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A COMPARISON OF ZIGZAG FUNCTIONS FOR THE BENDING, VIBRATION AND BUCKLING ANALYSIS OF MULTILAYERED COMPOSITE AND SANDWICH PLATES

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Abstract. In the framework of the Zigzag theories, an important role is played by the zigzag function. In the open literature, two kind of zigzag functions exist. A comparison of the advantages in adopting one zigzag function rather than the other is compulsory. In this work, with a general formalism, a displacement-based model wherein the First Order Shear Deformation (FSDT) kinematic is enriched by adding a zigzag contribution only to the in-plane displacements description, is developed. By selecting the zigzag function, two models arise from the general formulation. Comparison on the response predictive capabilities ensured by the two zigzag functions, is made. Results pertaining the elastostatic deformation, free vibrations and buckling load problems of square sandwich plates subjected to several load and boundary conditions are compared with exact Elasticity solutions, when available, or high-fidelity FE model.

1 INTRODUCTION

In modeling multilayered composite and sandwich structures, the main challenge is to capture the slope variation of the in-plane displacements distribution along the thickness direction of a beam/plate/shell component. The mismatch in mechanical properties between two adjacent layers requires to model a jump in the shear strain at layers interface in order to satisfy the equilibrium condition of the interlaminar transverse shear stresses. In the framework of the displacement-based models, two approaches are available: (i) Layer-Wise models [1], accurate but computationally expensive; (ii) Zigzag models, firstly introduced by Di Sciuva [2,3], where a zigzag contribution, accounting for the slope variation of the through-the-thickness in-plane displacements, is added to a smeared displacement field. The Zigzag models save the computational cost ensuring an accuracy comparable with that of the LW theories.

Assuming the same polynomial distribution, Zigzag models stand out based on the zigzag function adopted. In the open literature, two zigzag functions, ascribable to their original authors, exist: (i) Di Sciuva's type zigzag function [2-4]; (ii) Murakami's type zigzag function

[5]. The former is derived by enforcing the transverse shear stresses continuity at layers interface. Following the Di Sciuva' idea, several zigzag models arose [6-8]. Even though physically correct, the models developed have as main drawback a vanishing transverse shear stresses at the clamped edge [9]. To overcome this problem retaining the great accuracy in predicting the through-the-thickness distribution of in-plane displacements proper of the original Di Sciuva's work [2], Tessler, Di Sciuva and Gherlone [10] formulated the Refined Zigzag Theory (RZT). The RZT is a displacement-based approach wherein the FSDT kinematics is enriched by adding to the in-plane displacements a piece-wise and continuous linear contribution. The zigzag function is derived enforcing only a partial continuity condition of transverse shear stresses at layers interface. This model, even thought violates the equilibrium at interfaces, ensures an estimation of the transverse shear stresses, piece-wise constant along the thickness, correct in a average sense along with an accurate prediction of in-plane displacements. From this point, we refer to the RZT zigzag function (RZT-F) as Di Sciuva's type function. On the contrary, Murakami [5] developed a mixed first-order zigzag model via the Reissner Mixed Variational Theorem [11]. The Murakami's type zigzag function (MZZ-F) is derived considering the distribution of in-plane displacements proper of periodic laminates, that is laminates with stacking sequence (a/b/a/b/..). Then, the MZZ-F is not a physically-based zigzag function and its limits have been remarked by its same author [12]. Despite the author of the MZZ-F warned the scientific community about the drawbacks of his zigzag functions, the use of the MZZ-F in the formulation of first- and higher-order, both displacement-based and mixed, models is wide in the open literature and some examples of application are found in [13-15].

The need for a comparison between the two zigzag functions performances has been underrated for a long time. Recently, Gherlone [16] has covered this lack in literature by performing a deep investigation on the capabilities of the two zigzag functions both when used in a displacement-based approach and a mixed one. By means of extensive numerical investigation concerning the bending problem of multilayered and sandwich beams, the author remarks the advantages in adopting the RZT-F over the MZZ-F.

Aim of this paper is to extend the deep investigation performed by Gherlone [16] to the bending, free vibration and buckling load problems of sandwich plates. To compare the predictive capabilities of the two zigzag functions, two first-order zigzag models are taken into consideration. The first is the RZT; the second, called MZZ, consists in an enrichment of the FSDT kinematics, wherein to the in-plane displacements approximation, a zigzag contribution ruled by the MZZ-F is added. Moreover, by comparison with FSDT, adopting a suitable shear correction factors, the advantages in adding a zigzag contribution to the FSDT is assessed. Results are compared with the exact Elasticity solution, when available, and with high-fidelity FE models.

2 KINEMATICS AND FORMULATION

In this section, the kinematic assumptions proper of a first-order zigzag model are presented.

