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COMPUTATIONAL MODEL OF SEISMIC WAVE PROPAGATION IN PRESTRESSED FORMATION

E. V. LYS*, E. I. ROMENSKI †, V. A. CHEVERDA* AND M. I. EPOV*

* Trofimuk Institute of Petroleum Geology and Geophysics SB RAS
3, Acad. Koptyug avenue
630090, Novosibirsk, Russia

† Sobolev Institute of Mathematics SB RAS
4, Acad. Koptyug avenue
630090, Novosibirsk, Russia
email: evrom@math.nsc.ru

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Abstract. The new computational model for the seismic wave propagation in the initially prestressed media is proposed, the governing equations of which are written in terms of velocities, stress tensor and small rotations of element of the medium. The properties of wavefields in the prestressed medium are studied and some examples showing anisotropy of prestressed state are discussed. The staggered grid numerical method is developed for solving the governing equations of the model and numerical example is presented.

1 INTRODUCTION

Analysis of seismoacoustic wavefields is the basic tool for the study of internal structure of Earth and rock masses in the mining technology. The seismic wave properties can depend not only on the material characteristics of the formation (density and speeds of sound), but in addition on the existence of zones of non-hydrostatic stress field. These prestressed zones can be caused by many factors, such as geotectonic processes, gravity, temperature gradients, etc. In the mining area these zones, for example, can be caused by the underground excavation during shaft sinking. The impact of prestressed zones on seismic waves is a poorly studied problem and one can expect that the account of initial stress can have an influence on interpretation of the results of solution of inverse problems and seismic imaging.

The basis of the theory of elastic waves in prestressed elastic media goes back to the pioneer work of M. Bio [1]. An application of the theory to seismic problems was not systematic (see, for example, [2], [3] and references therein) and there is still an open area for research work.

We propose a new computational model for the small amplitude wave propagation in the prestressed medium, the simplified version of which is presented in [4]. The derivation of the model is based on the general theory of finite deformations and as a result, the governing equations in terms of velocities, stress and small rotations are formulated in the form of the first order hyperbolic system. The method of derivation is applicable for an arbitrary dependence of elastic energy on the invariants of strain tensor and the smallness of the initial strain tensor is not required in general. For the quadratic dependence of the elastic energy on the strain tensor the governing equations of the small amplitude wave propagation in the initially prestressed medium are derived. The properties of the wavefields in the prestressed medium are discussed. The second order accuracy in space and time staggered grid method is developed and a numerical test problem for the wave propagation in the unidirectionally stressed medium in the presence of stress gradient is presented.

2 DERIVATION OF GOVERNING EQUATIONS

2.1 Strain and stress tensors in the prestressed elastic medium

In this Section the method of derivation of governing equations for small amplitude wave propagation in the prestressed elastic medium is described. This method is based on the presented in [5] relationship between stress rate and strain rate in the hypoelastic representation of the hyperelastic model of solid. The governing equations are formulated in Lagrangian coordinates, but the method of derivation requires an introduction of Eulerian coordinates. In addition, it is necessary to define the reference unstressed configuration with its own coordinates of unstressed state.

Denote x^i Eulerian coordinates of the particle of the medium and x_0^i corresponding Lagrangian coordinates. Assume that the element of the medium in Lagrangian coordinates containing this particle is prestressed, that is nonzero stress field exists inside this element. Let us introduce coordinates ξ^i of the particle corresponding to the unstressed reference state of the element. Thus, the following parameters characterizing the deformation of element can be introduced: $(F^0)_j^i = \frac{\partial x_0^i}{\partial \xi^j}$, $(F)_j^i = \frac{\partial x^i}{\partial x_0^j}$. Here $(F^0)_j^i$ and $(F)_j^i$ are deformation gradients characterizing deformation from the unstressed reference configuration to the Lagrangian configuration and from the Lagrangian configuration to the current Eulerian configuration accordingly. Furthermore, the total deformation from the reference unstressed state to the current Eulerian state is characterized by the total deformation gradient $(F_{tot})_j^i = \frac{\partial x^i}{\partial \xi^j} = (F)_\alpha^i (F^0)_j^\alpha$. For our purpose it is more appropriate to use inverse to above defined deformation gradients:

$$(f^0)_j^i = \frac{\partial \xi^i}{\partial x^j}, \quad (f)_j^i = \frac{\partial x_0^i}{\partial x^j}, \quad (f_{tot})_j^i = \frac{\partial \xi^i}{\partial x^j} = (f^0)_\alpha^i (f)_j^\alpha$$

