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MODIFIED DYNAMIC OBSERVERS BASED ON GREEN FUNCTIONS METHOD TO SOLVE A 3D TRANSIENT IHCP

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Abstract. The inverse problem can be found in a large area of science and engineering and can be applied in different ways. The great advantage of this technique is the ability of obtaining the solution of a physical problem that cannot be solved directly. Different techniques of the inverse heat conduction problem (IHCP) can be found in literature. In the dynamic observer technique, the IHCP solution algorithms are interpreted as filters passing low-frequency components of the true boundary heat flux signal while rejecting high-frequency components in order to avoid excessive amplification of measurement noise [1]. The dynamic observers technique proposed by Blum and Marquardt [1], focused on the one-dimensional linear case, is here extended to solve an inverse multidimensional heat conduction problem. In order to deal with multidimensional thermal models, this work proposes an alternative way of obtaining the heat transfer function, G_H. The obtaining of this function represents an important role in the observer method and is crucial to allow that the technique be directly applied to three dimensional heat conduction problems. In this work, the heat conductor transfer function G_H is obtained by using the Green function concept. This new procedure allows flexibility and efficiency to solve multidimensional inverse problems. The inverse heat conduction problem is represented by an unknown heat flux heat that is partially imposed at a front surface of a sample while the other surface is kept at constant temperature. The reminiscent surfaces are exposed to convective medium. The heat flux is then estimated by using the modified dynamic observer techniques and temperature data from a sensor located at the sample far from the heat source. The novelty is the procedure used to obtain the heat transfer function. This work uses a polynomial fitting of the transfer function model G_H in time domain instead the linear adjust method. The technique is evaluated in two experimental cases.

1 INTRODUCTION

The inverse problem solution based on dynamic observers can be divided in two distinct steps: i) the obtaining of the transfer function model G_H; ii) the obtaining of the heat transfer functions G_Q and G_N and the building algorithm identification. A complete description of this technique can be found in the work of Blum and Marquardt [1] and Sousa [2].

2 FUNDAMENTALS

This section presents the new procedure used to obtain the heat transfer function. This work uses a polynomial fitting of the transfer function model G_H in time domain instead the linear adjust method.

2.1 Heat transfer function, G_H, identification using Greens function: 3D-transient applications.

The solution of a heat conduction equation can be given in terms of Green's function as in [2] by

$$T(x,y,z,t) = \int_{\tau=0}^{t} \left[G_H(x,y,z,t/\tau)\ Q^{+}(\tau) \right] d\tau \tag{1}$$

where

$$G_H(x,y,z,t/\tau) = \frac{\alpha}{k} \int_0^{x_h} \int_0^{z_h} G(x,y,z,t-\tau)\Big|_{y'=0} dx'\,dz' \tag{2}$$

and $G(x,y,z,t-\tau)$ represents the Green function of the thermal problem involved.

Equation (1) reveals that an equivalent thermal model can be associated with a dynamic model. It means, a response of the input/output system can be associated to Eq.(1) in the Laplace domain as the convolution product

$$T(x,y,z,t) = G_H(x,y,z,t-\tau) * q(\tau) \tag{3}$$

This dynamic system can be represented by Fig.(1).

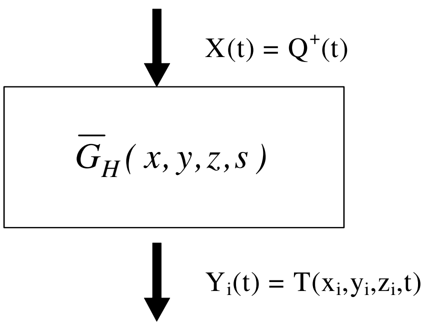


Figure. 2. Dynamic thermal model system

Equation (3) can also be evaluated in the Laplace domain as a single product

$$\bar{T}(x,y,z,s) = \bar{G}_H(x,y,z,s)\ \bar{Q}^{+}(s) \quad (4)$$

where the Laplace transform of a F(t) function is defined by

$$L[F(t)] = \bar{F}(s) = \int_t^{\infty} e^{-st'} F(t')dt' . \quad (5)$$

The heat transfer function $\bar{G}_H(x,y,z,s)$ can, then, be obtained through the auxiliary problem which is a homogenous version of the problem defined by Eq.(2) for the same region with a zero initial temperature and unit impulse source located at the region of the original heating.