Consider a laminated plate of uniform thickness *2h* with *N* perfectly bonded orthotropic layers as shown in Figure 1. The orthogonal Cartesian coordinate system $(\mathbf{x}, z)$ is taken as reference where the thickness coordinate *z* ranges from -*h* to *h*. The middle reference plane (or midplane) of the plate, S_m, is placed on the **x**-plane. The plate is bounded by a cylindrical edge surface, S, constituted by two distinct surfaces, S_u and S_σ, on which the geometrical and mechanical boundary conditions are enforced, respectively. Moreover, the intersection of the surface S and of the **x** -plane is the curve C which represents the perimeter of the midplane, S_m. As for the edge surface, the curve C is composed by two distinct curves, C_u and C_σ, originated by the intersection of S_u and S_σ with the **x** -plane, respectively.

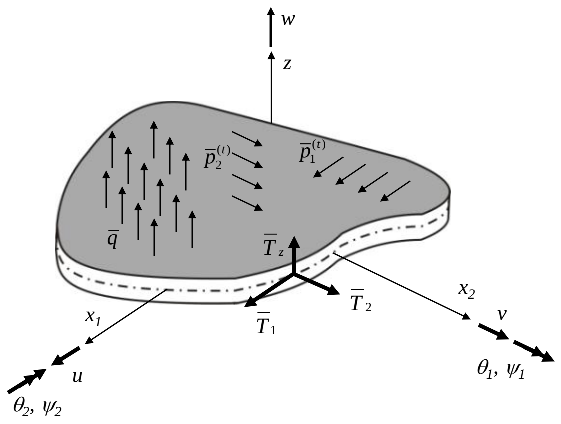

Figure 1. General plate notation.

Formally, the RZT and the MZZ share the same kinematic description:

$$\begin{aligned} &U_\alpha^{(k)}(\mathbf{x},z) = u_\alpha(\mathbf{x}) + z\theta_\alpha(\mathbf{x}) + f_\alpha^{(k)}(z)\psi_\alpha(\mathbf{x}); \qquad (\alpha = 1,2) \\ &U_z(\mathbf{x},z) = w(\mathbf{x}) \end{aligned} \tag{1}$$

wherein $U_\alpha^{(k)}$ and U_z are the in-plane displacements and the transverse one respectively, and the superscript *(k)* means that the quantities is related to the *k*th layer. The first-order zigzag model results as the superposition of the FSDT kinematic field and a piece-wise linear and continuous contribution, i.e. the zigzag contribution, given by the product of *a priori* known zigzag function, $f_\alpha^{(k)}(z)$, and the relative zigzag amplitude, $\psi_\alpha(\mathbf{x})$. To the five kinematic variables, proper of the FSDT ($u_\alpha(\mathbf{x})$, $\theta_\alpha(\mathbf{x})$, $w(\mathbf{x})$), the first-order zigzag model adds the two zigzag amplitudes, $\psi_\alpha(\mathbf{x})$, resulting in seven kinematic unknowns, independent of the number of layers.

Consistent with the displacement field in Eq.(1), the non-linear (in the Von Kàrmàn sense) in-plane and transverse shear strains are

$$2\varepsilon_{\alpha\beta}^{(k)} = u_{\alpha,\beta} + u_{\beta,\alpha} + z\left(\theta_{\alpha,\beta} + \theta_{\beta,\alpha}\right) + f_\alpha^{(k)}\psi_{\alpha,\beta} + f_\beta^{(k)}\psi_{\beta,\alpha} + w_{,\alpha}w_{,\beta}; \quad \gamma_{\alpha z}^{(k)} = \theta_\alpha + w_{,\alpha} + \beta_\alpha^{(k)}\psi_\alpha \tag{2}$$

The generalized Hooke's law for the *k*th orthotropic lamina, whose principal material directions are arbitrary with respect to the reference coordinate system reads

$$\sigma_{\alpha\beta}^{(k)} = C_{\alpha\beta\gamma\delta}^{(k)}\varepsilon_{\gamma\delta}^{(k)}; \quad \tau_{\alpha z}^{(k)} = Q_{\alpha\beta}^{(k)}\gamma_{\beta z}^{(k)} \tag{3}$$

where $C_{\alpha\beta\gamma\delta}^{(k)}$ and $Q_{\alpha\beta}^{(k)}$ are the transformed elastic stiffness coefficients referred to the $(\mathbf{x},z)$

coordinate system and relative to the plane-stress condition that assumes that transverse normal stress is negligibly small in relation to the in-plane stresses.

