (*x=0*), and consider the initial value Riemann problem with initial conditions $\vec{Q}(x,0) = \vec{Q}_l$, if $x < 0$, and $\vec{Q}(x,0) = \vec{Q}_r$, if $x > 0$. The solution to this Riemann problem has been studied in [2] for the particular case of stiffened gas EOS. We can consider this solution to be known. The calculation of the numerical flux follows the usual course.

Let us denote the Riemann problem solution as $\vec{Q}^R = \vec{Q}^R\left(\lambda, \vec{Q}_l, \vec{Q}_r\right)$, where $\lambda = x/t$ is the self-similar variable. Then, the numerical flux at the cell interface σ is determined in the following way:

$$\begin{aligned} &\vec{\Phi}_\sigma = \vec{F}\left(\vec{Q}_\sigma\right) - \vec{F}\left(\vec{Q}_\sigma^{C,+}\right) + \vec{F}\left(\vec{Q}_\sigma^{C,-}\right) - U_n\vec{Q}_\sigma, \ \text{if} \ V_C < 0 \\ &\vec{\Phi}_\sigma = \vec{F}\left(\vec{Q}_\sigma\right) - U_n\vec{Q}_\sigma, \ \textit{otherwise} \end{aligned} \tag{26}$$

where the arguments of the flux function are calculated from the Riemann problem solution as follows:

$$\begin{aligned} &\vec{Q}_\sigma = \vec{Q}^R\left(U_n + 0, \vec{Q}_i^\sigma, \vec{Q}_{\sigma(i)}^\sigma\right), \\ &\vec{Q}_\sigma^{C,+} = \vec{Q}^R\left(V_C + 0, \vec{Q}_i^\sigma, \vec{Q}_{\sigma(i)}^\sigma\right), \\ &\vec{Q}_\sigma^{C,-} = \vec{Q}^R\left(V_C - 0, \vec{Q}_i^\sigma, \vec{Q}_{\sigma(i)}^\sigma\right). \end{aligned} \tag{27}$$

Here V_c is the normal velocity of the solid contact which separates regions with different values of the porosity, α_i and $\alpha_{\sigma(i)}$. This solution is found in the process of solving the Riemann problem. Details can be found in [2].

4 NUMERICAL RESULTS

Here we show very preliminary results of calculations with the discrete model discussed in previous sections. In these calculations we use a reduced model where the exchange term $\vec{H}_e$ (except for $\dot{\alpha}_{def}$) is not taken into account. As a test we consider the impact problem in the one-dimensional approximation.

We simulate impact of a porous and solid (with no pores) material with the absolutely rigid wall. Initial values of parameters are chosen as $\rho_1^0 = 1.3125 g/sm^3$, $P_1 = 10^9 Pa$, $u_1 = -100 m/s$, $\beta = 0.75$, $\rho_2^0 = 0.4375 g/sm^3$, $P_2 = 10^9 Pa$, $v_1 = -100 m/s$, $\alpha = 0.25$, $Y = 0.5 GPa$, $\eta = 100\, Pa \cdot \sec$. A uniform grid consisting of 500 intervals is used. The left boundary of the computational domain corresponds to the rigid wall boundary conditions.

Figure 1 shows the difference in the wave structure that appears in the solid and porous material. In the right row figures we plot in black also the distribution of gas parameters in the case of pure gas impact (a piston problem). One can see the formation of the solid contact with a gap in the volume fraction. Behind this contact the pores volume fraction decreases, the gas is compressed and flows away from the wall. This results in additional contraction of pores near the wall with formation of the secondary contact wave.

Unloading process is presented in Figure 2. In this calculations the initial gas and solid velocity are directed away from the wall, $u_1 = v_1 = 100 m/s$. Tension of the material results in growing of pores, and the solid volume fraction is decreased. Comparison with the pure gas case is given in the right row figures. One can see the formation of two rarefaction waves in the gas phase in contrast to the pure gasdynamic case.