Similarly, the auxiliary thermal problem solution can be derived using Green function and the convolution properties as

$$T^{+}(x,y,z,t) = G_H(x,y,z,t-\tau) * 1 \quad (6)$$

Since the Laplace transform of 1 is

$$L[1] = \frac{1}{s} \quad (7)$$

then

$$\bar{T}^{+}(x,y,z,s) = \bar{G}_H(x,y,z,s)\ \frac{1}{s} \quad (8)$$

If the dynamic system is linear, invariant and physically invariable the response function $\bar{G}_H(x,y,z,s)$ is the same, independent of the input/output pairs and can be obtained by

$$\bar{G}_H(x,y,z,s) = s\ \bar{T}^{+}(x,y,z,s) \quad (9)$$

In order to complete the $\bar{G}_H(x,y,z,s)$ identification, the $\bar{T}^{+}(x,y,z,s)$ must be obtained at a specific position $r_i = (x_i, y_i, z_i)$.

A new simple and efficient procedure is proposed here to obtain $T^{+}(r_i,s)$. If Eq.(9) represents a cross correlation function of the two functions of stationary random process ***s*** and $\bar{T}^{+}(x,y,z,s)$, then $\bar{G}_H(x,y,z,s)$ will be independent of the absolute time ***t*** and will depend only on the time interval $\boldsymbol{t_a}$.

In this case, provided that the function $T^{+}(r_i,s)$ can be fit by a polynomials function in the sampled interval $[0, t_a]$ as

$$T^{+}(r_i,t) = a_1 t + a_2 t^2 + a_3 t^3 + \cdots \quad (10)$$

where $\boldsymbol{a_i}$ are the polynomial coefficients. Then , the Laplace transform of Eq.(10) gives

$$\bar{T}^{+}(r_i,s) = \frac{a_1}{s} + \frac{a_2}{s^2} + \frac{a_3}{2s^3} + \frac{a_4}{6s^4} + \cdots \tag{11}$$

Thus, from Eq. (9), G_H can be written as

$$\bar{G}_H(r_i,s) = s\ \bar{T}^{+}(r_i,s) = a_1 + \frac{a_2}{s} + \frac{a_3}{2s^2} + \frac{a_4}{6s^3} + \cdots \tag{12}$$

Once obtained the function $G_H(L,s)$ the next step is to obtain the estimators G_Q and G_N that are related to G_H as follows [1] and [2],

$$\bar{Q}^{+}(s) = G_Q(s)\ Q^{+}(s) + G_N(s)\ N \tag{13}$$

In Eq.(13) G_Q is referred to as the signal transfer function and G_N is referred to as the noise transfer function. The variable $Q^{+}(s)$ is the true value of heat flux in Laplace domain and N is the random noise due to the temperature measurements. Symbol (^) denotes estimates values. The relation between the heat transfer function G_H, G_Q and G_N is better described in the next section.

The optimization procedure can be resumed in the use of the two discrete-time difference equations

$$\hat{q}(k) = \sum_{i=0}^{n_n} b_i Y_M(k-i) - \sum_{i=1}^{n_n} a_i \hat{q}(k-i) \tag{14}$$

and

$$\hat{q}(k) = \sum_{i=0}^{n_n} b_i q(k-i) - \sum_{i=1}^{n_n} a_i \hat{q}(k-i) \tag{15}$$

In Eqs.(14) and (15) $\boldsymbol{a_i}$ and $\boldsymbol{b_i}$ are coefficients obtained through the equations

$$\hat{Q} = \underbrace{\frac{G_C G_h}{1+G_c G_h}}_{G_Q} Q + \underbrace{\frac{G_C}{1+G_c G_h}}_{G_N} N \tag{16}$$

and

$$G_N = G_Q G_H^{-1} \text{ or } \left|G_N(j\omega)\right| = \frac{\left|G_Q(j\omega)\right|}{\left|G_H(j\omega)\right|} \tag{17}$$