2.1 Refined Zigzag function

This section briefly recalls the key features of the RZT-F, herein quoted as $f_{RZT_\alpha}^{(k)}(z)$. By using the strain-displacement relations (Eqs. (2)), the shear strain deformation read as

$$\gamma_\alpha^{(k)} = \theta_\alpha + w_{,\alpha} + \beta_{RZT_\alpha}^{(k)}\psi_\alpha = \eta_\alpha + \left(1+\beta_{RZT_\alpha}^{(k)}\right)\psi_\alpha \tag{4}$$

where $\beta_{RZT_\alpha}^{(k)}$ denotes the first derivative with respect to the thickness coordinate of $f_{RZT_\alpha}^{(k)}(z)$, and the strain measure $\eta_\alpha \equiv \theta_\alpha + w_{,\alpha} - \psi_\alpha$ is introduced. According to Eqs.(3), the transverse shear stress follows

$$\tau_{\alpha z}^{(k)} = Q_{\alpha\chi}^{(k)}\eta_\chi + Q_{\alpha\chi}^{(k)}\left(1+\beta_{RZT_\chi}^{(k)}\right)\psi_\chi \tag{5}$$

The transverse shear stress in Eq.(5) are composed by two contribution: the first is zigzag function-independent, $Q_{\alpha\chi}^{(k)}\eta_\chi$, whereas the second, $Q_{\alpha\chi}^{(k)}\left(1+\beta_{RZT_\chi}^{(k)}\right)\psi_\chi$, is related with the zigzag function by means of its derivative with respect to the thickness coordinate, $\beta_{RZT_\chi}^{(k)}$. According to the RZT [10], the continuity condition at layers interfaces of the zigzag-dependent transverse shear stress (when $\chi = \alpha$) is enforced, that is the second contribution on the right hand side of Eq. (5) is stated to be constant and equal to G_α. In formula

$$Q_{\alpha\alpha}^{(k)}\left(1+\beta_{RZT_\alpha}^{(k)}\right) = G_\alpha \tag{6}$$

Solving Eq.(6), the first derivative of the zigzag function reads

$$\beta_{RZT_\alpha}^{(k)} = \frac{G_\alpha}{Q_{\alpha\alpha}^{(k)}} - 1 \tag{7}$$

where the constant G_α is a weighted –average transverse shear stiffness coefficient of their respective lamina-level coefficients $Q_{\alpha\alpha}^{(k)}$. For sake of brevity, only the final expression is given for G_α, that is

$$G_\alpha = \left(\frac{1}{2h}\int_{-h}^{h}\frac{dz}{Q_{\alpha\alpha}^{(k)}}\right)^{-1} \tag{8}$$

To completely define the zigzag function $f_{RZT_\alpha}^{(k)}(z)$, the zero-condition at the top and bottom plate surface is enforced, that is

$$f_{RZT_\alpha}^{(k)}(z=-h) = f_{RZT_\alpha}^{(k)}(z=h) = 0 \tag{9}$$

Readers interested to a detailed derivation of the RZT-F can refer to [10].

2.2 Murakami's zigzag function

Murakami [5] derived his zigzag function based on the distribution of the in-plane displacements along the thickness of periodic laminates. For this reason, the MZZ-F reflects a periodic nature, that is it oscillates between the values -1 and +1. The MZZ-F reads as

$$f_{MZZ_\alpha}^{(k)}(z) = (-1)^k \zeta^{(k)} \tag{10}$$

where $\zeta^{(k)} \in [-1,+1]$ is a local nondimensional thickness coordinate of kth lamina, defined as $\zeta^{(k)} \equiv \left(z - z_m^{(k)}\right)/h^{(k)}$ where $z_m^{(k)}$ is the distance of the kth layer mid-plane from the reference plane. Figure 2 compares the two zigzag functions for a three-layer laminate.

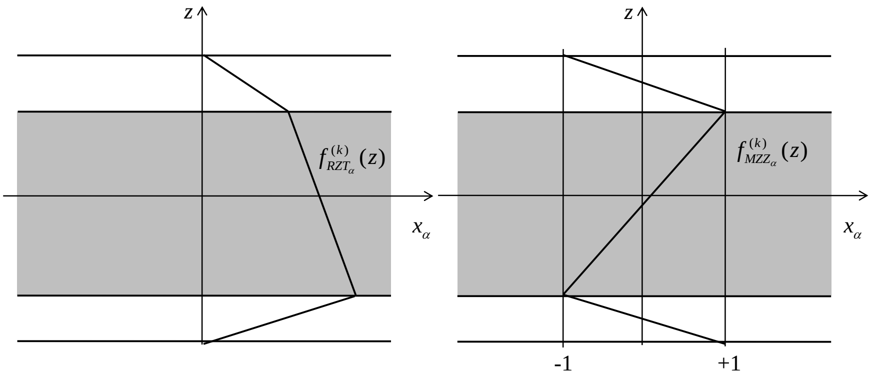

Figure 2. Comparison on the through-the-thickness distribution of zigzag functions.