Below the Finger strain tensor is used as a measure of deformation and the total strain from the unstressed state to the current configuration is characterizes by

$$(G)_{ij} = (f_{tot})_i^\alpha (f_{tot})_j^\alpha = (f)_i^\alpha (f^0)_\alpha^\beta (f^0)_\gamma^\alpha (f)_j^\gamma.$$

Thus, $(G)_{ij} = (f)_i^\alpha (G^0)_{\alpha\gamma} (f)_j^\gamma$, where $(G^0)_{\alpha\gamma} = (f^0)_\alpha^\beta (f^0)_\gamma^\alpha$ is the Finger strain tensor characterizing deformation from the unstressed state to Lagrangian configuration. Further we will use the matrix form of the Finger tensor, which reads as $G = f^T G_0 f$, where the superscript T denotes a matrix transposition.

For the derivation of governing equations we use the so-called hyperelastic model which is based on the fundamental laws of thermodynamics. If the specific elastic energy $E(G_{11},\dots,G_{33})$ is the known function of the strain tensor, then, according to [6], the Cauchy stress tensor in Eulerian configuration is given as

$$s_{ij} = -2\rho\frac{\partial E}{\partial G_{\alpha j}}G_{\alpha i}, \tag{1}$$

where $\rho = \rho_{00}/det(F_{tot})$ is the mass density and ρ_{00} is the density of the medium in the unstressed state. For the isotropic medium the elastic energy depends on invariants of the strain tensor. Note that the density can be represented as a function of the Finger tensor as

$$\rho = \rho_{00}\sqrt{detG} = \rho_{00}\sqrt{detG_0 det(f^T f)} = \rho_0\sqrt{det(f^T f)},$$

where $\rho_0 = \rho_{00}\sqrt{detG_0}$ is the Lagrangian density.

2.2 Equations of motion

The governing equations for the prestressed medium motion consist of the momentum conservation laws and evolution equations for the parameters characterizing deformation. Denoting u^i the velocity vector, the momentum equation in Eulerian coordinates can be written in a standard form and reads as

$$\frac{\partial\rho u^i}{\partial t} + \frac{\partial\left(\rho u^i u^k - s^{ik}\right)}{\partial x^k} = 0. \tag{2}$$

What concerns an evolution of the strain parameters, it requires thorough consideration. As a consequence of the definition of deformation gradient $(F)^i_j = \frac{\partial x^i}{\partial x_0^j}$ the following evolution equation in matrix form can be derived:

$$\frac{df}{dt} = -fU, \tag{3}$$

where $f = \left[(f)^i_j\right] = F^{-1}, U = \left[\frac{\partial u^i}{\partial x^j}\right]$ is the velocity gradient and $\frac{d}{dt} = \frac{\partial}{\partial t} + u^\alpha\frac{\partial}{\partial x^\alpha}$ is the material derivative. Note that the Finger tensor G_0, characterizing deformation from the reference unstressed state to Lagrangian configuration, does not change during the motion, i.e.

$$\frac{dG^0}{dt} = 0.$$

As a consequence of these two equations for f and G^0 the evolution equation for the Finger tensor can be derived

$$\frac{dG}{dt} = -GU - U^T G. \tag{4}$$

Our goal is to derive the small amplitude wave equations in terms of stress tensor. That is why we have to write out the evolution equation for s^{ij}. Such a derivation has been done in [5] for the isotropic elastic medium. Since the elastic energy is a function of the three invariants of strain tensor, one can prove with the use of Cayley-Hamilton theorem that stress tensor is a quadratic polynomial of strain tensor

$$s = A_0 I + A_1 G + A_2 G^2,$$

where A_0, A_1, A_2 are the functions of invariants of G.

From the above representation for s one can derive the following equation

$$\frac{ds}{dt} = -sU - U^T s + \beta_0 I\, trW + \beta_1 W + \beta_2 G\, trW + \beta_3 I\, tr(GW) +$$

$$\frac{1}{2}\beta_4(GW+WG)+\beta_5 G^2\, trW+\beta_6 G\, tr(GW)+\beta_7 I\, tr(G^2W)+ \tag{5}$$
$$\frac{1}{2}\beta_8(G^2W+WG^2)+\beta_9 G^2\, tr(GW)+\beta_{10} G\, tr(G^2W)+\beta_{11} G^2\, tr(G^2W).$$

Here $W=\frac{1}{2}(U+U^T)$ is the strain rate tensor in Eulerian coordinates, coefficients $\beta_0,\beta_1,\dots,\beta_{11}$ are functions of invariants of G and depend on the partial choice of elastic energy E. Thus, equations (2) - (5) can be used for the derivation of the small amplitude wave propagation in the prestressed isotropic medium with the arbitrary dependence of the elastic energy on three invariants of the strain tensor.

