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Generalized gradient flow structure of the Cahn-Hilliard-Biot model

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In this work, we present a model for single-phase flow through a two-phase deformable porous material where the solid phases are evolving and have different material properties [1]. The phase evolution of the system is governed by Landau theory and is closely related to the Cahn-Hilliard equation and the location of the solid phases are tracked by a phase-field function. The coupling of flow and deformation is governed by Biot's theory and the model can be seen as an extension of the Cahn-Larché equations which couples elasticity with the Cahn-Hilliard equation. There exist several applications of two-phase deformable porous materials that undergo phase-changes, and we mention specifically tumor growth where the two phases consist of cancerous and healthy cells and wood growth where heartwood and sapwood form the two evolving phases.

We show that this model has a generalized gradient flow structure, and therefore, is inherently thermodynamically consistent. Moreover, the generalized gradient flow structure is used to develop solution strategies for the nonlinear coupled system of equations, by taking inspiration from [2]. Furthermore, we build on the knowledge from [3, 4, 5] to tune and accelerate the convergence of the nonlinear- and decoupling solvers. Finally, we show numerical examples where we compare the solid phase evolution with and without flow through the medium with different material parameters and highlight the performance of the numerical solution algorithms.

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