

## DESIGN OF ARCHITECTURAL MEMBRANES WITH ISOGEOMETRIC ELEMENTS

B. Philipp<sup>1,\*</sup>, M. Breitenberger<sup>1</sup>, R. Wüchner<sup>1</sup> and K.-U. Bletzinger<sup>1</sup>

<sup>1</sup> Chair of Structural Analysis, Technische Universität München,  
Arcisstr. 21, 80333 Munich, Germany,  
{benedikt.philipp; michael.breitenberger; wuechner; kub}@tum.de, www.st.bgu.tum.de

**Key Words:** *textile membranes, tensile structures, form-finding, Isogeometric Analysis, IGA, CAD-CAE-integration.*

**Abstract.** The recently introduced Isogeometric Analysis presents interesting advantages compared to classical FE discretizations, mainly w.r.t. the use of the same model for the design and the analysis of structures. Architectural membranes with their smoothly curved silhouette on the one hand and the large number of iterative design steps between esthetical and engineering requirements are ideally suited for the use of the Isogeometric Analysis.

A brief introduction to the Isogeometric Analysis is given and the development of a membrane element, suitable for form-finding of architectural membranes, is outlined. As a well-established form-finding approach, the Updated Reference Strategy is used. The developed IGA-based membrane element is benchmarked at the example of Costa's minimal surface, making prove of the robustness and accuracy of the developed element and the applied form-finding approach. Advantages for the design of architectural membranes by using IGA elements are demonstrated and discussed. Remarks on the current state of CAD-CAE-integration are made. An outlook on future research in the field of Isogeometric Analysis for architectural membrane closes this contribution.

### 1 INTRODUCTION AND MOTIVATION

Architectural membranes offer a unique language of shapes, mainly characterized by their curved silhouette, underlining the light-weight, efficient nature of these structures. One of the core characteristics of architectural membranes is their load-bearing behavior: external and internal loads are transferred to the supports exclusively via tension. To ensure this load-bearing behavior, prestress in the membrane is required. Form-finding has the task to determine the shape that fits the prescribed prestress state and the given boundary conditions. Since form-finding usually is an iterative approach towards an architecturally desirable structure, the interlacing between design and structural engineering is extremely close. In this context the gap between *Computer-Aided Design (CAD)* and *Computer-Aided Engineering (CAE)* presents a major obstacle, since conversion and adaption between different "parallel", specialized geometric models requires considerable amounts of resources and obviously is prone to errors and deviations [1].

The *Isogeometric Analysis (IGA)* has been invented with the aim of overcoming this gap

by using the same basis functions for both the design model in CAD and the structural model in CAE [1]. In the present context of membrane structures, the Isogeometric Analysis therefore seems ideally suited for the very special requirements of architectural membranes. With their smoothly curved free form shapes, architectural membranes at their turn are ideally suited for the description by *Non-Uniform Rational B-Splines* (NURBS), the basis functions of the Isogeometric Analysis.

For these reasons geometrically non-linear IGA-based membrane and cable elements have been developed [2]. While the membrane element is formulated as a classical IGA element, close to successful shell formulations [3], the cable element is based on the recently developed *Isogeometric B-Rep Analysis* (IBRA), proposed in [4]. In the present contribution, these elements shall be used in the form-finding of architectural membranes at a first place, since form-finding represents the core part of membranes' design. As form-finding approach the *Updated Reference Strategy* (URS) [5] will be used.

The present contribution shall briefly sketch the necessary basis for formulating these elements and provide a basic understanding for the idea of the applied form-finding approach. Some benchmark examples will be presented. In the last part, the advantages related to the use of the Isogeometric Analysis in the architectural membranes' design and analysis shall be presented and discussed, thus underlining the potential of the developed combination of the Isogeometric Analysis and architectural membranes.