2.2 Equations for small amplitude wave propagation in the prestressed medium with the quadratic dependence of elastic energy on the strain tensor

In Section 2.1 the basic equations and relationships were formulated allowing one to derive governing equations for the small amplitude wave propagation in the isotropic medium with the elastic energy given as an arbitrary function of invariants of strain tensor. In this Section we present such equations for the case of quadratic dependence of energy on strain tensor.

Assume that the energy is given as a function of the Almansi strain tensor $\varepsilon=[\varepsilon_{ij}]=\frac{1}{2}(I-G)$ in the following form

$$E=\frac{\lambda}{2\rho_{00}}(\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33})^2+\frac{\mu}{\rho_{00}}(\varepsilon_{ij}\varepsilon_{ji}),$$

where $\boldsymbol{\lambda}$, μ are the Lame parameters. For the above choice of energy function the stress tensor computed by (1) takes a form

$$s=\frac{\rho}{\rho_{00}}(\lambda\, tr\varepsilon\, I+2\mu\,\varepsilon-2\lambda\, tr\varepsilon\,\varepsilon-4\mu\,\varepsilon^2). \tag{6}$$

Assume that the stress tensor is a sum of the initial stress Σ and its perturbation σ:

$$s=\Sigma+\sigma.$$

Further assume that the initially prestressed state of the medium is known and its stress field satisfies equilibrium equations in Lagrangian configuration

$$\frac{\partial\Sigma^{ij}}{\partial x_0^j}=0,$$

where Σ^{ij} is connected with the Almansi tensor of the prestressed state by the linearized relation (6) which reduces to the classic Hooke's law $\Sigma=\lambda\, tr\varepsilon_0\, I+2\mu\,\varepsilon_0$.

We will derive equations in Lagrangian coordinates but with the use of the Cauchy stress tensor referred to Eulerian coordinates in order to obtain equations in terms of symmetric stress tensor. Introduce the small deformation and small rotation tensors by the following relations:

$$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial V^i}{\partial x^j}+\frac{\partial V^j}{\partial x^i}\right),\quad \omega_{ij}=\frac{1}{2}\left(\frac{\partial V^i}{\partial x^j}-\frac{\partial V^j}{\partial x^i}\right),$$

where V^i is the displacement vector, $V^i=x^i-x_0^i$, so that $f_j^i=\delta_j^i-\frac{\partial V^i}{\partial x^j}$. The evolution equations for ε_{ij} and ω_{ij} read as

$$\frac{\partial \varepsilon_{ij}}{\partial t} = \frac{1}{2}\left(\frac{\partial u^i}{\partial x^j} + \frac{\partial u^j}{\partial x^i}\right), \quad \frac{\partial \omega_{ij}}{\partial t} = \frac{1}{2}\left(\frac{\partial u^i}{\partial x^j} - \frac{\partial u^j}{\partial x^i}\right). \tag{7}$$

Using all above definitions, transforming equations (2), (3), (5), (6), (7) to Lagrangian coordinates, assuming that $u^i, \varepsilon_{ij}, \omega_{ij}, \varepsilon_{0ij}$ are small, and neglecting all terms of order higher than the first one, we obtain the following system in terms of velocities, stress and rotations.

$$\rho_{00}(1 - tr\varepsilon_0)\frac{\partial u^i}{\partial t} = \frac{\partial \sigma^{ij}}{\partial x_0^j} + \left(\varepsilon_{j\alpha} + \omega_{j\alpha}\right)\frac{\partial \Sigma^{i\alpha}}{\partial x_0^j},$$

$$\begin{aligned}\frac{\partial \sigma}{\partial t} &= -\Sigma U_0 - U_0^T\Sigma - trW_0\,\Sigma + \lambda\, trW_0\ I + 2\mu\, W_0\ - \lambda\, tr\varepsilon_0\, trW_0\ I - \\ &\quad 2\mu\, tr\varepsilon_0\, W_0\ - 2\lambda\, tr(\varepsilon_0 U_0)\, I - 2\lambda\, trW_0\, \varepsilon_0 - 4\mu\, \varepsilon_0 W_0 - 4\mu\, W_0\varepsilon_0\,,\end{aligned} \tag{8}$$

$$\frac{\partial \omega_{ij}}{\partial t} = \frac{1}{2}\left(\frac{\partial u^i}{\partial x_0^j} - \frac{\partial u^j}{\partial x_0^i}\right), \quad \frac{\partial \varepsilon_{ij}}{\partial t} = \frac{1}{2}\left(\frac{\partial u^i}{\partial x_0^j} + \frac{\partial u^j}{\partial x_0^i}\right).$$

