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PRATICAL STATIC CALCULATION METHOD FOR ESTIMATING ELASTO-PLASTIC DYNAMIC RESPONSES OF SPACE FRAMES

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Abstract. The study deals with aluminum alloy spatial truss walls with fuse type joint connections subjected to horizontal earthquake waves. The fuse type connection prevents the brittle collapse induced by the member buckling and the tensile fracture of the weld connection. The proposed method using the property of the energy conservation can statically predict the horizontal maximum seismic response displacement at the top of the wall.

1 INTRODUCTION

This study deals with response and damage-controlled structures such as aluminum alloy latticed wall structures induced by the plastic elongation of the steel bolt among the joint connection. The truss wall structure system is composed of several elements such as ball joints, strut members and their connections. The spatial truss wall structure has two collapse mechanisms such as the progressive member buckling due to a compression stress and the tensile fracture of the web connection due to a tension stress. In our previous paper, we presented that the progressive member buckling induces the brittle collapse of the structure. This means that the structure looses the horizontal load carrying capacity, soon after the load reaches the maximum capacity. On the other hand, it could be known that an axial plastic elongation ductility of the steel brings about the response control of a structure due to the absorbing energy.

The purpose of this study is to investigate the energy absorbing mechanism within the joint connection bolt tensile collapse type structure system caused by the bolt plastic elongation due to an axial tension stress. As far as the one degree of freedom equivalent model is concerned, the modeling procedure of the simplified one degree of freedom vibration system is proposed in order to predict the earthquake elasto-plastic responses of the latticed wall using the earthquake response spectrum and the property of the energy conservation. In the modeling, the calculation method of predicting the equivalent one degree vibration system such as the horizontal effective stiffness K and the horizontal strength Q_{By} due to the steel connecting bolt yield state is presented. And the first horizontal natural vibration period T can be also calculated by using the proposed equations. The estimation results obtained by using the simplified vibration model are verified to be in close agreement with the elasto-plastic dynamic analysis results of the full numerical analysis model. Structural engineers can statically estimate the maximum dynamic horizontal displacement of the spatial truss wall structures subjected to earthquake motions by means of the proposed practical static calculation method.

2 MODELING OF SYSTEM TRUSS MEMBERS

Fig.1 illustrates a truss system composed of the strut, the hubs, the collars and the end plugs, which are extruded aluminum alloy (6061-T6). And the bolts are made of Cr-Mo Steel quenched and annealed. As far as the load transfer mechanism of the joint connection, a compression load is transferred through the collar and a tensile load is transferred through the steel bolt as shown in Fig.2. In this study, the system truss member is modeled by composing of two elements such as both the joint connection elements and the strut member element in Fig.3.

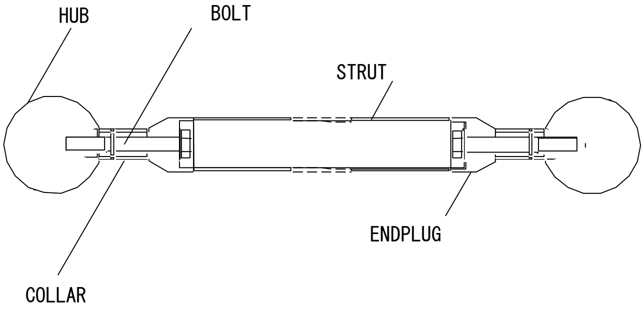


Figure 1: Truss system

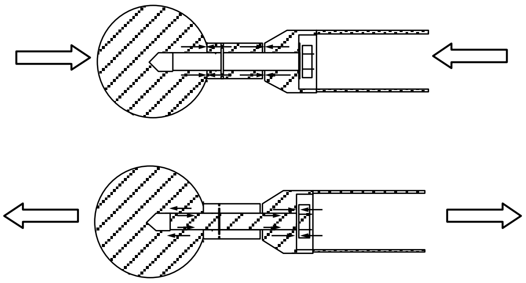


Figure 2: Load transfer mechanism

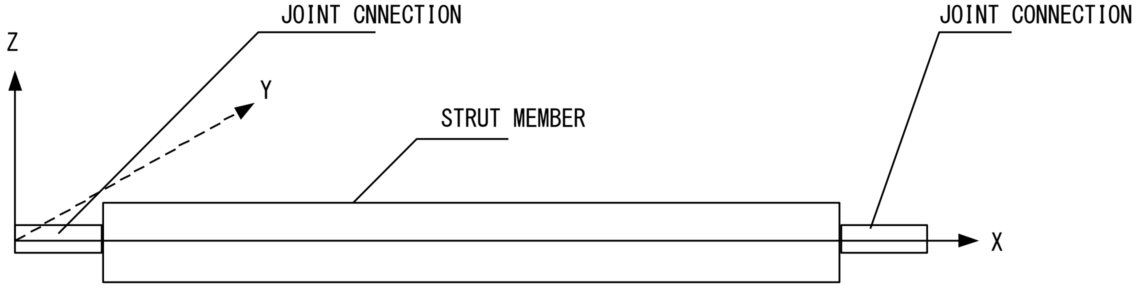


Figure 3: Components of truss system

3 STIFFNESS MATRICES OF TRUSS MEMBER FOR PRESICE ANALYSIS

Fig.4 shows the three axial spring elements such as the two joint connections and the strut. The stiffness matrix is introduced to reduce the all unknown displacements at the nodes 1, i1, i2 and 2 to express the displacements {u1, v1, w1} and {u2, v2, w2} at the both ends. The u, v and w are displacements with respect to the x, y and z directions in Fig.4, respectively.