3 GOVERNING EQUATIONS

The plate represented in Figure 1 is subjected to a transverse pressure loading, $\bar{q}(\mathbf{x},t)$, applied on the midplane S_m, to surface tractions, $\bar{p}_1^t(\mathbf{x},t)$ and $\bar{p}_2^t(\mathbf{x},t)$, acting on the top surface, and $\bar{p}_1^b(\mathbf{x},t)$ and $\bar{p}_2^b(\mathbf{x},t)$, acting on the bottom surface. Traction stresses, $(\bar{T}_1, \bar{T}_2, \bar{T}_z)$, are also prescribed on S_σ.The non-linear (in the Von Kàrmàn sense) plate governing equations and boundary conditions are derived from the D'Alembert's principle and read as [17]

$$\begin{aligned} &N_{\alpha\beta,\beta} + \bar{p}_\alpha = I_0 \ddot{u}_\alpha + I_1 \ddot{\theta}_\alpha + I_0^{f_\alpha} \ddot{\psi}_\alpha \qquad M_{\alpha\beta,\beta} - Q_\alpha + \bar{m}_\alpha = I_1 \ddot{u}_\alpha + I_2 \ddot{\theta}_\alpha + I_1^{f_\alpha} \ddot{\psi}_\alpha \\ &M_{\alpha\beta,\beta}^f - Q_\alpha^f = I_0^{f_\alpha} \ddot{u}_\alpha + I_1^{f_\alpha} \ddot{\theta}_\alpha + I_2^{f_\alpha} \ddot{\psi}_\alpha \qquad Q_{\alpha,\alpha} + \left(N_{\alpha\beta,\beta} w_{,\beta}\right)_{,\alpha} + \bar{q} = I_0 \ddot{w} \end{aligned} \tag{11}$$

where the force and moment stress resultants, the mass moments of inertia and the external loads are defined as

$$\begin{aligned} &\left(N_{\alpha\beta}, M_{\alpha\beta}, M_{\alpha\beta}^\phi\right) = \int_{-h}^{h} \sigma_{\alpha\beta}^{(k)} \left(1, z, f_\alpha^{(k)}(z)\right) dz; \quad \left(Q_\alpha, Q_\alpha^\phi\right) = \int_{-h}^{h} \left(\tau_{\alpha z}^{(k)}, \beta_\alpha^{(k)} \tau_{\alpha z}^{(k)}\right) dz \\ &\left(I_j, I_j^{f_\alpha}\right) \equiv \int_{-h}^{h} \rho^{(k)} z^j \left(1, f_\alpha^{(k)}(z)\right) dz; \;\; I_2^{f_\alpha} \equiv \int_{-h}^{h} \rho^{(k)} \left(f_\alpha^{(k)}(z)\right)^2 dz; \; \bar{p}_\alpha \equiv \bar{p}_\alpha^t + \bar{p}_\alpha^b; \;\; \bar{m}_\alpha \equiv h\left(\bar{p}_\alpha^t - \bar{p}_\alpha^b\right) \end{aligned} \tag{12}$$

The D'Alembert principle leads also to the variationally consistent boundary conditions

$$u_\alpha = \bar{u}_\alpha \text{ on } C_u \wedge N_{\alpha\beta} n_\beta = \overline{N}_{\alpha n} \text{ on } C_\sigma \quad \theta_\alpha = \bar{\theta}_\alpha \text{ on } C_u \wedge M_{\alpha\beta} n_\beta = \overline{M}_{\alpha n} \text{ on } C_\sigma$$
$$\psi_\alpha = \bar{\psi}_\alpha \text{ on } C_u \wedge M^f_{\alpha\beta} n_\beta = \overline{M}^f_{\alpha n} \text{ on } C_\sigma \quad w = \bar{w} \text{ on } C_u \wedge Q_\alpha n_\alpha + \left(N_{\alpha\beta} w_{,\beta}\right) n_\alpha = \overline{V}_{zn} \text{ on } C_\sigma \tag{13}$$

wherein the force and moment of the prescribed tractions are

$$\left(\overline{N}_{\alpha n}, \overline{M}_{\alpha n}, \overline{M}^f_{\alpha n}, \overline{V}_{zn}\right) \equiv \int_{-h}^{h} \left(\overline{T}_\alpha, z\overline{T}_\alpha, f^{(k)}_\alpha \overline{T}_1, \overline{T}_z\right) dz \tag{14}$$

By Introducing Eqs.(2) and Eqs.(3) in Eqs.(12) and integrating, the model constitutive equations are derived.

