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Robust discretizations for the biphasic Theory of Porous Media

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Porous materials consists of a solid skeleton, saturated by interstitial fluids, and are characterized by the interaction of deformation and a resulting fluid flow. Describing these interactions mainly three theoretical frameworks, namely the Biot theory [1], Mixture Theory (MT) [2] and the Theory of Porous Media (TPM) [3], are nowadays established. While Biot derived his theory from a macroscopic viewpoint, MT and TPM are based on a homogenization of the mixture, alongside with a thermodynamically consistent modeling of arising interaction terms.

The applicability of a general, poroelastic theory is ranging from soil mechanics up to the modeling of biological tissue. As model constants therefore vary over wide ranges, numerical algorithms, based on standard discretizations, often perform poorly due to their insufficient robustness. Well-known difficulties are locking, when nearly incompressible solids are considered, or pressure oscillations for low permeabilities. Increasing reliability and robustness, accurate approximations of total stress tensor and Darcy velocity are known to be crucially important. If computed from the primal approach in post-processing, those quantities possess a divergence which is not squarely integrable. They are not H(div)-conforming, and traction forces as well as fluid flux on a surface or interface cannot be evaluated. As these quantities are driving quantities, the standard methods are not accurate enough. After giving some concrete examples, this talk will demonstrate how to recover stress and fluid flux, following [4,5], and compare the results with the direct approximation of the dual quantities.

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