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Domain decomposition preconditioners for non-self-adjoint or non-positive-definite problems

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The matrices arising from the finite element discretization of problems such as high-frequency Helmholtz, time-harmonic Maxwell or convection-diffusion equations are not self-adjoint or positive definite. For this reason, it is difficult to analyze the convergence of Schwarz domain decomposition preconditioners applied to these problems. Note also that the conjugate gradient method cannot be used, and the analysis of the spectrum of the preconditioned matrix is not sufficient for methods suited for general matrices such as GMRES. In order to apply Elman-type estimates for the convergence of GMRES we need to prove an upper bound on the norm of the preconditioned matrix, and a lower bound on the distance of its field of values from the origin. Convergence results for the Helmholtz equation have been recently obtained in [3, 4] and for the Maxwell equation in [2]. In [1] we generalize to a a generic linear system the theory developed in [4] for the Helmholtz equation, and we identify a list of assumptions and estimates that are sufficient to prove the two bounds needed for the convergence analysis of GMRES, with the SORAS (Symmetrized Optimized Restricted Additive Schwarz) preconditioner.

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