4 NUMERICAL RESULTS

In order to compare the two zigzag functions, the bending, free vibrations and buckling problems of square sandwich plates are solved.

Mechanical properties of materials and stacking sequences taken into consideration are listed in Tables 1 and 2.

Table 1. Mechanical properties of materials; elastic moduli are expressed in GPa, the density in kg/m^3.

Material	$E_1^{(k)}, E_2^{(k)}, E_3^{(k)}$	$\nu_{12}^{(k)}, \nu_{13}^{(k)}, \nu_{23}^{(k)}$	$G_{12}^{(k)}, G_{13}^{(k)}, G_{23}^{(k)}$	$\rho^{(k)}$
C	50,10,10	0.25, 0.25, 0.25	5,5,5	-
N	10^{-5}, 10^{-5},75.85×10^{-3}	0.01,0.01,0.01	22.5×10^{-3},22.5×10^{-3},22.5×10^{-3}	-
B	276, 6.9, 6.9	0.25, 0.25, 0.3	6.9, 6.9, 6.9	681.8
H	0.5776,0.5776, 0.5776	0.0025, 0.0025, 0.0025	0.1079, 0.1079, 0.22215	1000
G	19,1,1	0.32,0.32,0.49	0.52.0.52,0.338	-
P	3.2×10^{-5}, 2.9×10^{-5},0.4	0.99, 3×10^{-5}, 3×10^{-5}	2.4×10^{-3}, 7.9×10^{-2}, 6.6×10^{-2}	-

Table 2. Laminate stacking sequences (from bottom to top surface).

Laminate	Normalized lamina thickness, $2h^{(k)}/2h$	Lamina materials	Lamina orientation
L1	(0.1/0.8/0.1)	(C/N/C)	(0°/Core/0°)
L2	(0.05/0.05/0.8/0.05/0.05)	(C/C/N/C/C)	(0°/90°/ Core /90°/0°)
L3	$(0.5t_f/0.5t_f/t_c/0.5t_f/0.5t_f)$	(B/B/ H/ B/ B)	(90°/0°/Core/90°/0°)
L4	$(0.1t_f/0.1t_f)_5/t_c/(0.1t_f/0.1t_f)_5$	$(G/ G)_5$/ P/$(G/ G)_5$	$(0°/90°)_5$/Core/$(90°/0°)_5$

4.1 Linear bending

The numerical results reported in this section pertain the linear boundary value problem of bending of simply supported square sandwich plates subjected to bi-sinusoidal transverse pressure. Two sandwiches are considered (laminate L1 and L2), made by the same materials for faces and core but with different stacking sequence. Figure 3 compares the through-the-thickness distribution of normalized in-plane displacements and normalized transverse shear stresses obtained by the RZT and the MZZ. In order to assess the benefit given by the inclusion of a zigzag function in the FSDT kinematic, results obtained with the FSDT adopting a suitable shear correction factor are also presented. The reference solution is the exact Elasticity solution as derived by Pagano [18].