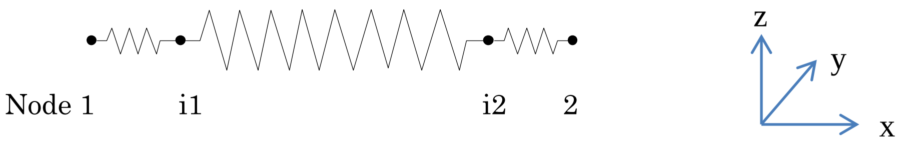


Figure 4: Truss elements of truss system

Joint Connection (1 – i1) : The relationship between the axial displacement and force of the joint connection (1 – i1) can be given by Eq. (1)

$$\left[K_T^{BL}\right]\left\{\begin{array}{c}\Delta D_1\\ \hdashline \Delta D_{i1}\end{array}\right\}+\left\{\begin{array}{c}F_1\\ \hdashline F_{i1}\end{array}\right\}=\left\{\begin{array}{c}Q_1+\Delta Q_1\\ \hdashline Q_{i1}+\Delta Q_{i1}\end{array}\right\} \tag{1}$$

Strut Member (i1 – i2) : The relationship between the axial displacement and force of the strut member (i1 – i2) can be given by Eq. (2).

$$\left[K_T^{S}\right]\left\{\begin{array}{c}\Delta D_{i1}\\ \hdashline \Delta D_{i2}\end{array}\right\}+\left\{\begin{array}{c}F_{i1}\\ \hdashline F_{i2}\end{array}\right\}=\left\{\begin{array}{c}Q_{i1}+\Delta Q_{i1}\\ \hdashline Q_{i2}+\Delta Q_{i2}\end{array}\right\} \tag{2}$$

Joint Connection (i2 – 2) : The relationship between the axial displacement and force of the joint connection (i2 – 2) can be given by Eq. (3).

$$\left[K_T^{BR}\right]\left\{\begin{array}{c}\Delta D_{i2}\\ \hdashline \Delta D_2\end{array}\right\}+\left\{\begin{array}{c}F_{i2}\\ \hdashline F_2\end{array}\right\}=\left\{\begin{array}{c}Q_{i2}+\Delta Q_{i2}\\ \hdashline Q_2+\Delta Q_2\end{array}\right\} \tag{3}$$

Where △D is the incremental displacement vector, F is the internal force vector and Q is the external load vector at the nodes.

3.1 Reduced member stiffness matrices

The reduced member stiffness matrices expressed by the node displacements and stresses at the both ends can be derived from the above Eq. (1), (2) and (3) as follows.

$$\left\langle [A]-[B][C]^{-1}\left[B^T\right]\right\rangle\left\{\begin{array}{c}\Delta D_1\\ \Delta D_2\end{array}\right\}=\left\{\begin{array}{c}Q_1+\Delta Q_1-F_1\\ Q_2+\Delta Q_2-F_2\end{array}\right\} \tag{4}$$

$$\left[\begin{array}{c:c} A & B \\ \hdashline B^T & C \end{array}\right]\left\{\begin{array}{c} \Delta D_1 \\ \Delta D_2 \\ \hdashline \Delta D_{i1} \\ \Delta D_{i2} \end{array}\right\}+\left\{\begin{array}{c} F_1 \\ F_2 \\ \hdashline 0 \\ 0 \end{array}\right\}=\left\{\begin{array}{c} Q_1+\Delta Q_1 \\ Q_2+\Delta Q_2 \\ \hdashline 0 \\ 0 \end{array}\right\} \tag{5}$$

4 HYSTERETIC MODELS OF COMPONENT PARTS OF SYSTEM TRUSS

4.1 Strut member restoring force characteristic model

The system truss composes of the tubular strut member, the ball joints and the joint connection bolts. We present a modeling procedure of the strut member enough to predict the elasto-plastic buckling behavior.

The strut member is modeled to be buckled due to a compression axial force and be fractured at the welded parts due to a tension axial force. Fig.5 shows the hysteresis curves used for the dimensionless slenderness ratio divided by the dividing slenderness ratio. The maximum compressive stress σ_{cr} of the member is calculated by using Eq.(6).