The transfer function G_Q is chosen to have the behavior of type I chebychev filter and its frequency response magnitude assume the form

$$G_Q(s) = \frac{k_{cheb}}{(s-s_{Cheb,1})(s-s_{Cheb,2})\cdots(s-s_{Cheb,n_Q})} \tag{19}$$

The poles $s_{cheb,I}$ are computed using MATLAB software package. As mentioned, more details of the observer procedure can be found in references [1,2].

$$G_N = G_Q G_H^{-1} \text{ or } \left|G_N(j\omega)\right| = \frac{\left|G_Q(j\omega)\right|}{\left|G_H(j\omega)\right|} \tag{20}$$

2 RESULTS AND DISCUSSION

The dynamic observer technique is used here to solve a cutting orthogonal application. This section presents the IHCP thermal problem that occurs during a cutting orthogonal test..

The prejudicial effects of the high temperatures generated in machining operations basically involve premature wear of the tools and the formation of imperfections on the piece surface, in the form of geometric and dimentional deviatons. On the other side, there are benefits, beacuse of the high temperatures that modify the mechanical properties of the material, leading to redutcion in machining power. For such reasons, the comprehension of the mechanisms involved in the heat generation and distribution in machining processes is fundamental for the development of technologies in interest of cutting edge preservation and machined surface quality improvement, aiming machining costs reduction. In this cutting application, the tests involved dry machining of grey iron cylinders, using uncoated carbide tools. The temperatures of the insert were measured with five T-type 28 AWG thermocouples each. The thermocouples were attached using capacitive discharge welding at the insert. Figure 2 shows the experimental apparatus and Figure 3 shows a scheme for visualizing the positions of the thermocouples attached at carbide tools.

