

ISOGEOMETRIC AND HIGH-ORDER BOUNDARY ELEMENT METHODS

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ABSTRACT

The boundary element method (BEM) has historically offered engineers an alternative technique to finite elements that has offered attractions for certain classes of problem including infinite domains as found in acoustic scattering or those involving discontinuous and singular fields in fracture mechanics. The emergence of the isogeometric approach in computational mechanics has recently led to a rekindling of interest in the BEM for general problems in solid mechanics due the requirement for a surface-only mesh that provides a direct link with geometric surface modelling technology.

Isogeometric Analysis with the Finite Element Method (IGAFEM) [1] reformulates the FE problem by seeking the solution using a basis that is defined by the CAD geometry discretisation. Most commonly this takes the form of NURBS basis functions or, more recently, T-splines [2]. Both discretizations offer considerable benefits in that (i) the exact geometry is used, rather than a piecewise polynomial approximation, (ii) the requirement for

meshing is bypassed (iii) more accurate solutions can be obtained for a given number of degrees of freedom. While IGAFEM is most attractive for the analysis of shells, its use for solids presents more challenges. However, applying the isogeometric idea to the BEM, in a technique we can call IGABEM, these challenges no longer apply.

IGABEM research is in its infancy, but researchers are demonstrating the significant benefits of circumventing mesh generation, higher accuracies obtained through exact geometries and attractive convergence properties [3]. There are many exciting prospects both from an engineering and mathematical standpoint, with many questions remaining on topics such as convergence, integration error and the application of the technique to other engineering applications. What is clear, though, is that the surface modelling descriptions provided by CAD lend themselves naturally to BEM and this link represents a significant advance for both the mathematical and engineering communities.

In addition to IGABEM, we widen the scope of the proposed minisymposium to high-order boundary element methods. Recent findings provide the theoretical foundation for the development of high order implementations which lead to accurate solutions with optimal numerical costs. In what concerns the numerical realization, most fundamental is the choice of appropriate boundary element spaces on curved surfaces. The embedding of curved surface patches is non-canonical as the metric is not Euclidean. Thus, one of the fundamental topics concerning high-order boundary element methods in the context of isogeometric analysis is a clear understanding of how the non-canonical embedding of the surface patches goes along with the duality pairings relating the trace spaces.

IGABEM and associated high order methods are emerging as an exciting area, and we expect significant interest to have developed within two years, in time for a vibrant minisymposium at WCCM Barcelona.

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