

MODELING MICROCIRCULATION AT THE MESOSCALE USING MIXED-DIMENSIONAL PDES

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The microcirculation exemplifies the mesoscale in physiological systems, bridging larger and smaller scale phenomena. Multiscale mathematical models represent a valuable tool to investigate and understand such phenomena, where a brute force computational approach is not viable yet. Accurate microcirculation models must comprise both interstitial and vascular compartments, along with their complex morphology. Transport phenomena across the vascular wall couple these two main compartments. In addition, several nonlinear effects are required to properly model microvascular flow.

A naive modeling approach based on the standard application of computational fluid dynamics tools would fail to consider the complex interaction between the microcirculation and the tissue at a reasonable computational cost. The mesoscale approach addressed here is the ideal framework for the interaction of all these effects in a reasonably complex vascular network. The modeling framework that we adopt stems from the seminal work on the method of Green's functions applied to microcirculation [1]. However, we depart from such an approach because the phenomena addressed before are formulated in the continuum setting in our model. More precisely, a model reduction technique is applied to the equations defined in the vasculature, transforming the network of vessels into a one-dimensional (1D) graph. Its coupling with the three-dimensional (3D) surrounding environment is addressed by the mathematically sound coupling operators described and analyzed in [2]. The main advantage of this approach consists in its modularity: the vascular and tissue regions can be discretized independently and the numerical solution of the former is greatly simplified by the dimensionality reduction. We will demonstrate this property on a particular example involving the microvascular oxygen transfer [3].

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