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A multiscale hybrid-mixed method for Helmholtz problems in periodic structures

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We propose and analyze a multiscale hybrid-mixed (MHM) method [1] for time-harmonic wave propagation problems in two-dimensional periodic structures. As noted in [2], the MHM method exactly preserves certain plane waves, which turns out to be of particular importance in the setting we consider here.

To fix the ideas, the incoming plane wave is characterized by its frequency $k > 0$ and its incident angle $\theta \in (-\pi/2, \pi/2)$, and the periodic structure is described by the periodicity scale $\ell_1 > 0$. The periodicity of the structures is modeled by the "quasi-periodic" boundary condition $u(0, \cdot) = e^{ik\ell_1 \sin\theta} u(\ell_1, \cdot)$, which will be the main focus of the talk. Specifically, the solution may feature quasi-resonances when $k\ell_1(1 + \sin\theta) = 2n\pi$ for $n \in \mathbb{N}$. In turn these quasi-resonances may affect the stability and accuracy of both perfectly matched layers (PML) and numerical schemes.

In the first part of the talk, I will describe how we employ PML to take into account the non-local radiation condition onto a bounded domain that is suitable for volume-based discretization. The originality of the analysis is that it is fully explicit with respect to k, ℓ_1 and θ. I will also present numerical experiments illustrating the theory and suggesting that the results are sharp.

The second part of the talk will be devoted to the MHM discretization. Importantly, I will show that unlike standard finite element methods, the stability of the MHM method is not affected by quasi-resonances. This last result is linked to the fact that the method exactly reproduces plane waves that are related to quasi-resonant modes. I will illustrate these findings with numerical experiments, highlighting the enhanced stability of MHM over standard finite elements.

REFERENCES

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