The 8th European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS Congress 2022
5-9 June 2022, Oslo, Norway

Goal-oriented adaptive MLMC for elliptic random PDEs

Joakim Beck[1,2], Yang Liu[1,2,*], Erik von Schwerin[1,2], and Raúl Tempone[1,2,3]

[1] Computer, Electrical and Mathematical Sciences and Engineering,
4700 King Abdullah University of Science and Technology (KAUST),
Thuwal 23955-6900, Kingdom of Saudi Arabia.
[2] KAUST SRI Center for Uncertainty Quantification in Computational Science and Engineering
[3] Alexander von Humboldt Professor in Mathematics for Uncertainty Quantification,
RWTH Aachen University, 52062 Aachen, Germany.

Keywords: *Goal-oriented adaptivity, MLMC, Matérn random fields, elliptic random PDE*

Multilevel Monte Carlo methods (MLMC) can dramatically reduce the computational cost of Monte Carlo simulations where each sample is computed using a discretization based numerical method, for example, when computing the expected value of a quantity of interest (QoI) depending on the solution to a partial differential equation with stochastic data.

Goal-oriented adaptive finite element refines the mesh based on the error contribution to the QoI. This is efficient, for instance, when the geometry presents a singularity, such as a non-convex domain.

The purpose of this work is to combine MLMC and adaptive finite element solvers, to efficiently solve a boundary-value problem of an elliptic partial differential equation with random coefficients on a non-convex domain. The QoI is a linear functional of the PDE solution, and the coefficient field is efficiently sampled from a regular coefficient random field. The adaptive refinement algorithm is based on [1]. This work can also be seen as an extension of [2].

REFERENCES

[1] Moon, Kyoung-Sook., Erik von Schwerin, Anders Szepessy, and Raúl Tempone. "Convergence rates for an adaptive dual weighted residual finite element algorithm." BIT Numerical Mathematics 46, no. 2 (2006): 367-407.

[2] Hoel, Håkon, Erik von Schwerin, Anders Szepessy, and Raúl Tempone. "Implementation and analysis of an adaptive multilevel Monte Carlo algorithm." Monte Carlo Methods and Applications 20, no. 1 (2014): 1-41.