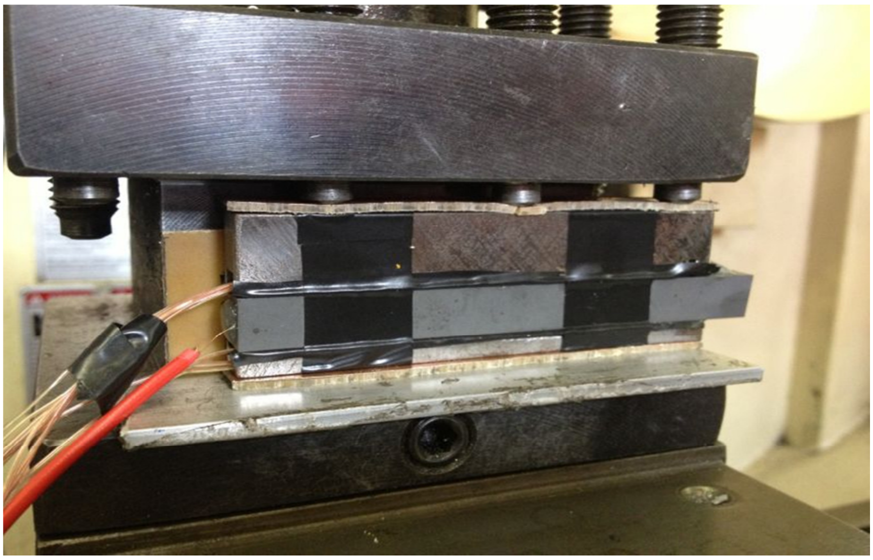


Figure 2 . Tool-thermocouple experimental set-up

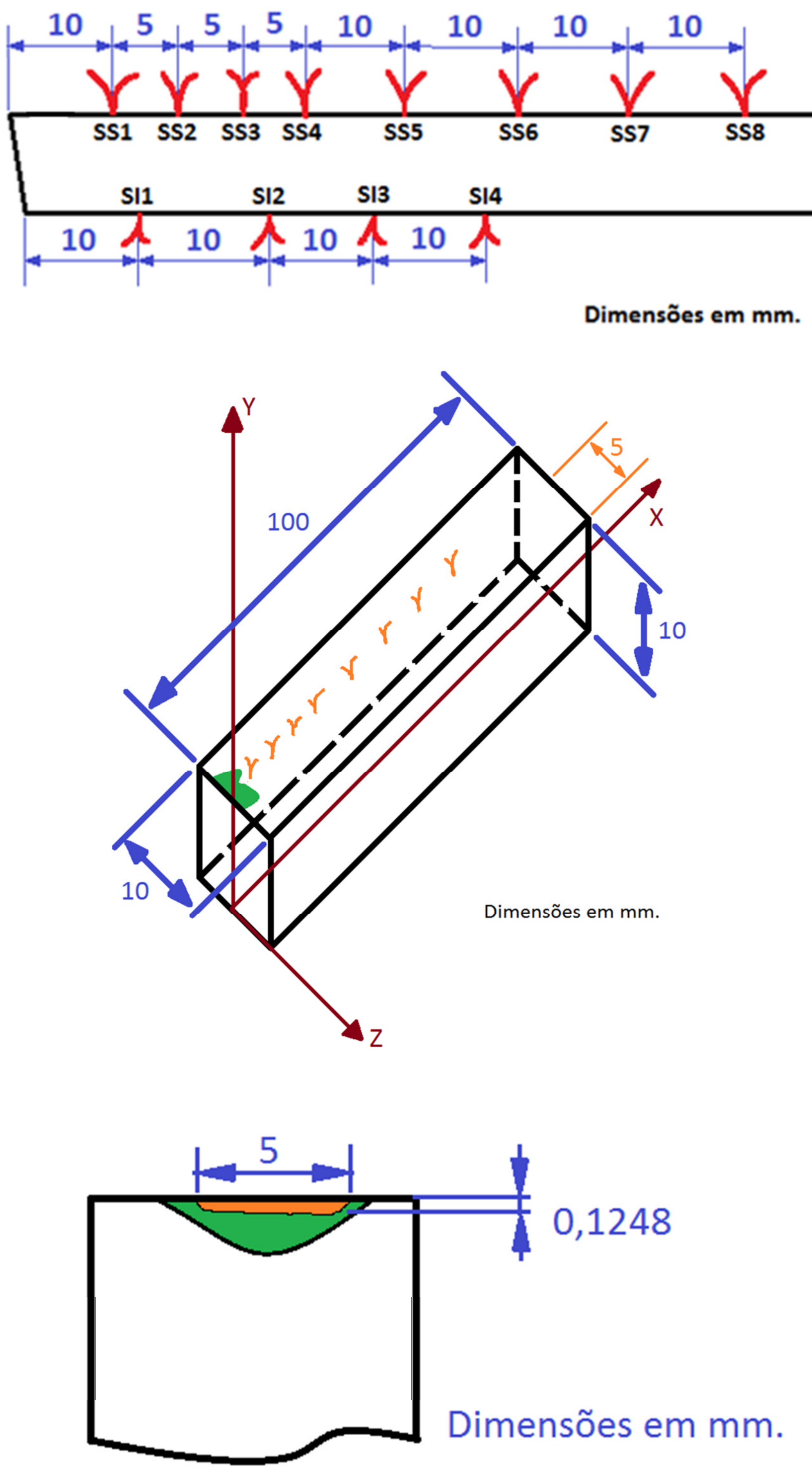


Figure 3. Tool geometry and thermocouple positions

For the heat flux estimation the method uses temperature measurements at the bottom surface of the tool, opposite to its rake face. Figure 4 shows these temperatures.