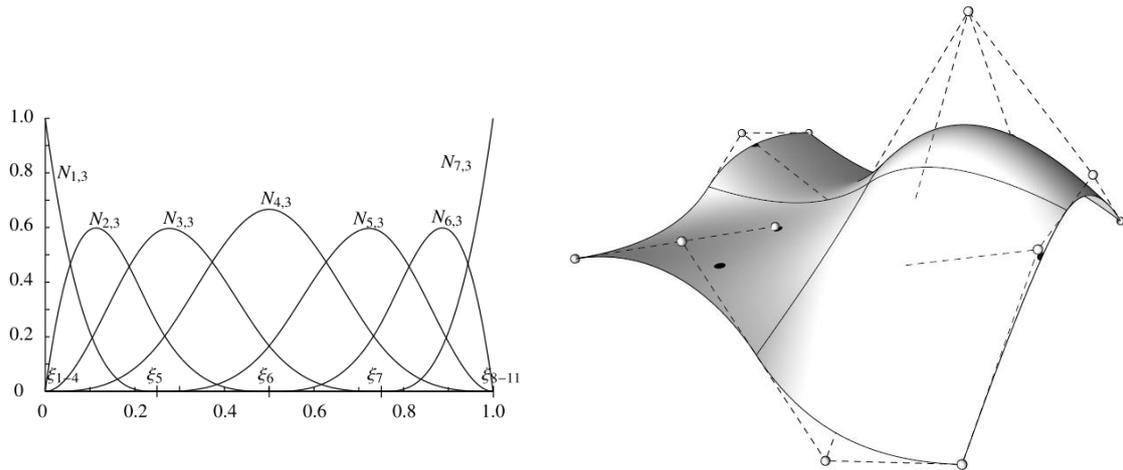
## 2 THE ISOGEOMETRIC ANALYSIS

The isogeometric analysis (IGA) is a relatively recent branch of finite elements that allows performing analysis on finite elements that use the *Non Uniform Rational B-Splines* (NURBS) as basis for discretization. The term IGA was introduced by Hughes *et al.* [1] and expresses the use of the same mathematical description for both the analysis model and the geometry model of a structure. While in classical finite element analysis (FEA) typically low-order interpolating – often linear – functions describe the geometry in a facette-type manner, IGA allows using the original design basis from CAD.

### 2.1 IGA basics

The B-Spline and NURBS functions fulfil the required properties for the use in finite element formulations [1]. In contrast to classically used shape functions, they are non-interpolating (cf. Fig. 1). With NURBS as basis functions the description of free form surfaces as well as of basic geometric entities like circles or cones is possible, which favors their universal use.

This non-interpolating characteristic of NURBS becomes problematic, when sudden changes of low continuity like in kinks are part of the shapes to be modelled. Coupling and refinement strategies as well as trimming techniques have been investigated in order to overcome these problems and have successfully been applied to a wide range of structures [6]. Recent developments like the *Isogeometric B-Rep Analysis* (IBRA), allow describing the complete range of geometries in a closed and consistent framework [4].

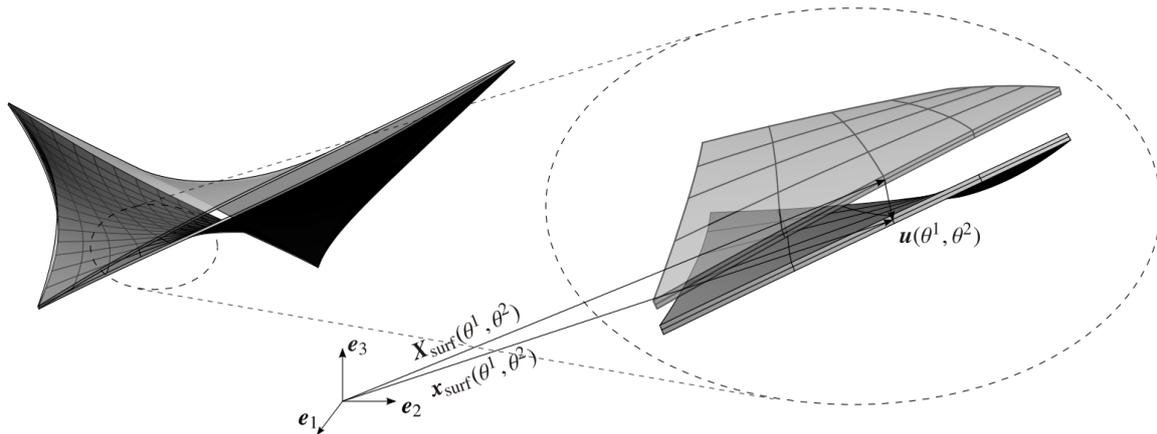


**Figure 1:** B-Spline basis functions with an open knot vector  $\Xi = [0, 0, 0, 0, 0.25, 0.5, 0.75, 1, 1, 1, 1]$  (left), and NURBS-based surface with its control points (right) [2].

In the last decade, a wide range of IGA-based element formulations has been developed, *e.g.* shell elements like in [3,7] or beam elements like in [8]. In the present contribution, a membrane element formulated based on NURBS shape functions as well as a cable element embedded in the 2D-discretization of the membrane is briefly described.

