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ANALYSIS AND APPROXIMATION OF MIXED-DIMENSIONAL PDES ON 3D-1D DOMAINS COUPLED WITH LAGRANGE MULTIPLIERS

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Coupled partial differential equations (PDEs) defined on domains with different dimensionality are usually called mixed-dimensional PDEs. We address mixed-dimensional PDEs on three-dimensional (3D) and one-dimensional (1D) domains, which gives rise to a 3D-1D coupled problem.

Such a problem poses several challenges from the standpoint of existence of solutions and numerical approximation, see for example [1]. For the coupling conditions across dimensions, we consider the combination of essential and natural conditions, which are basically the combination of Dirichlet and Neumann conditions. To ensure a meaningful formulation of such conditions, we use the Lagrange multiplier method suitably adapted to the mixed-dimensional case [2]. The well-posedness of the resulting saddle-point problem is analyzed.

Then, we address the numerical approximation of the problem in the framework of the finite element method. The discretization of the Lagrange multiplier space is the main challenge. Several options are proposed, analyzed, and compared, with the purpose of determining a good balance between the mathematical properties of the discrete problem and flexibility of implementation of the numerical scheme. The results are supported by evidence based on numerical experiments. The application of this technique to microcirculation [3,4], the modeling of reinforced materials [5] and subsurface wells will be discussed.

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