Here $U_0 = \left[\frac{\partial u^i}{\partial x_0^j}\right]$ is the Lagrangian velocity gradient, $W_0 = \frac{1}{2}(U_0 + U_0^T)$ is the Lagrangian strain rate tensor. The initial Almansi strain tensor can be expressed via initial stress Σ as $\varepsilon_0 = \frac{1}{2\mu}\left(\Sigma - \frac{\lambda}{3\lambda+2\mu} I\, tr\Sigma\right)$. The equation for small deformation ε is included to system (8) in order to avoid the derivation of relationships between stress tensor s, small rotation tensor ω, and small deformation tensor ε.

3 PROPERTIES OF WAVEFIELDS IN PRESTRESSED MEDIA

It is obvious that system (8) and conventional linear elasticity equations for isotropic media are different. Coefficients of equations (8) depend on the values of initial stress tensor and their spatial derivatives. It turns out that this difference drastically changes the character of elastic waves and leads to their anisotropy and dispersion. To prove this fact one can consider the second order equations system for velocities which can be derived from (8) by differentiating velocity equations with respect to t and exclusion of stress derivatives with the use of equation for s:

$$\rho_0 \frac{\partial^2 u^i}{\partial t^2} = C_{ijkl}\frac{\partial^2 u^l}{\partial x_0^j \partial x_0^k} + B_{ijk}\frac{\partial u^j}{\partial x_0^k}. \tag{9}$$

Here $B_{ijk} = \partial\Sigma^{ik}/\partial x_0^j$, moduli C_{ijkl} depend on the initial stress tensor Σ^{ik}. It is obvious that the initial stress results in the anisotropy of the medium. Moreover, the term containing first derivatives of the velocities in (9) can result in attenuation and dispersion of the waves.

As an example we consider the unidirectionally stressed state with the initial stress field given as $\Sigma^{11} = -P,\ \Sigma^{ij} = 0\ (ij \neq 11)$. In this case elastic moduli are computed as

$$c_{11} = \lambda + 2\mu + 6P + \frac{4\lambda(\lambda+\mu)}{\mu(3\lambda+2\mu)}P\,, \quad c_{12} = \lambda + P + \frac{\lambda(\lambda+3\mu)}{\mu(3\lambda+2\mu)}\ P\,,$$

$$c_{21} = \lambda + \frac{\lambda(\lambda+3\mu)}{\mu(3\lambda+2\mu)}\ P\,, \quad c_{13} = \lambda + P + \frac{\lambda(\lambda+3\mu)}{\mu(3\lambda+2\mu)}\ P\,,$$

$$c_{31} = \lambda + \frac{\lambda(\lambda + 3\mu)}{\mu(3\lambda + 2\mu)} P, \quad c_{14} = c_{41} = c_{15} = c_{51} = c_{16} = c_{61} = 0,$$

$$c_{22} = \lambda + 2\mu - P - \frac{2\lambda^2}{\mu(3\lambda + 2\mu)} P, \quad c_{23} = c_{32} = \lambda - \frac{\lambda(2\lambda - \mu)}{\mu(3\lambda + 2\mu)} P,$$

$$c_{24} = c_{42} = c_{25} = c_{52} = c_{26} = c_{62} = 0, \qquad c_{33} = \lambda + 2\mu - P - \frac{2\lambda^2}{\mu(3\lambda + 2\mu)} P,$$

$$c_{34} = c_{43} = c_{35} = c_{53} = c_{36} = c_{63} = 0, \quad c_{44} = \mu - \frac{2\lambda}{3\lambda + 2\mu} P,$$

$$c_{45} = c_{54} = c_{46} = c_{64} = c_{56} = c_{65} 0, \quad c_{55} = c_{66} = \mu + P - \frac{2\lambda}{3\lambda + 2\mu} P,$$

where the common relation between C_{ijkl} and c_{mn} is used by the following correspondence of indices: $11 \to 1, 22 \to 2, 33 \to 3, 23{,}32 \to 4, 13{,}31 \to 5, 12{,}21 \to 6$.