## 2.2 IGA elements

For the form-finding and analysis of architectural membranes during the design process, at a first place membrane elements and cable elements are needed. The geometrically non-linear membrane element is formulated starting with a kinematic description of the deformation, see Fig. 2. An advantageous property of the membrane and cable elements is their load bearing exclusively via tension. Therefore only a description of the displacements  $\mathbf{u}$  and the resulting strains  $\boldsymbol{\varepsilon}$  is necessary, bending respectively curvature don't occur.



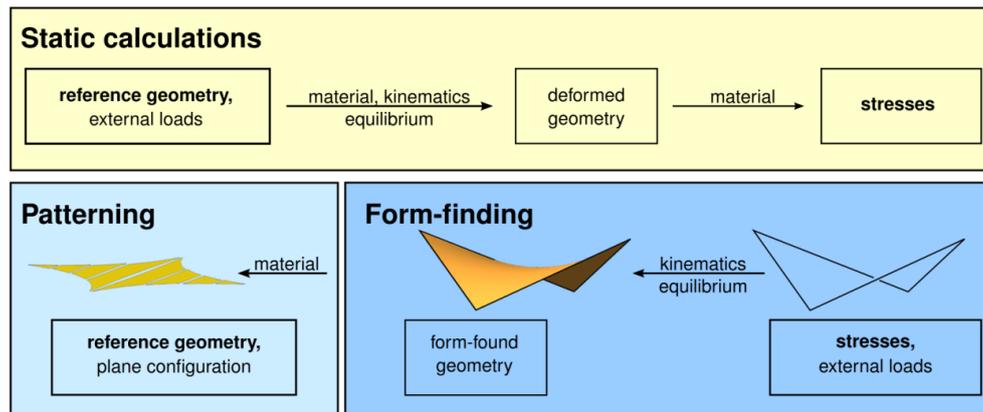
**Figure 2:** Position  $\mathbf{X}_{\text{surf}}$  and  $\mathbf{x}_{\text{surf}}$  of an observed point in the reference configuration and current configuration, respectively. Between the position vectors in the two configurations is the displacement vector  $\mathbf{u}$ .  $\theta^1$  and  $\theta^2$  are the surface coordinates, describing the points position in the parameter space [2].

Based on the kinematic description of the deformation process the strains in the elements are linked to the stresses by the material law. At a first time, linear elastic material behavior is assumed, without losing generality in the element formulation.

Related to the applications in the analysis of architectural membranes, prestress is a crucial structural property. For the case of isotropic prestress, the desired prestress can directly be applied to the surface, regardless of the element orientation. For the case of anisotropic prestress, the projection of the prestress on the structure is an important issue, addressed *e.g.* in [2,9].

### 3 FORM-FINDING WITH THE URS

The most important step in the design of architectural membrane structures certainly is the form-finding. The goal of any form-finding procedure is to define a shape that is in equilibrium with given boundary conditions and a defined prestress state. In principle, form-finding might be considered a very special application of non-linear structural analysis. In contrast to classical structural analysis, where the stress state is determined based on load-dependent displacements, in form-finding this procedure is inverted: the desired stress state is applied as a prescribed parameter, and the shape that brings this state into equilibrium is determined. Therefore form-finding is often termed as “the inverse problem” (cf. Fig. 3).



**Figure 3:** The inverse problem of form-finding, opposed to classical structural analysis [10].

The main challenge in form-finding is related to an in-plane non-uniqueness of shape parameterization, related to the definition of the problem. This instability becomes evident in the singular stiffness matrix of the problem, when solving the resulting non-linear equations in an iterative Newton-Raphson procedure. In general there are various approaches to overcome this singularity, see *e.g.* [11-13] for different solution approaches. In the present context the *Updated Reference Strategy* (URS), as proposed in [5] is used.

The equilibrium in the still unknown, current configuration is formulated with the help of the principle of virtual work as

$$-\delta W_{\text{original}} = -\delta W_{\text{cur}} = -(\delta W_{\text{int}} + \delta W_{\text{ext}}) = \int_{\Omega} (\boldsymbol{\sigma} : \delta \mathbf{e}) \, dv - \int_{\Omega} (\mathbf{p} \cdot \delta \mathbf{u}) \, dv = 0, \quad (1)$$

where the integration domain  $\Omega$  is the current volume,  $\boldsymbol{\sigma}$  the Cauchy stress tensor and  $\delta\mathbf{e}$  the virtual Euler-Almansi strains. The external loading  $\mathbf{p}$  contributes to the virtual work along its virtual displacements  $\delta\mathbf{u}$ .

