

Parameter-robust methods for the Biot-Stokes interfacial coupling without Lagrange multipliers

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Coupled Biot-Stokes systems appear in all fields of science and engineering. From biomechanics where we study the brain, to large scale engineering projects working with groundwater or oil. Naturally, these systems, of different materials and vastly different scales, will differ hugely in model parameters. Thus, there is need for a Biot-Stokes formulation that is robust with respect to parameter variations. In this talk, I will present a stable Biot-Stokes formulation, robust in all parameters and a corresponding preconditioner.

The system for the Stokes velocity-pressure and Biot displacement-total pressure-fluid pressure is solved using a five-field mixed-primal finite element scheme building on the work in [1]. Following the method presented in [2], I will present adequate inf-sup conditions for the formulation. One of its distinctive features is that its stability is established robustly in all material parameters.

For this perturbed saddle-point problem, I will also present robust preconditioners using appropriately weighted operators in fractional Sobolev and metric spaces at the interface. The performance is corroborated by several test cases, including the application to interfacial flow in the brain.

REFERENCES

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