

A coupled projection scheme for the Navier–Stokes/Allen–Cahn model

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In this work we consider the unsteady flow of two immiscible and incompressible fluids without assumptions on their densities (i.e. Boussinesq is not assumed here). The Allen–Cahn model is used to describe the interface between fluids. A Navier-Stokes system with variable density and viscosity is driving the dynamic of the flow and its interface.

Projection methods are known for their efficiency for Newtonian fluids. However their use in the context of variable density or viscosity is problematic (but not impossible, see [2, 1]). We propose in this work a time-discrete formulation of the coupled Navier–Stokes/Allen–Cahn equations based on a projection method. The scheme is based on two ingredients: a novel projection method for heterogeneous fluid and the concept of coupled projection scheme [3].

First we establish this time-discrete formulation to be well posed and energy stable. Next we propose a new family of iterative schemes for the actual approximation of solutions. In the last part of our presentation, using the finite element method for spatial discretization we estimate the order of accuracy based on manufactured solution. Finally we will illustrate the validity of the scheme through numerical results on bi-dimensional and three-dimensional tests.

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