On Figure 1 one can see the plane waves velocity distribution for the unidirectionally stretched medium with $\Sigma^{11} = \rho_{00} V_p^2/50,\ \Sigma^{ij} = 0\ (ij \neq 11)$ (left) and compressed medium with $\Sigma^{11} = \rho_{00} V_p^2/50,\ \Sigma^{ij} = 0\ (ij \neq 11)$ (right) with parameters $V_p = \sqrt{(\lambda + 2\mu)/\rho_{00}} = 3000 m/s,\ V_s = \sqrt{\mu/\rho_{00}} = 2000\ m/s,\ \rho_{00} = 2000 kg/m^3$. Dots correspond to the wave velocity distribution in the initially unstressed medium. The anisotropy in longitudinal and shear wave propagation in the prestressed medium is clearly seen in both cases.

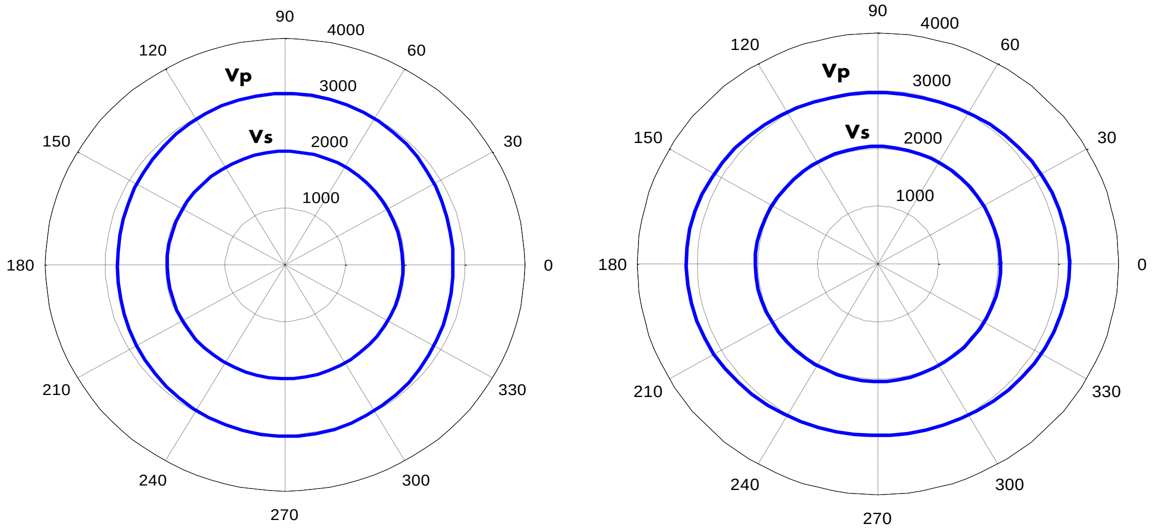

Figure 1: Velocity distribution for the unidirectionally stretched (left) and compressed (right) media.

FINITE DIFFERENCE STAGGERED GRID NUMERICAL METHOD

As a numerical tool for solving differential equations (8) the staggered grid finite difference method has been developed, which is similar to proposed in [7] and has the second order accuracy in space and time. Below we denote spacial coordinates as $x_1 = x_0^1$, $x_2 = x_0^2$ and do not distinguish upper and inferior indices. On Figure 2 the definition of the staggered grid is presented. The velocities and densities are computed at the points marked by circles, while stress, strain and rotation tensors as well as elastic moduli are related to the points marked by squares. On Figure 3 the structure of finite differences is shown: at the midpoint of dot line between circles or squares the arithmetic mean of corresponding values is used. Thus, the finite difference method for (8) reads as

$$\rho_0 \frac{u_i^{n+1} - u_i^n}{\tau} = D_{x_j}\sigma_{ij}^{n+1/2} + \left(\varepsilon_{j\alpha}^{n+1/2} + \omega_{j\alpha}^{n+1/2}\right) D_{x_j}\Sigma_{ij}^{n+1/2},$$

$$\frac{\sigma_{ij}^{n+1/2} - \sigma_{ij}^{n-1/2}}{\tau} = C_{ijkl} D_{x_l} u_k^n,$$

$$\frac{\varepsilon_{ij}^{n+1/2} - \varepsilon_{ij}^{n-1/2}}{\tau} = \frac{D_{x_i}u_j^n + D_{x_j}u_i^n}{2}, \quad \frac{\omega_{ij}^{n+1/2} - \omega_{ij}^{n-1/2}}{\tau} = \frac{D_{x_i}u_j^n - D_{x_j}u_i^n}{2}.$$

