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Learning reduced subgrid-scale models

Wouter Edeling* [1], **Daan Crommelin** [1,2]

[1] Centrum Wiskunde & Informatica - Scientific Computing group, Science Park 123, 1098 XG Amsterdam, The Netherlands, wouter.edeling@cwi.nl

[2] Korteweg-de Vries Institute for Mathematics, Science Park 105-107, 1098 XG Amsterdam, The Netherlands, D.T.Crommelin@uva.nl

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It is well known that the wide range of spatial and temporal scales present in turbulent flow problems represents a currently insurmountable computational bottleneck, which must be circumvented by a coarse-graining procedure. The effect of the unresolved fluid motions enters the coarse-grained equations as an unclosed forcing term, denoted as the subgrid-scale (SGS) term. The SGS term is a dynamically evolving field, with complex spatial patterns. Therefore, despite the coarse graining procedure, the SGS term still contains a high number of unclosed (and therefore unknown) degrees of freedom.

One can attempt to decrease the number of unknowns by creating a Reduced-Order Model (ROM), for instance one based on extracting (POD) modes from a database of reference SGS snapshots. Instead of creating a ROM that aims to represent the full SGS term, we propose a new type of ROM that is tailor-made to capture spatially-integrated quantities of interest (QoIs). Examples of such QoIs include climate-like statistics of the full model, for instance the global energy of the system. We will show that if we restrict our interest to these small QoI, we do not need a SGS term with a large number of unclosed degrees of freedom. Instead, we can derive a ROM (which we call a reduced SGS model), that strikes a balance between physical insight and data-driven modelling, and which significantly reduces the amount of training data that is needed to model the remaining (small) unclosed component [1]. This small remaining component is then the only part for which we construct a surrogate model via machine learning.

We derive the new ROM for two-dimensional turbulence in a doubly periodic square domain, and show that it produces the same statistics for our quantities of interest as the exact (full-field) SGS term, which is extracted from a high-resolution reference model. Additionally, we discuss the challenges of training the reduced SGS term, while it is coupled to the partial differential equations of the (macroscopic) flow quantities.

REFERENCES

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