A buckling length is needed to calculate a slenderness ratio. The slenderness ratio of the member buckling load is taken to be the effective slenderness ratio considering the joint connection restraint.

$$\begin{aligned} &\lambda/\Lambda \le 1.0 \ : \ \sigma_{cr} = F/\left\{1-0.5(\lambda/\Lambda)^2\right\} \\ &\lambda/\Lambda > 1.0 \ : \ \sigma_{cr} = F/\left\{2\,(\lambda/\Lambda)^2\right\} \end{aligned} \tag{6}$$

$$\Lambda = \sqrt{\pi^2 E/0.5F} = 37.17/\sqrt{F} = 81.1$$

Where F is the basic strength σy, λ is the member slenderness ratio and Λ is the dividing slenderness ratio, which is taken to be 81 in the study.

The yield stress σ_y and Young’s modulus E are set at 210Mpa and 70Gpa, respectively. The member buckling occurs at the maximum compressive stress σ_{cr} due to a compression axial stress and the weld fracture occurs at $0.71\sigma_y$ due to a tension axial stress.

4.2 Joint connection restoring force characteristic model

The joint connection composes of hubs, collars and end plugs in Fig.1.

Fig.2 shows the axial force transfer mechanism under a compression and a tensile stress, respectively.

Fig.6 shows the steel bolt hysteresis model of the relationship between the axial force and displacement.

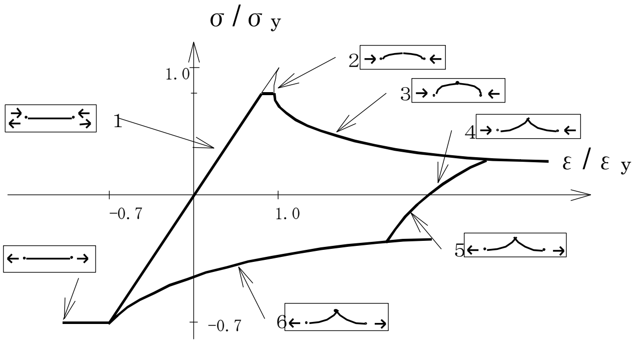


Figure 5: Hysteresis model of strut member

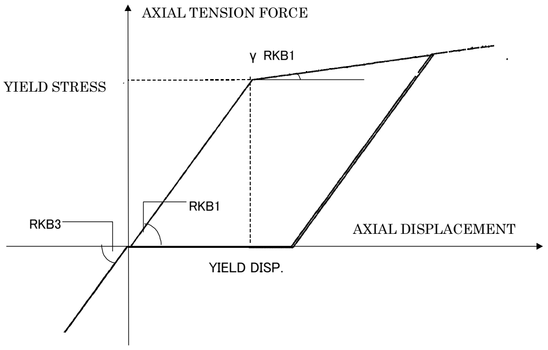


Figure 6: Hysteresis model of steel bolt

5 STATIC ELASTO-PLASTIC ANARYSIS OF SPATIAL TRUSS WALL

The static elasto-plastic analysis of the spatial truss walls with the fuse type connection is carried out to verify the ductile collapse mechanism induced by the tensile yield of the steel bolt in the connection. The above mentioned hysteresis models of the elements are used to obtain the relationship between the stress and the strain of the elements in the step by step pushover analysis.

5.1 Configuration and mechanical properties of static analysis model

The global configuration of the analysis model is shown in Fig.7. The structure has the 1m x 2m grid and the 1m depth of the wall type. The wall has 4.5m x 3.75m geometry. As far as the boundary condition is concerned, all the bottom nodes of the wall are pin support. All the top nodes of the wall can move for the horizontal axis (Y axis) and are restrained for the vertical and out of plane axes (Z and X axes), as shown in Fig.7. The member properties such as the sectional area A (mm^2), the radius of gyration i (mm), the length L (mm) and the slenderness ratio λ are shown in Table 1and 2.

Table 1: Mechanical properties of aluminum struts (L : member length, λ : member slenderness ratio, Λ : member dividing slenderness ratio, A : member sectional area, i: radius of gyration) Λ=81

ALUMINUM STRUT		L (mm)	λ	λ/Λ	A (mm^2)	i (mm)
Chord	Φ150x10	x: 1858 y: 1483	x: 37.4 y: 29.9	x: 0.462 y: 0.368	4398.2	49.6
Web	Φ150x10	1381.3	27.8	0.343	4398.2	49.6

Table 2: Mechanical properties of steel connection bolts (A: sectional area, L : length)

STEEL BOLT	A (mm^2)	L (mm)	Minimum Elongation (%)	Standard Strength (N/mm^2)	Yield Axial Stress (kN)
Chord Φ30.0	706.9	196	16	640	452.4
Web Φ30.0	254.5	196	16	640	452.4