Here D_{x_i} are the difference approximation of spatial derivatives:

$$D_{x_1}(x_1, x_2)u_i = \frac{1}{h_1}\left[\frac{u_i(x_1 + h_1/2\,, x_2 + h_2/2) + u_i(x_1 + h_1/2\,, x_2 - h_2/2)}{2} - \frac{u_i(x_1 - h_1/2\,, x_2 + h_2/2) + u_i(x_1 - h_1/2\,, x_2 - h_2/2)}{2}\right] \approx \frac{\partial u_i}{\partial x_1}(x_1, x_2),$$

$$D_{x_2}(x_1, x_2)u_i = \frac{1}{h_2}\left[\frac{u_i(x_1 - h_1/2\,, x_2 + h_2/2) + u_i(x_1 + h_1/2\,, x_2 + h_2/2)}{2} - \frac{u_i(x_1 - h_1/2\,, x_2 - h_2/2) + u_i(x_1 + h_1/2\,, x_2 - h_2/2)}{2}\right] \approx \frac{\partial u_i}{\partial x_2}(x_1, x_2).$$

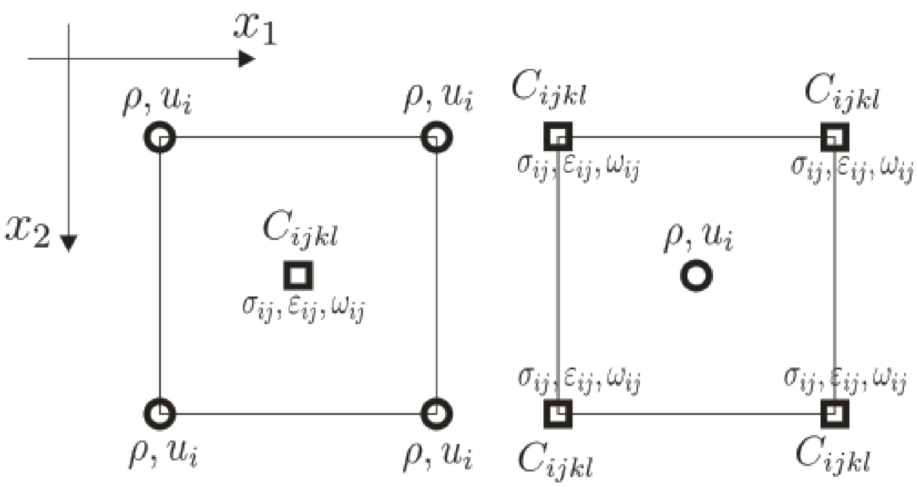

Figure 2: Staggered grid definition. The density and velocities are related to circles. Stress, strain, rotation tensors and elastic moduli are related to squares.

One can prove that the stability condition for this method in the two-dimensional case is similar to that formulated in [7]: $\tau \leq \frac{h}{\max V_{p\alpha}}$, $h = \sqrt{h_1^2 + h_2^2}$, where $V_{p\alpha}$ is the speed of longitudinal wave at grid points.

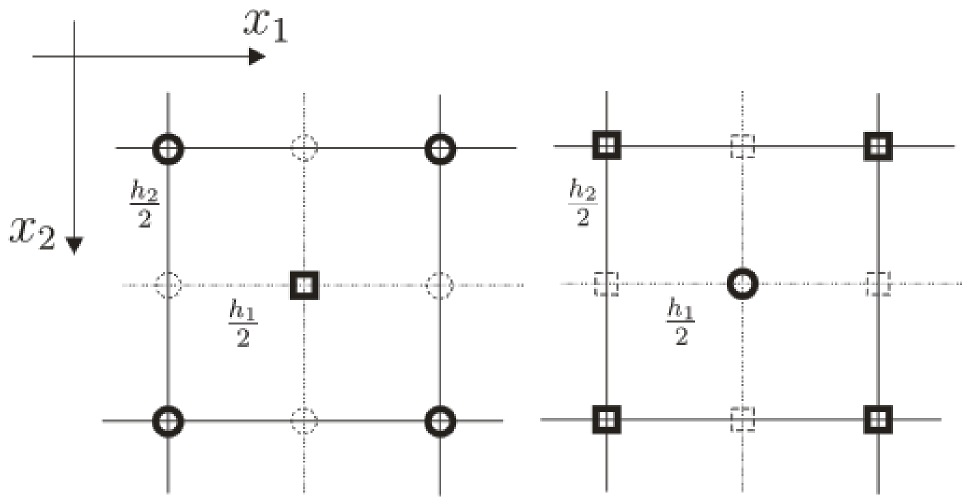

Figure 3: The structure of finite differences. At the midpoint of dot line between circles or squares a mean value of corresponding variables is used.

