

# MULTIDIMENSIONAL RANK-ONE CONVEXIFICATION OF INCREMENTAL DAMAGE FORMULATIONS: ALGORITHMIC TREATMENT, IMPLEMENTATION ASPECTS, AND NUMERICAL ANALYSIS

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Various continuum mechanical formulations suffer from mesh-dependent solutions due to a loss of convexity of the underlying incremental energy density, e.g. in the context of continuum damage mechanics, which will be the focus application in this talk. Incremental variational formulations in the sense of e.g., [1] enable the analysis of generalized convexity conditions known for hyperelasticity in the context of dissipative materials and thus, they provide a basis for convexification to overcome the issue of mesh-dependency. In the context of continuum damage mechanics at finite strains, a convexified (relaxed) formulation has been constructed for one-dimensional fiber damage in [2]. Aside from avoiding mesh-dependencies, such relaxed formulations provide a direct interpretation in terms of microstructure nucleation and evolution. In this contribution, such a convexification scheme is extended to the multidimensional case by constructing the rank-one convex hull, for which an extended, parallelized algorithm based on [3] is presented. As a first step, we focus on two spatial dimensions and analyze our algorithm in several numerical benchmark tests.

## REFERENCES

- [1] M. Ortiz and E.A. Repetto, Nonconvex energy minimization and dislocation structures in ductile single crystals, *Journal of the Mechanics and Physics of Solids*, Vol. **47**, pp. 397–462, 1999.
- [2] Daniel Balzani and Michael Ortiz, Relaxed Incremental Variational Formulation for Damage at Large Strains with Application to Fiber-Reinforced Materials and Materials with Truss-like Microstructures, *International Journal for Numerical Methods in Engineering* Vol. **92**, pp. 551–570, 2012.
- [3] Sören Bartels, Linear convergence in the approximation of rank-one convex envelopes, *ESAIM: Mathematical Modelling and Numerical Analysis*, Vol. **38**, pp. 811–820, 2004.