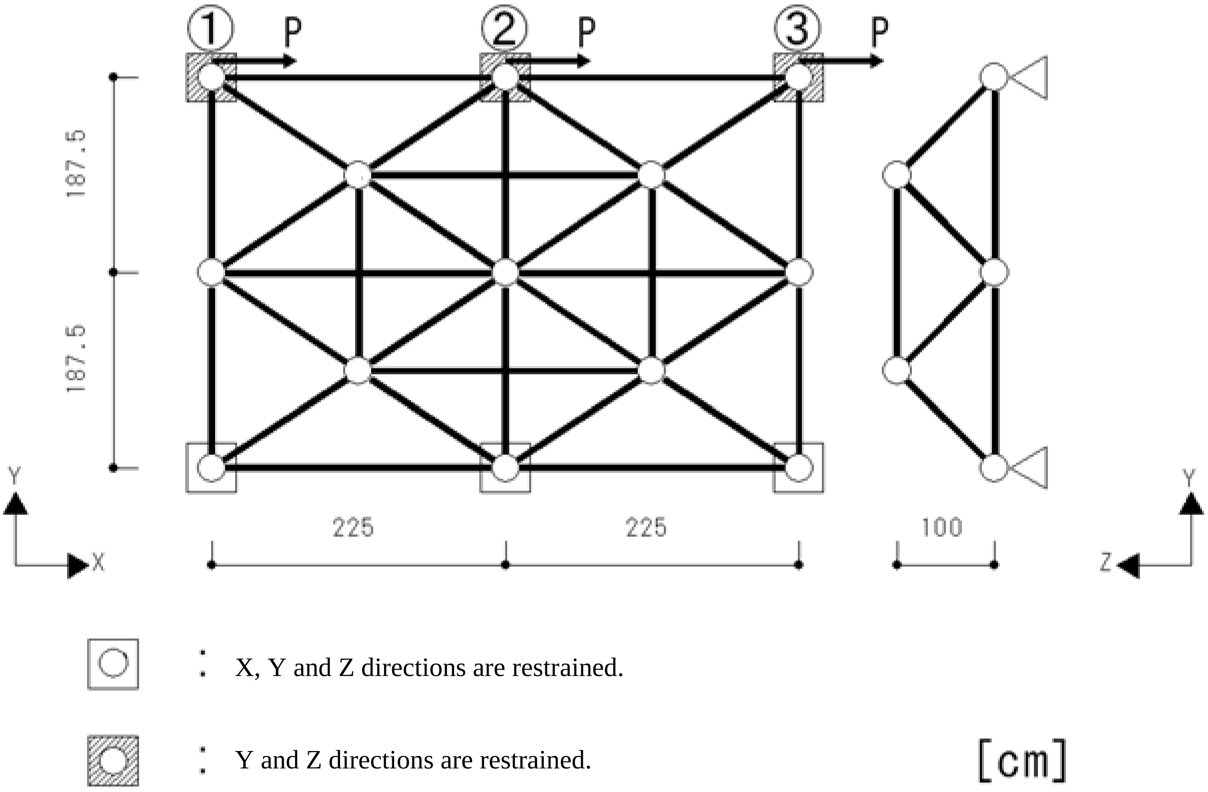


Figure 7: Analysis model of double layer truss wall

5.2 Elasto-plastic analysis of the truss wall

Fig.8 shows the relationship between the horizontal load P and the horizontal displacement D at the node 2 of the analysis of the double layer truss wall in Fig.7. The region from the origin point to the turning point results in a linear elastic relationship. This is a reason why all of the members such as the chords and the webs remain in the elastic region. The stress of the steel bolt reaches the yield stress 235N/mm^2 at the turning point of the horizontal load P=408(kN). The yield elongation of the truss wall appears beyond the turning point. The member buckling doesn’t occur to the terminal point as shown in Fig.8. This means that the brittle collapse is avoided by means of the fuse type elongation such as the tensile yield elongation of the steel bolt.