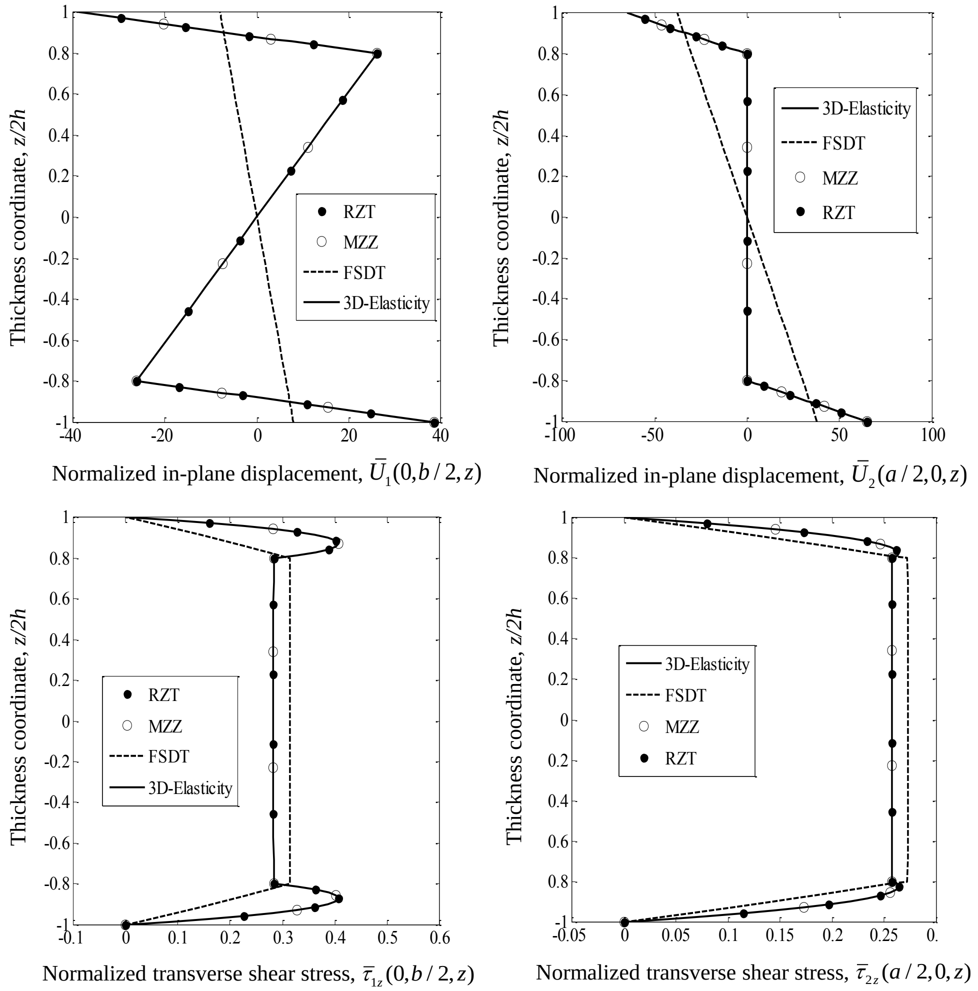

Figure 3. Laminate L1, *a/2h*=6: through-the-thickness distribution of normalized in-plane displacement, $\bar{U}_\alpha = \left(10^4 D_{11}/q_0 a^4\right) U_\alpha^{(k)}$ and normalized transverse shear stress, $\bar{\tau}_{\alpha z} = \left(2h/q_0 a^2\right)\tau_{\alpha z}^{(k)}$. The FSDT solution is obtained with $k_x^2 = k_y^2 = 0.022$.

If compared with the reference solution, both RZT and MZZ provides accurate results for the in-plane displacements and transverse shear stresses, obtained by integration of the equilibrium equations. Instead, the FSDT is not able to follow the exact solution.

By changing the stacking sequence, the laminate L2 is obtained. For this laminate, distribution along the laminate thickness of normalized in-plane displacements and integrated transverse shear stresses are shown in Figures 4.