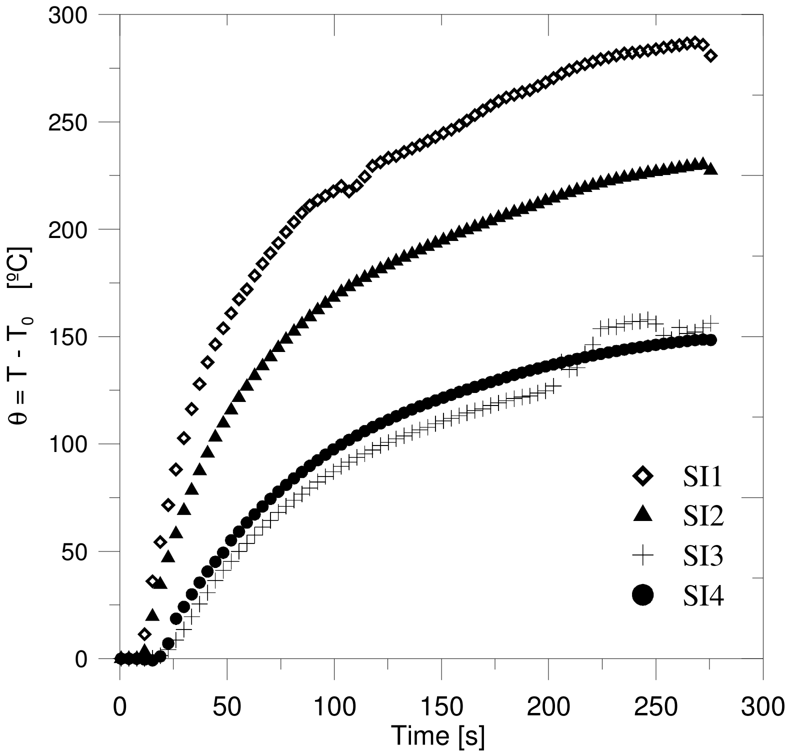


Figure 5. Measured temperatures at the bottom surface

Using this temperatures and the Eqs.(14) and (15) the heat flux at interface chip-tool can be estimated. The direct thermal model is a 3D transient heat conduction with a heat flux heating partially a surface with all reminiscent surface submitted to a constant convection heat transfer coefficient (h = 20 W/m²K). The values of thermal properties used to calculate these temperatures are $\lambda = 100$ W/m.K and $\alpha = 27\text{x } 10^{-06}$ m^2/s. The simple time was 0.524 s. The direct themal model was solved using the comercial software COMSOL. Figure 6 shows the heat flux estimation using dynamic observer and the well-known sequential method [3].

Dispersion of results may be credited to chip-tool contact area approximation and other unavoidable sources of error such as uncertainty in measurements or in thermal properties considered.

The assumption of constant heat transfer coefficient is another error source. Yet it can be found little influence on the results for values of h between 5 to 30 W/m2K.

The temperature at the chip-tool contact area is shown in Fig. 7.