4 WAVE PROPAGATION IN THE UNIDIRECTIONALLY PRESTRESSED MEDIA IN THE PRESENCE OF THE GRADIENT OF INITIAL STRESS

In this Section a numerical test problem aimed to demonstrate an influence of the initial stress on the wave field generated by the Ricker wavelet is considered. The computations are made by the staggered grid method presented in the previous Section. The test problem is formulated as follows. The computational domain $(x_1, x_2) \in [0, L] \times [0, L]$, $L = 800m$ is a square, in which the initial stress is given as $\Sigma^{11} = P(x_2)$, $\Sigma^{ij} = 0, (ij \neq 11)$, where $P(x_2)$ is the linear function of x_2: $P(x_2) = \frac{C}{10}\left(1 - \frac{2x_2}{L}\right)$. It is obvious that the above stress field satisfies equilibrium equations. The maximal tensile stress $\Sigma^{11} = C/10$ is on the bottom of the computational domain and the maximal compression with $\Sigma^{11} = -C/10$ is on the top of the domain. The parameters of the medium are $V_p = 3000m/s$, $V_s = 2000m/s$, $\rho_{00} = 2000kg/m^3$, and $C = \rho_{00} V_p^2$. The source of elastic waves with 60 Hz dominant frequency is located in the centre of the domain. On Figure 4 the snapshot of x_2 stress component σ_{22} is presented. It obvious that the wave velocities decrease towards to the bottom and increase towards to the top of computational domain. This effect is caused by the effect of compression of the upper part of the domain and tension of the lower part. On Figure 5 and Figure 6 the seismograms recorded by receivers in the upper and lower parts of the domain are shown. The receivers are located on the horisontal straight lines corresponding to $160m$ and $640m$ on vertical axis. Red curves correspond to the prestressed medium, black curves

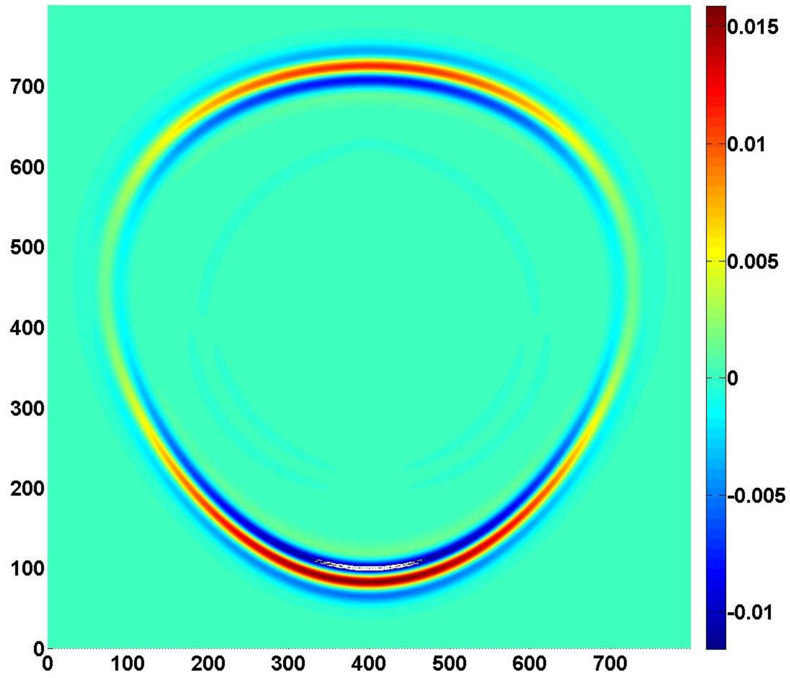

Figure 4: The snapshot of the vertical component of stress tensor.

correspond to the unstressed elastic medium. It is clearly seen the dependence of wave velocities on the spacial direction.

5 CONCLUSIONS

- The method of derivation of the governing equations for the small amplitude wave propagation in the initially prestressed medium is proposed.
- Governing equations for elastic waves in prestressed medium are derived in the case of quadratic dependence of elastic energy on the strain tensor.
- Properties of wavefield in the unidirectionally prestressed medium are discussed and it is demonstrated that the initial stress can have a significant influence on the character of wave propagation.
- The existence of prestressed zones must be taken into account for the forward modeling and inversion of seismic waves.