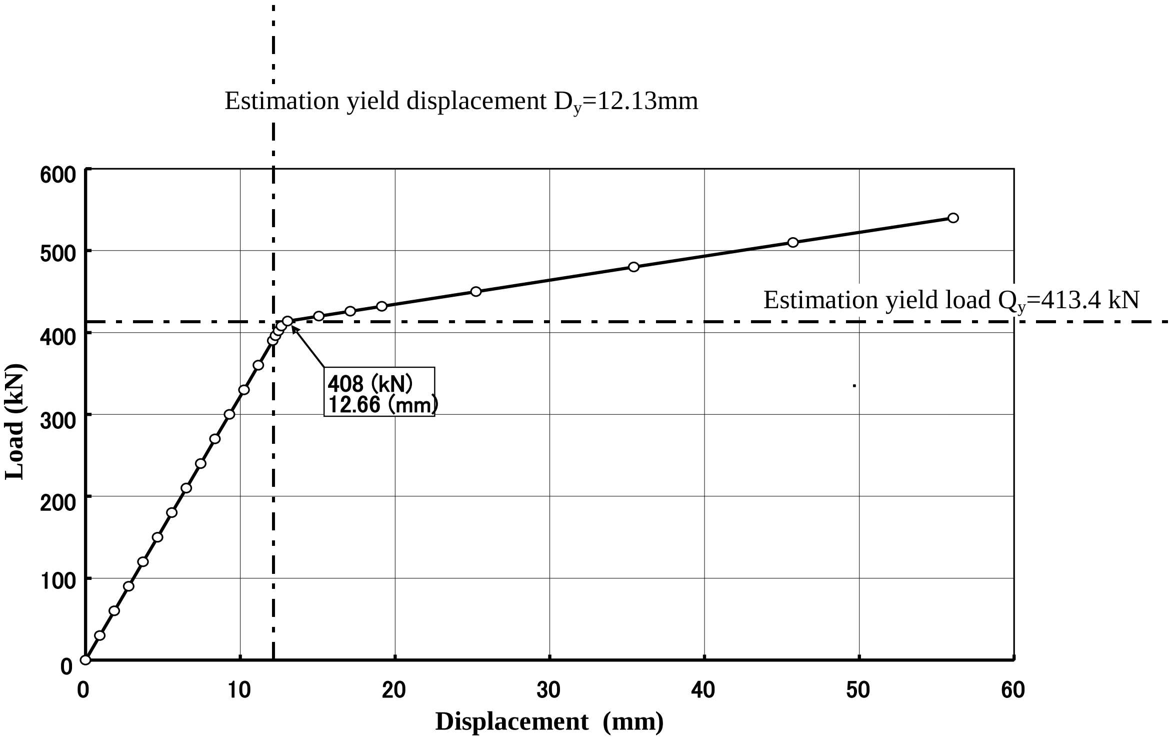


Figure 8: Static analysis result of relationship between the horizontal load and displacement

5.3 Practical calculation methods of the yield load Q_{By} and the corresponding displacement D_y induced by the yield of the steel bolt in the connection

The calculation methods of predicting the horizontal resistant capacity, such as the horizontal displacement and load at the top level of the wall are proposed by means of the virtual work theory. And the yield load Q_{By} and the corresponding displacement D_y induced by the yield of the steel bolt in the connection are obtained by means of the proposed calculation method.

Consider the wall type structures subjected to the plane horizontal load. The shear deformation may be dominant in comparison with the bending one. Consequently, it can be assumed that the web members equally resist the shear force. Under the assumption, the shear stress per half column of a grid is given by

$$p = Q / (2 \cdot m) \tag{7}$$

The axial stress of the web member is given by $N = p / \alpha$.

The yield horizontal shearing load Q_{By} is introduced in the following equation.

$$Q_{BY}=2 \cdot m \cdot \alpha \cdot N_{BY} \tag{8}$$

Where N_{BY} is the steel yield stress.

Using the principle of the virtual work, the horizontal displacement D_y corresponding to the web joint connection bolt yield is given by

$$D_y = D_s + 2\times D_B$$
$$= \frac{n}{m}\cdot\frac{Q_{By}}{E_S\cdot A_S\cdot\alpha^2}\cdot l_S + 2\times\left(\frac{n}{m}\cdot\frac{Q_{By}}{E_B\cdot A_B\cdot\alpha^2}\cdot l_B\right) \tag{9}$$

Where m is the grid number along the shear force direction, n is the grid number along the orthogonal direction to the shear force direction. E_s, l_s and A_s are the Young's modulus, the member length and the sectional area of the strut member, respectively. E_B, l_B and A_B are the Young's modulus, the member length and the sectional area of the joint connecting bolt, respectively. α is the cosine of the web to the lateral direction. D_s is the component induced by the strut elongation and D_B is the component induced by the joint connection bolt elongation subjected to the horizontal load Q_{By}. Q_{By} brings about the yield of the joint connection steel bolts.

6 DYNAMIC ELASTO-PLASTIC ANALYSIS OF SPATIAL TRUSS WALL

As far as the numerical analysis method is concerned, the hysteric material behavior and the geometric non-linearity are considered in the dynamic analysis of the truss structure under the horizontal (X direction) motions such as El Centro NS. And the peak ground accelerations of the three earthquake motions are taken to be 5.61 (m/sec^2).

The Newmark β method is used in the numerical integration method. Since it has been known that a case of β=1/4 will be unconditionally stable for most nonlinear problems, β=1/4 is used in this study. The Rayleigh damping is used and both of the first and second damping factors are taken to be 0.02. The details of the analysis method are described in the presented paper [1].