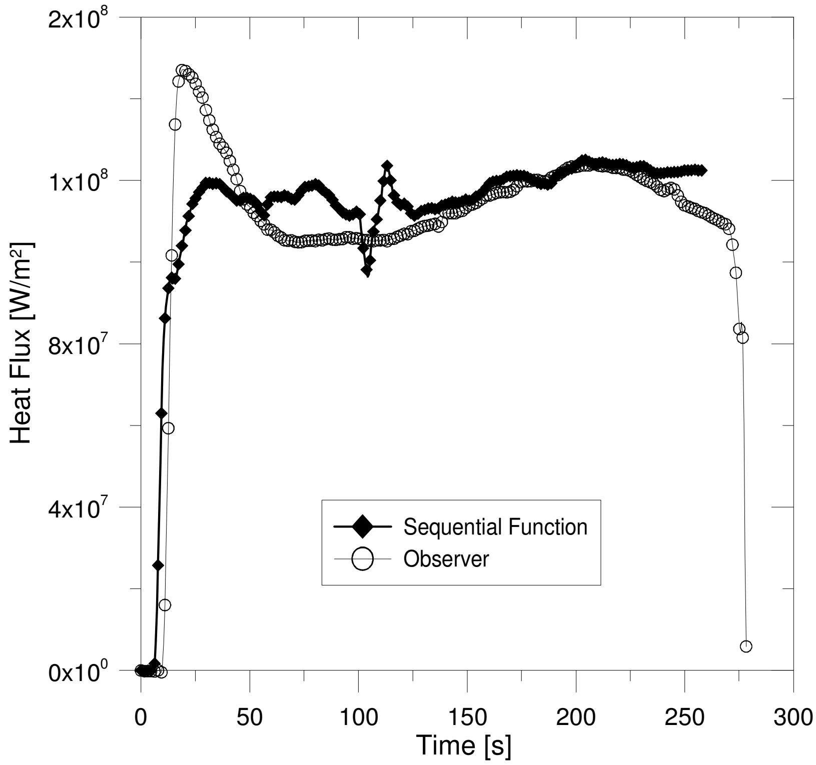


Figure 6. Heat flux estimated using temperature measured from SI2

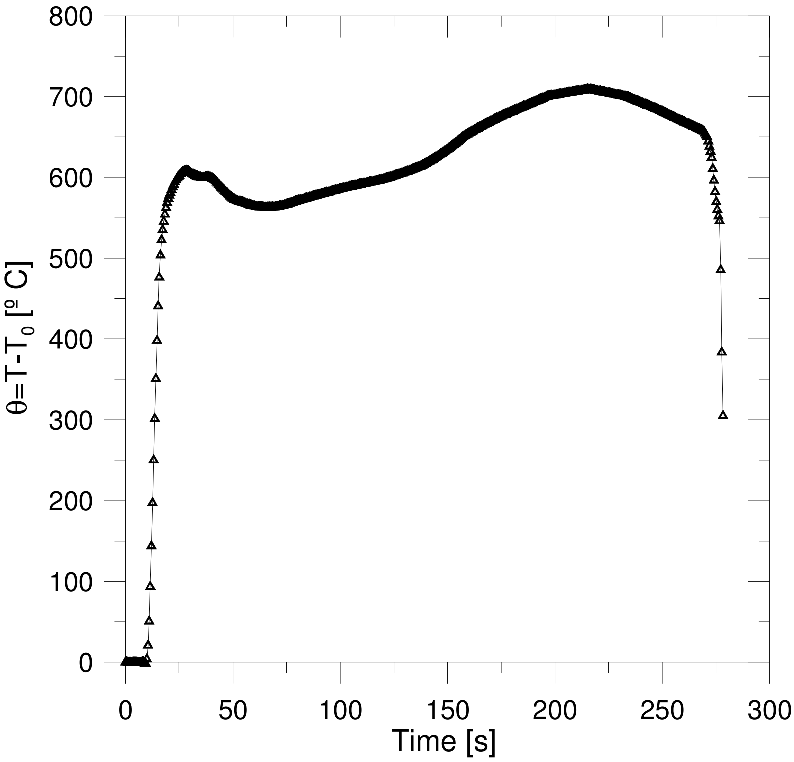


Figure 7. Interface chip-tool estimated temperature

The maximum temperature found in this experiment was around 700 °C. The validation of the results can be obtained by comparison between the calculated and measured temperature at the rake face. Figure 8 shows this comparison for thermocouples SS5, SS6, SS7 and SS8. These thermocouples were not used to obtain the heat flux. They were used specifically for validation proposes. This comparison is an evidence of that methodology used is adequate. Small discrepancies can be attributed to various source of error such as:

i) theoretical model uncertainties (thermal properties of the tools and constant heat transfer coefficient assumptions); ii) measurement uncertainties (chip-tool contact area, thermocouple fixation, calibration, response time and position); iii) uncertainty due to numeric approaches (integration, discretization and convergence).