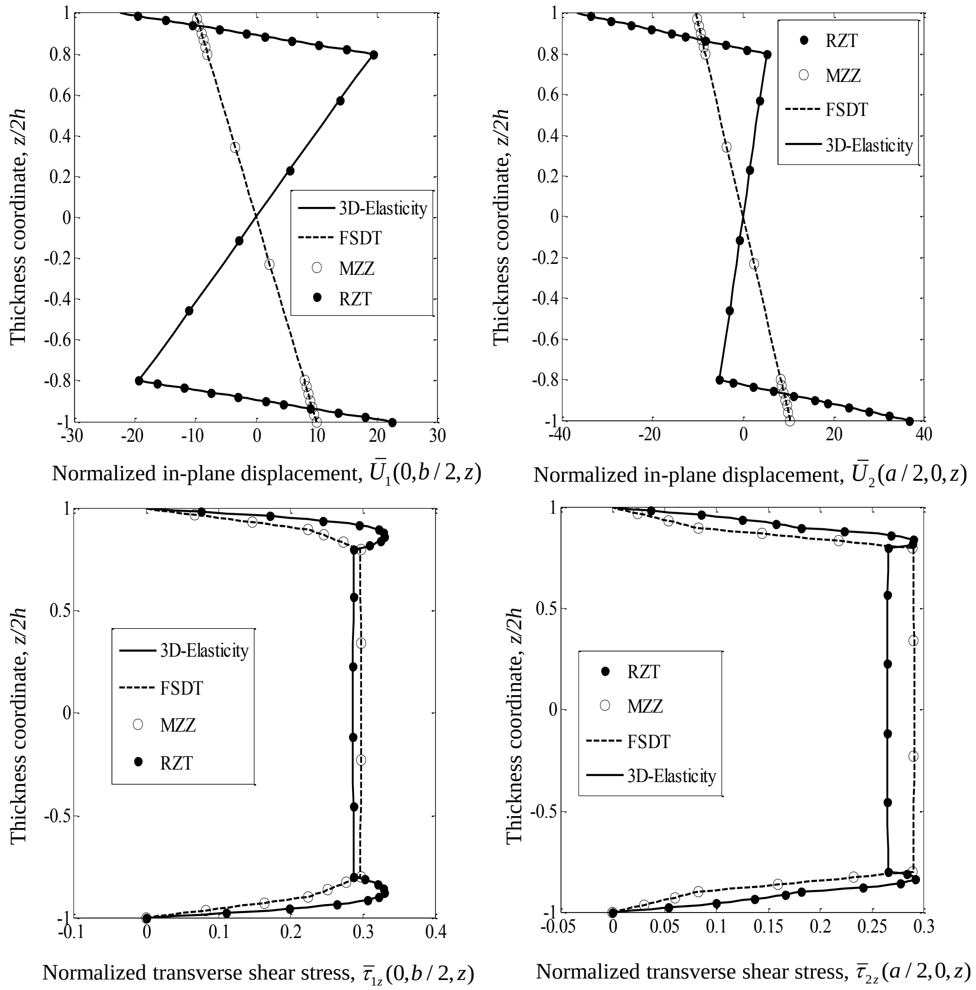

Figure 4. Laminate L2, *a/2h*=6: through-the-thickness distribution of normalized in-plane displacement, $\bar{U}_\alpha = \left(10^4 D_{11}/q_0 a^4\right) U_\alpha^{(k)}$ and normalized transverse shear stress, $\bar{\tau}_{\alpha z} = \left(2h/q_0 a^2\right)\tau_{\alpha z}^{(k)}$. The FSDT solution is obtained with k_x^2= 0.0242, k_y^2=00225.

Considering results in Figure 4, the RZT preserves its great accuracy if compared with the Elasticty solution demonstrating an adaptive behavior. On the contrary, MZZ leads to significant error since the solution fits with the FSDT one in every *z* location.

For laminate L2, a comparison on the normalized maximum deflection for several values of the span-to-thickness ratio, *a/2h*, is in Table 3. Solution obtained with the FSDT adopting an adequate value of the shear correction factor [17] and a unit one is listed. Comparison performed in Table 3 demonstrates the great accuracy of RZT for every value of *a/2h* since its solution is in agreement with the reference one. The FSDT improves its accuracy if a suitable shear correction factor is adopted, even if the through-the-thickness distributions of in-plane displacements and transverse shear stresses are far from the exact pattern (see Figure 4). The MZZ predicts the same deflections than the FSDT model with unitary shear correction factor. In fact, the only difference between this two model relies on the zigzag contribution. If the zigzag function used is not well established, the zigzag contribution tends to vanih thus leading the MZZ to coincide with the FSDT model with $k_x^2 = k_y^2 = 1$.

Table 3. Laminate L2: normalized maximum deflection, $\bar{w} = (10^2 D_{11} / q_0 a^4) w(a/2, b/2)$; k^2 is the shear correction factor.

$a/2h$	3D-Elasticity	RZT	MZZ	FSDT	
				$k_x^2 = 1;\ k_y^2 = 1$	$k_x^2 = 0.0242$ $k_y^2 = 0.0225$
6	8.038	8.040	0.573	0.573	8.243
10	3.254	3.253	0.456	0.456	3.217
20	1.118	1.118	0.406	0.406	1.097
50	0.507	0.507	0.392	0.392	0.503
100	0.419	0.419	0.390	0.390	0.418

The bending problems solved in this section remark the periodic nature of the MZZ-F and its limited applicability only to periodic laminates (like the L1 lamiante) whereas the MZZ-F becomes complitely inadequate for those laminates without a periodic stacking sequence (like the L2 laminate).

4.2 Free vibrations

In this section, the problem of undamped free vibration problem of fully clamped square sandwich plate is solved. For this kind of boundary condition, the exact RZT and MZZ solution do not exist and an approximate one has been developed by using the Rayleigh-Ritz method. For the details, refer to [17].

Table 4 lists the first six natural frequencies obtained by means of the RZT, MZZ model and the FSDT adopting both an adequate and a unit value shear correction factor. Results from a three-dimensional finite element analysis [19], are assumed as reference in the comparison.

Results in Table 4 demonstrate the superior predictive capabilities of RZT that preserves its accuracy changing the boundary conditions, stacking sequence and nature of the problem. The FSDT model is able to provide results close to the reference ones only if an appropriate shear correction factor is used; when the unit value is assumed for k_x^2 and k_y^2, relative error greater than the 140% since the first natural frequency is obtained. The erroneous overestimation of the natural frequencies concerns also the MZZ that provides the same results than the FSDT with unit shear correction factors. As for the bending problem of laminate L2, laminate L3 is not a periodic one then the MZZ-F is not adequate to simulate the elastic behavior of this laminate.

Table 4. Laminate L3, *a/2h*=10, core-to-face thickness ratio, t_c/t_f=8: comparison on the first ten non-dimensional circular frequencies, $\bar{\omega}_{mp} = 100\omega_{mp} a\sqrt{\left(\rho_c/E_{1f}\right)}$, where ρ_c is the mass density of the core and E_{1f} is the longitudinal Young's modulus of the face.