6.1 Configuration and mechanical properties of dynamic analysis model

The global configuration of the analysis model is shown in Fig.9. The structure has the 2m x 2m grid and the 1m depth of the wall type. The wall has 16m x 8m geometry. As far as the boundary condition is concerned, all the bottom nodes of the wall are pin support. All the top nodes of the wall can move for the horizontal axis (Y axis) and are restrained for the vertical and out of plane axes (Z and X axes), as shown in Fig.9. The member properties such as the sectional area A (mm^2), the radius of gyration i (mm), the length L (mm) and the slenderness ratio λ are shown in Table 3and 4.

6.2 Seismic coefficient q_y of steel bolt yield state and maximum response displacement

The first natural period of the prototype of the model is set to be 0.219second for a shearing shape mode of carrying out the eigenvalue analysis. And the latticed wall structure has a shear resistance mechanism, because of the high bending stiffness in comparison with the shear stiffness for the aspect ratio. Therefore, it is assumed that the web members equally resist the shear force. Under the assumption, the analysis models are designed to be the tensile yield collapse mode of the steel bolt among the connection. The static seismic resistant coefficient q_y of the latticed wall due to the web bolt yield state is also determined varying the yield stress of the steel bolts before carrying out the dynamic analysis.

The purpose of the elasto-plastic dynamic analysis is to calculate the maximum response horizontal displacement at the top of the wall, and verify the collapse mechanism. This study deals with realizing a damage-controlled structure induced by a ductile failure induced by the plastic elongation of the steel bolt among the joint connection. It is one of the significant matters that the latticed wall structure collapse mode is not caused by brittle failure types such as a sequential member buckling, before the ductile collapse occurs.

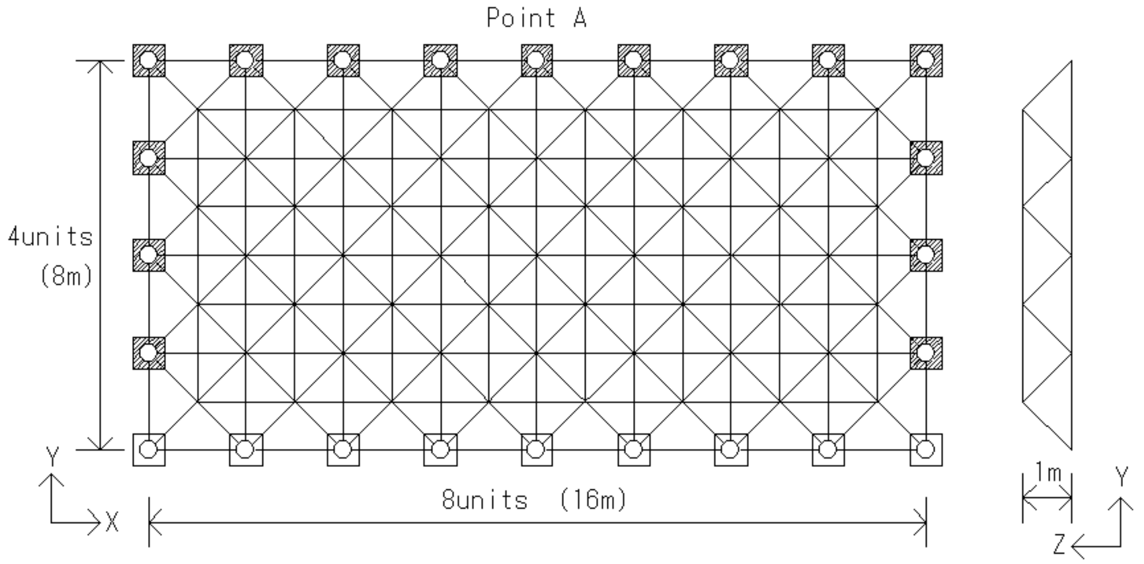
6.3 First natural period T_E for horizontal mode shape

The first horizontal natural period is given by

$$T_E = 2\pi\sqrt{\frac{M}{K}} \tag{10}$$

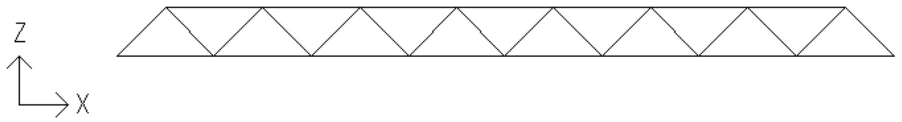
Where M is the mass of the single degree of freedom vibration system and K is the horizontal stiffness of the wall. K is the horizontal stiffness rigidity given by

$$K = \frac{Q_{By}}{D_y} = \frac{m}{n}\cdot\frac{\alpha^2}{\left(\dfrac{l_S}{E_S\cdot A_S}+\dfrac{2l_B}{E_B\cdot A_B}\right)} \tag{11}$$



(a) X and Y plane

(b) Y and Z plane



(c) X and Z plane

▢ : X, Y and Z directions are restrained.

▨ : Y and Z directions are restrained.