In order to stabilize the shape in the in-plane direction, a stable formulation of the principle of virtual work for the internal contributions in the reference configuration,  $-\delta W_{\text{regularization}}$ , is introduced,

$$-\delta W_{\text{regularization}} = -\delta W_{\text{int}} = \int_{\Omega_0} (\mathbf{S} : \delta\mathbf{E}) dV, \quad (2)$$

where now  $\mathbf{S}$  is the 2<sup>nd</sup> Piola-Kirchhoff stress tensor and  $\delta\mathbf{E}$  the virtual Green-Lagrange strain tensor. The integration domain – in analogy to the virtual work in the current configuration – now is the reference domain  $\Omega_0$ .

The two descriptions of equilibrium from Eqs. (1) and (2) are then blended, introducing the so-called homotopy factor  $\lambda$ :

$$-\delta W_{\text{URS}} = -\lambda \cdot \delta W_{\text{original}} - (1-\lambda) \cdot \delta W_{\text{regularization}} \quad (3)$$

In Equ. (3) the principle of virtual work for the use in the URS,  $-\delta W_{\text{URS}}$ , is formulated. As long as  $\lambda$  is sufficiently small to stabilize the form-finding problem the system of equations is guaranteed to converge. For the extreme choice of  $\lambda=0$  the pure regularization term is solved. After each form-finding step, the resulting geometry is “updated” to be the starting configuration of the next form-finding step, thus giving the method its name.

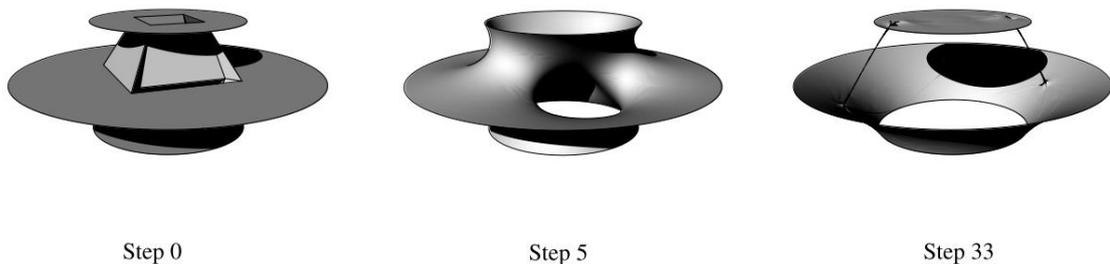
Each of the form-finding steps basically corresponds to a structural analysis in which the prestress is applied to the membrane and – if applicable – the cable. Convergence is achieved, when the prestress state at the end of a form-finding step corresponds to the desired prestress.

## 4 FORM-FINDING WITH ISOGEOMETRIC ANALYSIS

The developed IGA based membrane and cable elements are tested w.r.t. their ability for form-finding in several test cases. These presented examples range from academic benchmarks of minimal surfaces up to real-life structures.

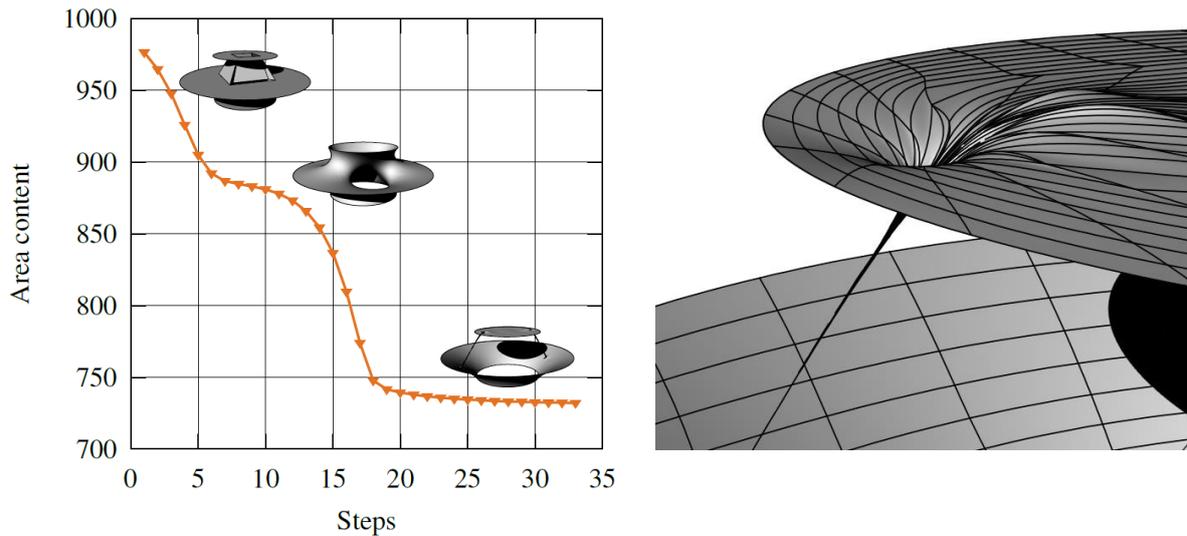
### 4.1 Benchmark-type example: Costa’s minimal surface

