The 8th European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS Congress 2022
5-9 June 2022, Oslo, Norway

Nonlinear Reduced Modelling based on Optimal Transport Metrics

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Keywords: *Model reduction, metric spaces, Wasserstein Barycenter, hyperbolic problem.*

The modeling of complex systems described by Partial Differential Equations (PDEs) often involves many parameters that describe the physical or geometric properties of the system. In this context, model reduction methods are designed to approximate quickly and accurately the solution of these equations when the parameters of the PDEs vary. Important model reduction techniques such as proper orthogonal decompositions (POD) and reduced bases rely on linear sub-spaces of Hilbert and Banach spaces. These linear methods have been used successfully for elliptic and parabolic problems, but are no longer efficient for problems with strong advection effects that carry shocks and discontinuities: the Kolmogorov n-width decays slowly for this class of problems [1]. To overcome this limitation, it is therefore necessary to study non-linear approximations methods and work with non-Euclidean metric spaces such as the Wasserstein space of probability distributions [2]. In this work, we focus on the development of such model reduction methods using tools derived from optimal transport theory [3, 4].

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