Mode: *m,p*	3D FE [19]	RZT	MZZ	FSDT	
				$k_x^2=1;k_y^2=1$	$k_x^2=0.0834;$ $k_y^2=0.1445$
1,1	11.22	11.44	27.76	27.76	11.18
2,1	16.68	16.46	45.03	45.03	16.09
1,2	18.96	19.81	45.71	45.72	19.05
3,1	22.71	22.92	58.11	58.11	22.20
2,2	23.53	23.16	67.32	67.32	22.36
3,2	28.07	28.19	68.77	68.77	27.11

4.3 Linear buckling

In this section, the linearized problem of buckling of a simply supported square sandwich plate subjected to a compressive load, $\bar{N}_1$, is solved. It is assumed that the plate remains flat during the pre-buckling equilibrium state and that the external in-plane stress resultants vary neither in magnitude nor in direction during buckling. For details on the solution procedure, readers can refer to [17].

Table 5. Laminate L4: comparison of uni-axial overall buckling load parameter, $\bar{n}_1 = \bar{N}_1^{cr} b^2 / \left(E_{2f} h^3\right)$ where $\bar{N}_1^{cr}$ is the uniform uni-axial critical load and E_{2f} is the transverse Young's modulus of the face.

$t_f/2h$	0.025			0.05			0.1		
a/2h	5	10	20	5	10	20	5	10	20
3D [20]	1.503	2.238	2.554	2.082	3.737	4.659	2.605	5.608	7.897
RZT	1.539	2.263	2.566	2.115	3.765	4.681	2.628	5.633	7.921
MZZ	1.676	2.334	2.588	2.509	4.108	4.806	3.517	6.905	8.474
FSDT									
$k_x^2 = k_y^2 = 1$	1.682	2.337	2.589	2.622	4.122	4.811	4.029	6.952	8.491
k_x^2, k_y^2	1.539	2.263	2.566	2.116	3.767	4.682	2.620	5.638	7.926
	(k_x^2=0.820, k_y^2=0.782)			(k_x^2=0.697, k_y^2=0.643)			(k_x^2=0.541, k_y^2=0.479)		

In Table 5, results on the overall uni-axial buckling load parameters obtained with the RZT, MZZ and FSDT are listed. A 3D solution [20] is taken as reference in the comparison. The results investigate also the effect of the face-to-thickness ratio, $t_f/2h$, and the span-to-thickness ratio, $a/2h$.

The RZT results fits with the reference solution on the entire range of face-to-thickness and span-to-thickness ratio considered. The FSDT allows prediction of the buckling load parameter of the same RZT accuracy only when a suitable shear correction factor is implemented. Instead, the FSDT with a unit shear correction factor leads to errors, up to 7,5 %, that decrease with the reduction of the face-to-thickness ratio as the sandwich tends to become a single layer plate, made only by the core. When the MZZ model is used, results in agreement with the FSDT with a $k_x^2 = k_y^2 = 1$ are obtained as result of the non periodicity of the sandwich. Then, also in this case, the MZZ-F reveals not adequate zigzag function.

5 CONCLUSIONS

The aim of this paper is to compare the capabilities of the two zigzag functions available in literature (the Refined Zigzag Theory function, RZT-F, and the Murakami's zigzag one, MZZ-F) in predicting response of sandwich plates, both simply supported and clamped, covering a lack in the open literature.

The zigzag functions are involved in the formulation of a general first-order zigzag model, wherein the First-Order Shear Deformation (FSDT) kinematic is enriched by adding a zigzag contribution, given by the product of a zigzag amplitude and a zigzag function. Depending on the zigzag function adopted, two first-order zigzag models are obtained.

Results reported for the bending, free vibrations and buckling load problems demonstrate superior predictive capabilities of the first-order zigzag model employing the RZT-F over that adopting the MZZ-F. The MZZ-F provides results accurate as those of the RZT-F only for periodic laminates. When the periodic structure of the stacking sequence is removed, the first-order zigzag model using the MZZ-F leads to an heavily underestimation of the maximum deflection and overestimation of natural frequencies and buckling loads. It is worth to note that when the non periodic laminates are considered, the first-order zigzag model adopting the MZZ-F leads to the same results provided by the FSDT adopting a unit shear correction factor. In fact, if the zigzag function is not adequate, the zigzag rotation becomes small enough to get the zigzag correction almost ineffective on the FSDT. Then, the first-order zigzag model using the MZZ-F and the FSDT, with a unit shear correction factor, tend to be the same model.

By considering the numerical results presented in this work, contrary to the wide use established in the open literature, the authors strongly recommend the adoption of the Refined Zigzag function, instead of the Murakami's one, in modeling the laminate and sandwich structures.

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