An extensive series of benchmarks, presented in [2], proofs the correct formulation of the developed elements. As an example in the present contribution, the form-finding of Costa’s minimal surface is demonstrated, a rather recent and quite complex example of a minimal surface.



**Figure 4:** Form-finding of Costa’s minimal surface. Starting configuration (left), intermediate stable solution (middle), and “collapsed” final solution (right) [2].

Starting with a rough approximation (cf. Fig. 4, left), mainly representing the topology of Costa's minimal surface, the form-finding produces the usually known shape represented in Fig. 4, middle. It is interesting to note that this shape is just an intermediately stable surface. This can be seen in the graph in Fig. 5, left: The surface area stays approximately constant for some form-finding steps, before ultimately the shape collapses to the global minimum, represented by a single disk and a section of a catenoid (cf. Fig. 4, right). This collapse of the shape comes with extreme distortion of the membrane elements, basically they are degenerated to lines as can be seen in Fig. 5, right.

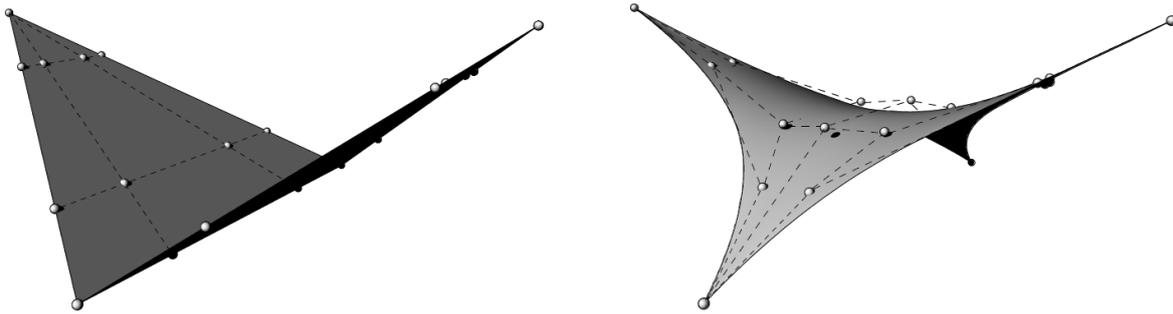


**Figure 5:** Form-finding of Costa's minimal surface: Evolution of the surface area throughout the form-finding steps (left), and heavily distorted elements (right) [2].

Besides classical element benchmarking, the chosen example reveals a first advantage of using IGA in a CAD-integrated form-finding environment. For the modelling of complex structures the supporting tools of a CAD-oriented environment are obviously of help. Recovering a full geometric description of a structure like Costa's minimal surface from a facette-discretized FEM surface is at least very expensive and error-prone.

#### 4.2 Real-life example: a four-point sail with edge cables

The form-finding of a four-point sail with isogeometric membrane and cable elements demonstrates the applicability and their interesting characteristics, see Fig. 6. Even more than it is the case for other structures, the design of architectural membranes is an iterative procedure, looping through different boundary conditions, prestress ratios and form-finding analyses. In this context two aspects are of major interest: Since the form-finding has to be repeated several times, a fast computation due to the very low number of necessary control points as can be seen in Fig. 6 is attractive. Additionally it is of advantage that the common model for the design and for the analysis stays intact during the form-finding. This allows for modifications based on the resulting shape in order to iteratively approach a shape that fulfils esthetical as well as technical requirements.



**Figure 6:** Form-finding of a four-point sail with isogeometric membrane and cable elements. Both the starting configuration (left) and the form-found geometry (right) are discretized by the few control points [2].

