

On the tracking of eigensolutions to parametric partial differential equations

Moataz M. Alghamdi¹, Daniele Boffi² and Francesca Bonizzoni³

¹ King Abdullah University of Science and Technology (KAUST), Thuwal 23955-6900,
Kingdom of Saudi Arabia, moataz.alghamdi@kaust.edu.sa

² King Abdullah University of Science and Technology (KAUST), Thuwal 23955-6900,
Kingdom of Saudi Arabia and University of Pavia, Italy, daniele.boffi@kaust.edu.sa

³ University of Augsburg, Germany, francesca.bonizzoni@math.uni-augsburg.de

Keywords: *Parameter Dependent Eigenvalue Problems, Finite Elements, A Posteriori Analysis, Cost Function, Mode Tracking*

Several models arising from science and engineering can be expressed as eigenvalue problems depending on various parameters. While the numerical approximation of partial differential equations depending on stochastic parameters has been investigated widely, less attention has been devoted to eigenvalue problems. Before any reasonable discretization, the regularity of the solutions should be discussed. Eigenvalues may cross and a fundamental task is to properly track all correct modes. Also for this reason, only few investigations have been carried on so far, mainly related to isolated eigenvalues [1, 2].

We consider an elliptic eigenvalue problem with a multi parametric dependence and we design an adaptive algorithm which minimizes the number of solves to guarantee the correct matching of the eigenvalues within a prescribed tolerance. Our non intrusive scheme can be considered as a greedy parametric model reduction. It combines a technique involving an a priori cost functional [3] with a suitable a posteriori strategy, capable of detecting whether the matching of the corresponding eigenvalues was performed correctly.

The performance of the method is tested for various applications in order to assess its behavior when several parameters are present and intersections of eigenvalues occur.

REFERENCES

- [1] R. Andreev and C. Schwab, Sparse tensor approximation of parametric eigenvalue problems. In: Graham I., Hou T., Lakkis O., Scheichl R. (eds) *Numerical Analysis of Multiscale Problems. Lecture Notes in Computational Science and Engineering*, Vol. **83**, pp. 203–241, 2012.
- [2] I. Fumagalli and A. Manzoni and N. Parolini and M. Verani, Reduced basis approximation and a posteriori error estimates for parametrized elliptic eigenvalue problems. *ESAIM Math. Model. Numer. Anal.*, Vol. **50**, pp. 1857–1885, 2016.
- [3] F. Nobile and D. Pradovera, Non-intrusive double-greedy parametric model reduction by interpolation of frequency-domain rational surrogates. *ESAIM Math. Model. Numer. Anal.*, Vol. **55**, pp. 1895–1920, 2021.