Figure 9: Analysis model of double layer truss wall for dynamic analysis

Table 3: Mechanical properties of aluminum struts (L : member length, λ : member slenderness ratio, Λ : member dividing slenderness ratio, A : member sectional area, i: radius of gyration) $\Lambda=81$

ALUMINUM STRUT		L (mm)	λ	λ/Λ	A (mm^2)	i (mm)
Chord	Φ180 x 12	x: 1560 y: 1560	x: 26.2 y: 26.2	x: 0.323 y: 0.323	6333.5	59.5
Web	Φ150x12	1332.1	27.2	0.335	5202.5	49.0

Table 4: Mechanical properties of steel connection bolts (A: sectional area, L : length)

STEEL BOLT		A (mm ^{2})	L (mm)	Minimum Elongation (%)	Standard Strength (N/mm ^{2})	Yield Axial Stress (kN)
Chord	Φ34.0	706.9	220	16	640	452.4
Web	Φ15.0	176.7	200	16	640	113.1

6.4 Results of dynamic elasto-plastic analysis

The elasto-plastic dynamic analyses are carried out using the full model as shown in Fig.9 subjected to El Centro NS earthquake motion. In the analyses, a significant parameter such as the seismic resistance coefficient q_y of the analysis model is set varying the yield stress of the steel connection bolt as shown in Fig. 1. And the vibration characters such as the natural period T for the in-plane horizontal direction and the damping factor h are taken to be 0.219 second and 0.02 for the all models, respectively.

The purpose of the analysis is to find out the steel bolt mechanical properties in order to make certain the ductile ultimate state induced by the bolt yield mechanism.

Fig.10 shows the relationship between the seismic resistant coefficient q_y and the horizontal maximum response displacement at the top of the wall. In the figure, a dotted line is drawn using the property of the energy conservation and the elastic earthquake response spectrum by the equivalent single degree of freedom model. The detail of the property is described the following section. And the mark ● are plotted the horizontal maximum response displacements corresponding the seismic resistance coefficient q_y of the latticed wall structures subjected to El Centro NS. The latticed wall structures with q_y ranging from ductility factor (Eq. 14) $\mu=1$ to 1.7 bring about the steel bolt yield state by the peak ground acceleration. It is seen that the prediction method using the property of the energy conservation shows a good agreement with the numerical full scale analysis results.