A more general discussion of the advantages of the use of IGA in the context of architectural membranes will be presented in the following section.

## 5 APPLICATION OF IGA IN THE DESIGN OF ARCHITECTURAL MEMBRANES

In the design of architectural membranes, one of the major advantages introduced by the use of IGA-based elements certainly is the possibility to use the CAD-environment to directly create the geometry model for the form-finding and structural analysis, as it has been explained above.

With their smoothly curved shape, membrane structures are ideally suited to be modeled with a very limited number of control points when using NURBS as basis functions. This low number of necessary degrees of freedom results – among other advantages – in an impressive speed of computation, without losing quality in the geometry description (see [2] for details).

### 5.1 Geometry-related applications of IGA in the design of architectural membranes

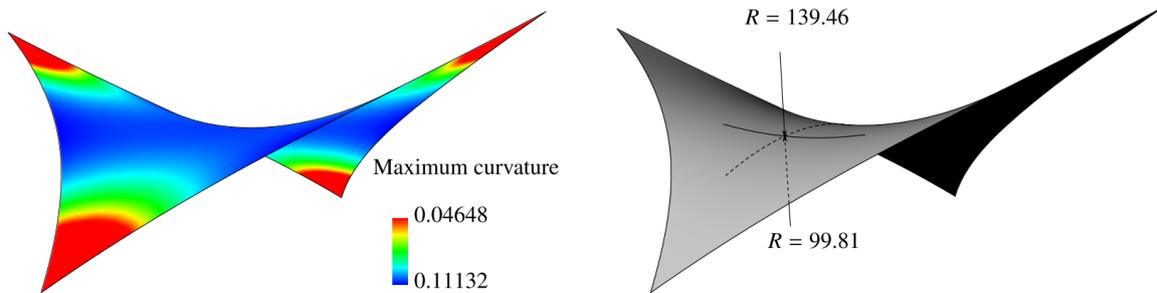
By using NURBS as basis for the geometry description throughout the form-finding and follow-up structural analyses a complete and continuous mathematical description of the membrane's shape is obtained. This continuous description is very interesting for computation approaches, since information like the direction of the surface normal or the tangent to an edge are at the base of many algorithms like *e.g.* analyses taking into account follower loads or sliding cables, respectively.

In terms of design of architectural membranes a continuous description of the surface allows for precise and fast visualization of the membrane in its future surrounding. Without this continuous surface description, a reliable prediction of effects like light reflection or translucency related visual impact is at least very complicated, usually also subject to considerable deviations. Especially for architectural membranes which have very high esthetical requirements and potential, this factor is very important. In this context the existence of a continuous yet reliable geometry could bring important advantages.

The assessment of membrane structures against dynamic loads, induced by wind for instance, is of special importance since the lightweight characteristic and the special load-

bearing behavior make membranes prone to dynamic excitations. One verification approach that is proposed *e.g.* in the French membrane design recommendations [14], consists of assessing the curvature in extreme load cases and checking, whether the sign of curvature stays constant throughout the deformation process. This evaluation of curvature is equivalent to the verification of the curvature radius as shown in Fig. 7, right. If the curvature's sign changes between different loading scenarios, the membrane risks to have gone slack between these load steps, *i.e.* it has lost – at least temporarily – its prestressed state. In case this loss in prestress is related to dynamic loading like wind (*i.e.* fluttering occurs), it may result in a fatigue failure of the membrane, one of the most common failure scenarios [14,15].

In classical FE-environments the evaluation of curvature changes may be quite challenging, since no direct description of the structure's geometry exists. With the presented IGA-membrane and the related continuous surface description, an evaluation of the curvature or any other surface property is possible straightforward, directly resulting from the surface description. Many of the common surface properties are already supported by conventional CAD-environments that are able to treat the NURBS-based geometry description.



**Figure 7:** Evaluation of curvature properties of an architectural membrane: Maximum curvature along the surface (left) and corresponding curvature radius at a selected position (right).

Additionally, curvature-related properties (see Fig. 7, left) are often major design-goals, since they have an important influence on the structure's appearance. With the presented framework, these properties now can easily be verified.