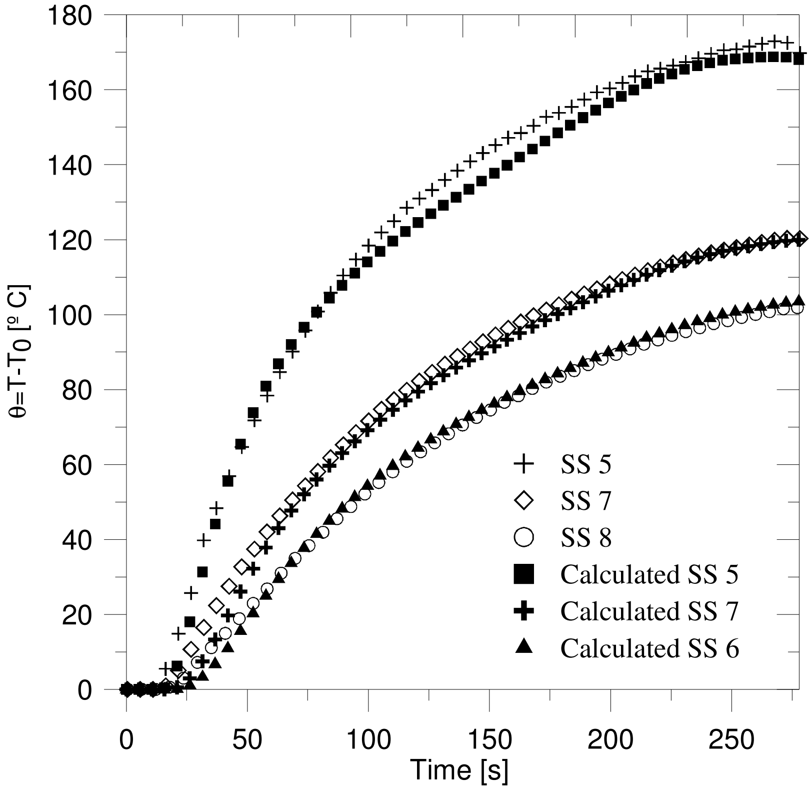
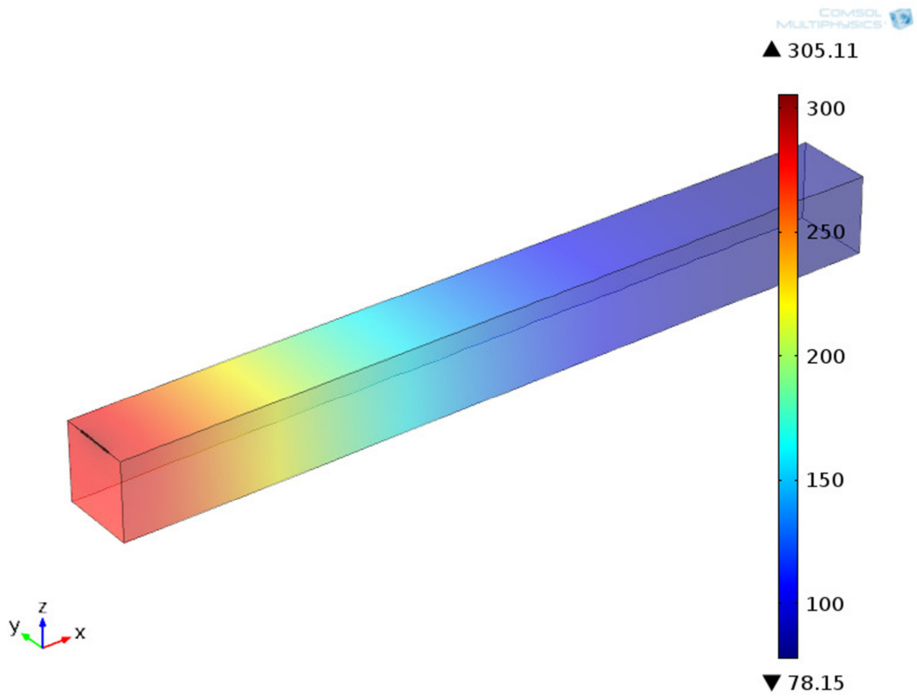


Figure 8. Comparison between calculated and measured temperatures at rake face

Since the heat flux has been estimated at chip-tool interface, the temperature field on the tool can be calculated. This field can be seen in Figure 9.

It can be observed that the heat flux is recovered in both heating and cooling regions. However some oscillations can be observed. The great advantage of dynamic observer technique is the easy and fast numerical implementation for 3D model. The robustness and low computational cost and low error sensitivity give to this procedure a great potential in inverse techniques application.



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4 CONCLUSIONS

The dynamic observer technique based on analytical Green function is presented here. The procedure used to obtain the analytical heat transfer gives more flexibility and robustness to the inverse procedure.

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