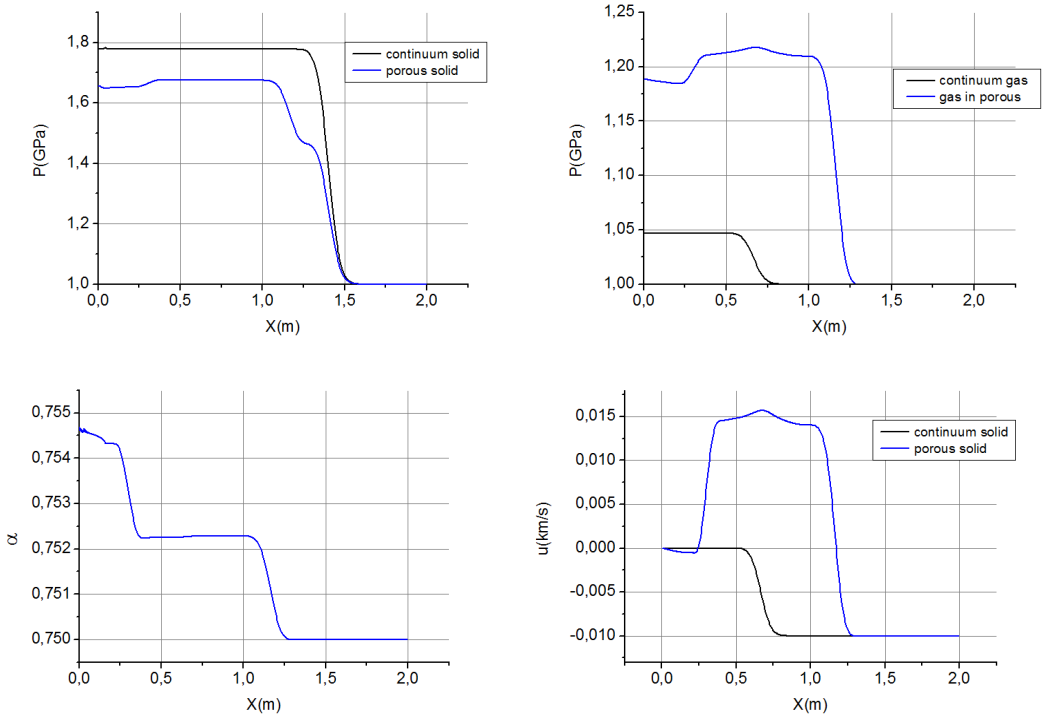

Figure 1: Impact problem. Flow parameters for t=0.25 ms: solid medium (black), porous medium (blue).

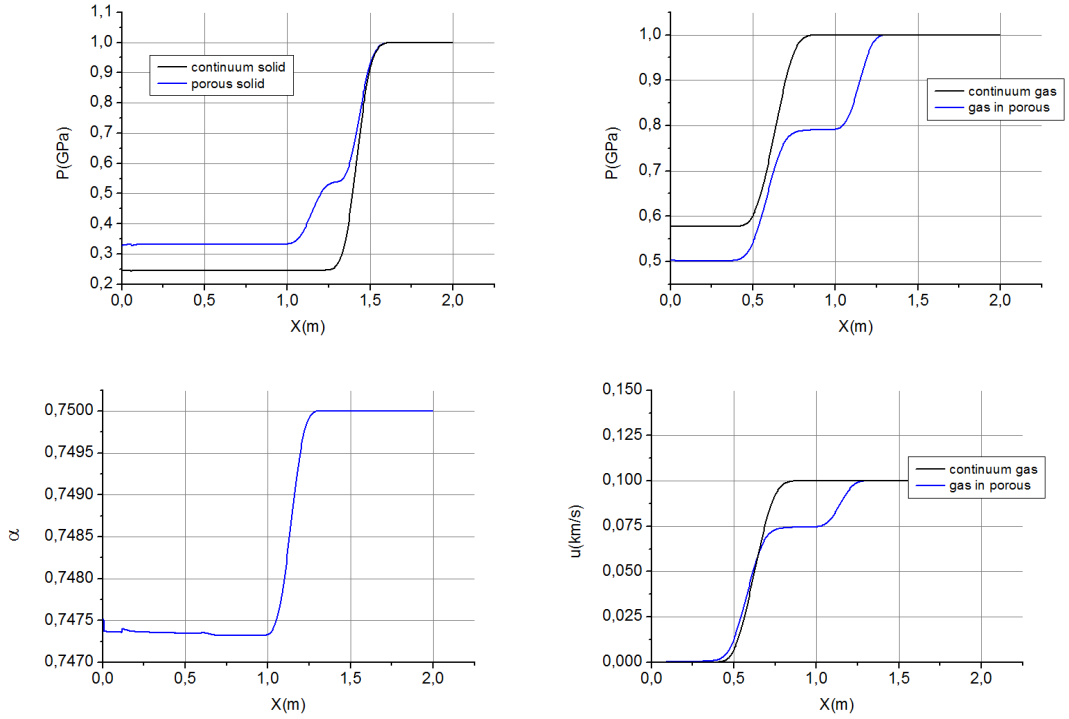

Figure 2: Impact problem. Flow parameters for t=0.25 ms: solid medium (black), porous medium (blue).

5 CONCLUSIONS

- A numerical model has been developed for elasto-plastic porous solid and gas dynamics, which can be considered as the extension to the two-phase hydrodynamical model of Baer-Nunziato.
- The Godunov method has been generalized for the system of governing equations of elastic-plastic two-phase flow. A novel treatment of non-conservative (nozzling) terms has been proposed which has the straightforward implementation in the multidimensional case.

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