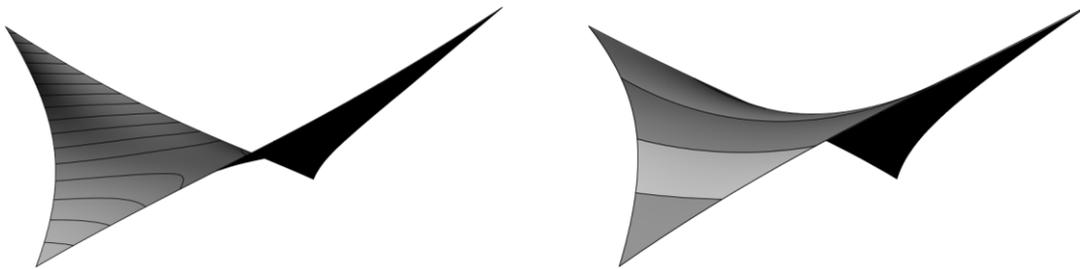
Since membrane structures usually are subject to large displacements, important deformations may occur for different loading scenarios. In order to prevent an increasing accumulation of snow or water, an assessment against ponding is prescribed in many design guides [14,15]. This verification basically is realized by an assessment of positive drainage in all relevant load cases. With the closed surface description of the IGA-membrane and the tools provided by CAD environments, these contour lines may easily be verified and visualized for each scenario (cf. Fig. 8, left).

As a last example for the advantage of a closed surface description of the form-found shape the definition of seam lines, separating the individual patterns of an architectural membrane, may be cited. Usually these seams between the patterns follow geodesic lines, *i.e.* the shortest possible distance on the curved surface [15]. Since the seam lines have an

important impact on the visual effect of the structure, this definition of the geodesic lines is a very important design step.

For FE-discretized surfaces, the determination of geodesic lines as a starting point for the cutting pattern generation is rather complex and often leads to distorted meshes inside the respective patterns. This is especially disadvantageous, since the following cutting pattern generation in general is a rather ill-posed problem, now additionally suffering from bad mesh quality [16].

In contrast to that approach, the definition of seam lines based on the closed surface description of the IGA-membrane is straightforward in a CAD environment and doesn't affect the quality of the geometry description (cf. Fig. 8, right). The visual impact can directly be evaluated in the CAD-environment, thus leading towards better cutting pattern distribution. Based on the obtained seam distribution, the individual patterns could then be meshed in order to enter existing cutting pattern generation approaches. At a long range, the extension towards cutting pattern generation directly based on the derived IGA-membrane elements seems quite promising.

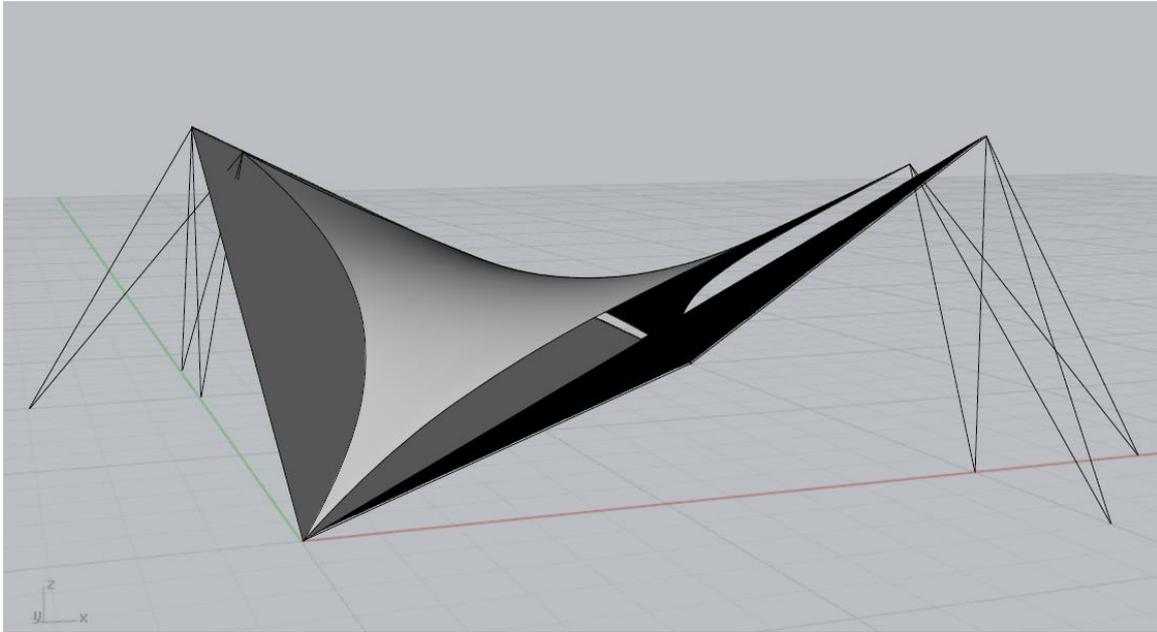


**Figure 8:** Visualization of geometry related entities on a four point sail with prestressed edge cables: contour lines for the assessment against ponding, visualized under snow load (left), and geodesic lines for placing the seam lines between individual patterns (right).

