

Physics-constrained deep learning-based reduced order models for parametrized PDEs

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Conventional reduced order models (ROMs) anchored to the assumption of modal linear superimposition, such as proper orthogonal decomposition (POD), may reveal inefficient when dealing with nonlinear time-dependent parametrized PDEs, especially for problems featuring coherent structures propagating over time. To enhance ROM efficiency, we propose a nonlinear approach to set ROMs by exploiting deep learning (DL) algorithms, such as convolutional neural networks. In the resulting DL-ROM, both the nonlinear trial manifold and the nonlinear reduced dynamics are learned in a non-intrusive way by relying on DL algorithms trained on a set of full order model (FOM) snapshots, obtained for different parameter values. Performing then a former dimensionality reduction on FOM snapshots through POD enables, when dealing with large-scale FOMs, to speedup training times, and decrease the network complexity, substantially. Accuracy and efficiency of the resulting POD-DL-ROM technique are assessed on a very broad range of examples, where new queries to the POD-DL-ROM can be computed in real-time. Finally, preliminary results about the integration of physics laws in the POD-DL-ROM neural network definition are shown.

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