

## Averaged models for two-phase flow at the pore scale: The effect of surface tension and contact angle dynamics

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We consider the flow of two immiscible fluids in pores of a porous medium. The fluids are separated by a freely moving interface in contact with the pore wall, which is driven by the flow and surface tension. We discuss a model in the idealized setting of a thin tube of varying width. The contact line model incorporates Navier-slip boundary conditions and a dynamic and possibly hysteretic contact angle law. Assuming a scale separation between the typical width and the length of the pore, matched asymptotic expansions can be applied to derive effective models for the two-phase flow. These models form a system of differential algebraic equations for the interface position and the total flux. The result is Darcy-type equations for the flow, combined with a capillary pressure - saturation relationship involving dynamic effects. We provide numerical examples to illustrate these. In the context of capillary rise in tubes, the effective model extends the classical Lucas–Washburn model by incorporating a dynamic contact angle and effective slip. We further extend this model to account for inertia. Comparison of models to experimental data shows good agreement only, if the dynamic contact angle is taken into account.

Finally, an outlook to our ongoing work covers the upscaling from pore scale to Darcy scale. To this end, we include the effective model as pore-throat model in a dynamic pore-network simulation. Averaging over the pore network then yields the macro-scale behaviour including the effect of surface tension and contact angle dynamics.

## REFERENCES

- [1] S. B. Lunowa, C. Bringedal, I. S. Pop, *On an averaged model for immiscible two-phase flow with surface tension and dynamic contact angle in a thin strip*, Studies in Applied Mathematics 147 (2021), pp. 84–126. doi.org/10.1111/sapm.12376
- [2] S. B. Lunowa, A. Mascini, C. Bringedal, T. Bultreys, V. Cnudde, I. S. Pop, *Dynamic effects during the capillary rise of fluids in cylindrical tubes*, UHasselt CMAT preprint UP-21-07. www.uhasselt.be/Documents/CMAT/Preprints/2021/UP2107.pdf