## 5.2 CAD-CAE-integration

It is evident that a closer integration of CAD and CAE presents important advantages. At a first glance one might consider the visualization and evaluation possibilities enabled by the use of the CAD environment. In addition, the modeling in fully developed CAD-environments might be easier compared to many of the known preprocessors, cf. *e.g.* the modelling of the starting configuration for Costa's minimal surface in Fig. 4, left.

Moreover the integration of both design and analysis in one single environment is very favorable in case of modifications: Since CAD and CAE use a common geometry description, enriched by the respective specific data, any modification in the geometry is straightforward integrated in all other design and analysis steps. For architectural membranes this aspect is of special interest: Since esthetical and mechanical decisions cannot be separated, their design usually takes several iteration loops until all technical and esthetical requirements are finally met. Here a manual update of the various models is quite time-consuming, expensive and rather error prone. Working in one single common framework (cf. Fig. 9 for an example) therefore seems quite attractive.



**Figure 9:** Form-finding of a four-point sail with the *Rhinceros*®-plugin, developed at the authors' chair [2].

Nevertheless one should mention that this integration of CAD and CAE also represents an important challenge. Keeping the data for both the architectural design (like *e.g.* texture, reflection properties, translucency,...) and structural engineering (like *e.g.* stiffness, ultimate stresses,...) as well as the common geometrical properties like thickness and – especially – keeping it consistently, requires a careful setup of the environment.

## 6 CONCLUSIONS AND OUTLOOK

In the present contribution a first introduction to the design of architectural membranes with isogeometric elements has been given, especially focusing on the advantages that are related to the use of this relatively recent technique.

Very briefly the basics of the isogeometric analysis have been outlined. Based on this technique, the development and formulation of IGA membrane and cable elements has been sketched, incorporating the requirements for the use in the simulation of architectural membranes. These elements – like IGA-elements in principle – allow keeping the original, continuous geometry description from CAD. Furthermore they are very well suited to describe the smoothly curved shape of membrane structures. The developed IGA-elements have extensively been benchmarked for accuracy and performance.

In the design of architectural membranes form-finding is the central step. In the present contribution, the Updated Reference Strategy as a form-finding approach has been sketched in its principles. The challenging benchmark of form-finding of Costa's minimal surface has proven the developed elements to be correct and entirely capable of modeling tensile structures in form-finding and structural analysis.

The real-life example of a four-point tent has served as demonstration for various

advantages of the use of IGA-elements in architectural membrane's design. These advantages are mainly related to the continuous surface description of the membrane's shape that stays intact throughout all design steps. As examples the evaluation of geometry related properties like reflection effects, curvature evaluations, assessment against ponding and the determination of seam lines along geodesic lines have been discussed. The advantages and challenges related to a closer CAD-CAE-integration have briefly been outlined. This integration has already been realized in the form of an IGA-*Rhinoceros*®-plugin that is continuously developed at the authors' chair.

Future work might include the integration of stress adaption for the case of anisotropic prestress ratios as presented in [9,16]. This would allow for solutions for tensile structures requiring compromises concerning the desired prestress state in order to be able to find stable equilibrium.

A generic extension to the presented framework is the development for a cutting pattern generation on the basis of a NURBS-description of the individual patterns. Even taking the first step of dividing the membrane into strips along the geodesic lines in the IGA-surface and then creating a common FE-mesh out of these strips could help to increase the convergence behavior of established algorithms for cutting pattern generation. Nevertheless a complete framework for cutting pattern generation of membranes with IGA-elements seems very promising and shall be explored.

Additionally further integration of CAD and CAE but also further exploration of the coupling of elements based on different geometry descriptions, *i.e.* coupling classical FE-elements and IGA-elements, still provides interesting routes of research.

**Note:** All structural computations in this contribution have been performed in the FE-code *CARAT++*, continuously developed at the Chair of Structural Analysis at the Technische Universität München. The mentioned IGA-*Rhinoceros*®-plugin has been used, where *CARAT++* is linked.

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