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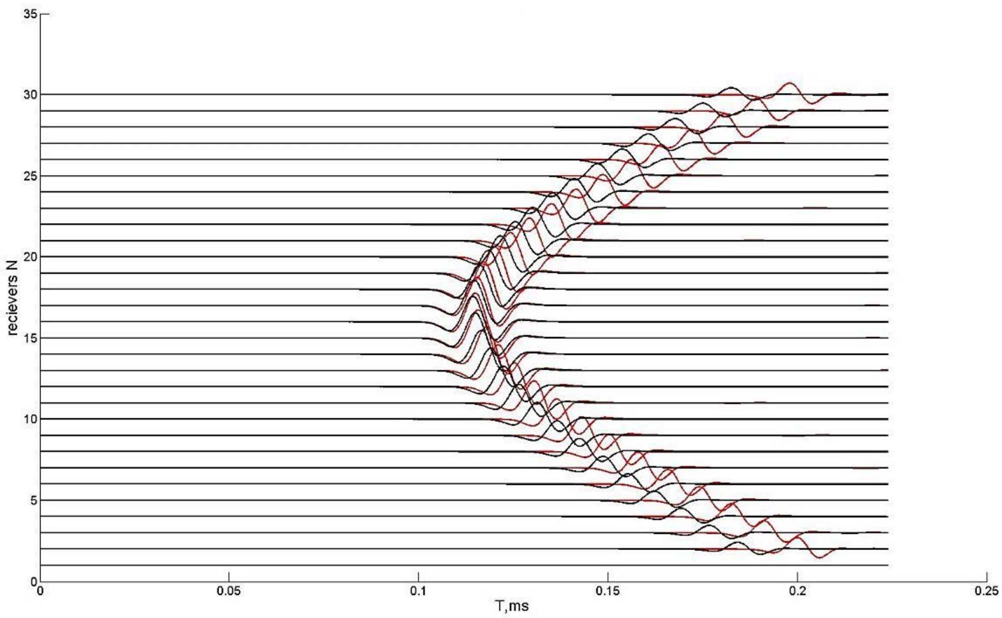

Figure 5: Seismograms recorded by receivers in the upper region. Red lines correspond to the prestressed medium, black lines correspond to the unstressed medium.

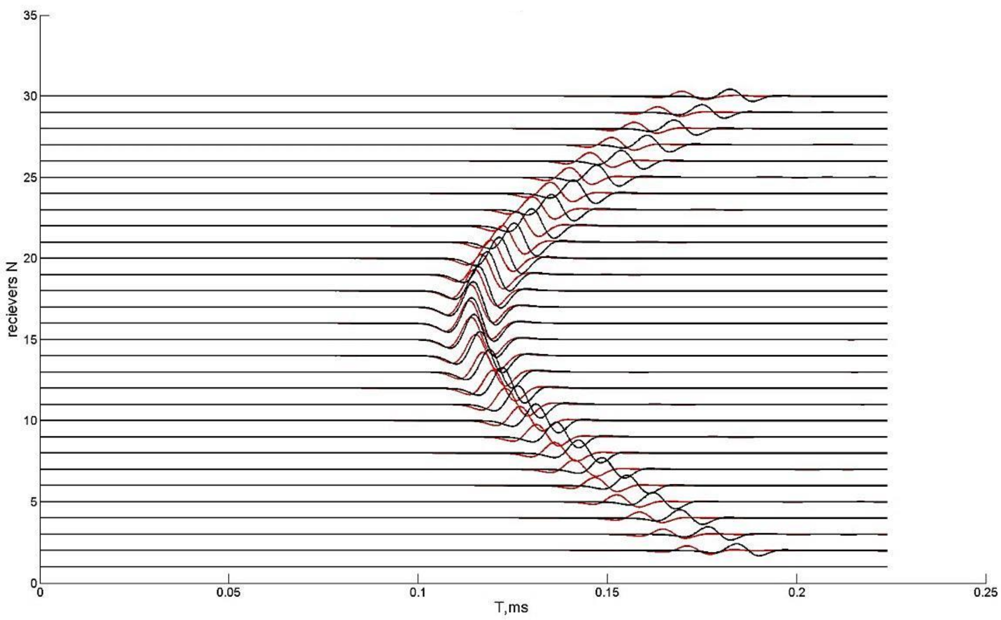

Figure 6: Seismograms recorded by receivers in the lower region. Red lines correspond to the prestressed medium, black lines correspond to the unstressed medium.

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