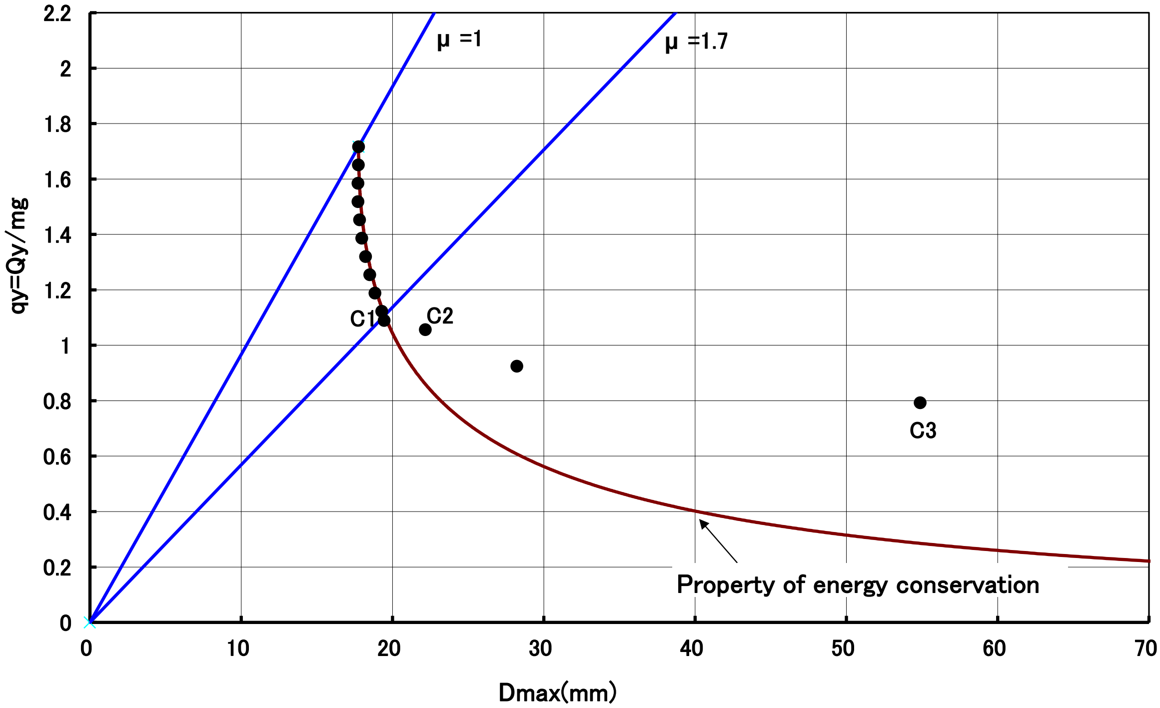


Figure 10: Seismic coefficient of yield state and maximum response displacement

7 CONCEPT OF EQUIVALENT SINGLE DEGREE OF FREEDOM VIBRATION MODEL AND PROPERTY OF ENERGY CONSERVATION

7.1 Property of energy conservation by the single degree of freedom vibration equivalent model

Consider a relationship between the yield strength Q and the maximum response displacement δ of the single degree of freedom vibration model in Fig.12. δ_N of the elasto-plastic structure is calculated by using the assumption of the property of the energy conservation between the triangular part of the elastic strain energy and the trapezoidal part as shown in Fig.11. A broken line in Fig.11 is drawn using the above mentioned property.

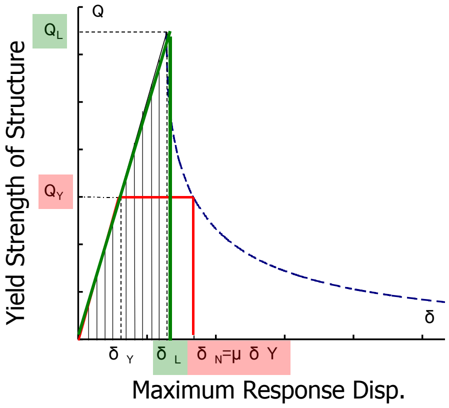


Figure 11: Property of energy conservation

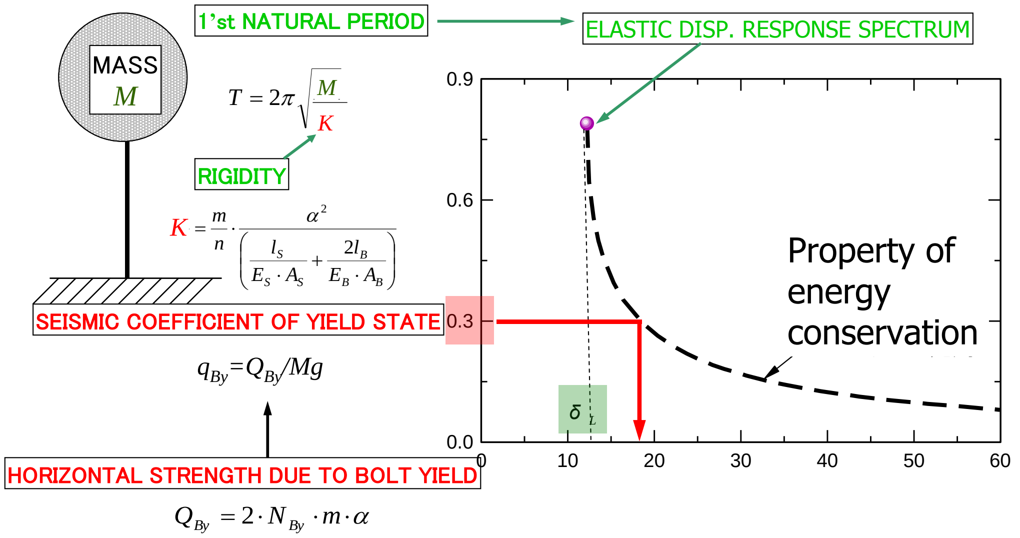


Figure 12: Concept of modeling for predicting the horizontal maximum response displacement of the wall

Fig.12 shows a concept of predicting the maximum horizontal response displacement of the ductile collapse type truss wall induced by the joint connection bolt elongation subjected to earthquake motions by using the equivalent single degree of freedom vibration model and the property of the energy conservation as follows.

First, the elastic earthquake maximum displacement δ_L in Fig.12 is given by the response spectrum using the equivalent natural period T_E (Eq.10) and the equivalent rigidity K (Eq.11). Second, the elasto-plastic earthquake maximum displacement δ_N induced by the connection steel bolt yield ultimate state can be estimated the following equations using the property of energy conservation (Fig.12). The horizontal shear force Q_{BY} can be calculated by using Eq.(8) corresponding to the steel bolt yield stress N_{BY}. δ_Y can also be calculated by Eq.(9).

$$\delta_N = \frac{\delta_Y}{2}\left\{\left(\frac{Q_L}{Q_{BY}}\right)^2 + 1\right\} \tag{12}$$

Where Q_L is the elastic horizontal shear force corresponding to the seismic response displacement spectrum δ_L. Q_L is derived by means of K(Eq.11) multiplied by δ_L.

7 CONCLUSIONS

The study has investigated the two ultimate state types of the double layer truss wall structures such as the joint connection bolt yield of the web member, and the web member buckling by the dynamic elasto-plastic analysis. As a result, the proposed method using the property of the energy conservation can statically predict the horizontal seismic maximum displacement at the top of the wall in the case of the steel bolt yield type collapse mechanism.

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