The 8th European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS Congress 2022
5-9 June 2022, Oslo, Norway

Stability of 2-domain wall for the Landau-Lifshitz-Gilbert equation in a nanowire with Dzyaloshinskii-Moriya interaction

Raphaël Côte[12], Guillaume Ferriere*[13]

[1] Institut de Recherche Mathématique Avancée, UMR 7501 Université de Strasbourg et CNRS
7 rue René-Descartes, 67000 Strasbourg, France
[2] cote@math.unistra.fr
[3] guillaume.ferriere@math.unistra.fr

Keywords: *Landau-Lifshitz-Gilbert equation, Domain walls, Multi-solitons, Long-time behavior, Asymptotic stability*

We consider the Landau-Lifshitz-Gilbert equation with Dzyaloshinskii-Moriya interaction describing the evolution of the magnetization of a ferromagnetic nanowire, given by

$$\partial_t m = m \wedge H(m) - \alpha m \wedge (m \wedge H(m)), \qquad m = m(t,x) : \mathbb{R} \times \mathbb{R} \to \mathbb{S}^2, \qquad \text{(LLG)}$$

with $\alpha > 0$ and where H is the magnetic field $H(m) = \partial E(m) + h(t)e_1$. h is the external magnetic field and $\partial E(m)$ is the variation of the energy E :

$$E(m) = \frac{1}{2} \int \Big(|\nabla m|^2 + 2\gamma \partial_x m \cdot (e_1 \wedge m) + (1 - m_1^2) \Big) \,\mathrm{d}x,$$

with the condition $|m| = 1$.

Some explicit solutions connecting $\pm e_1$ to $\mp e_1$, called *domain walls* and analogous to the *Walker wall* from the equation without Dzyaloshinskii-Moriya interaction and external field ([1]), are known to be asymptotically stable in H^1 (from a work of Raphaël Côte and Radu Ignat which has not been published yet, see [2] for a physical point of view). Such a result follows from the fact that these domain walls are local minimizers of the energy E, which decreases in time when m is close to a domain wall in H^1.

In the case where the conditions at $\pm\infty$ are the same (for instance $-e_1$), another explicit solution is the constant $m = -e_1$, which is expected to be asymptotically stable. As a following of the work of Raphaël Côte and Radu Ignat, we show that it is not the only one: the sum of two domain walls (one connecting $-e_1$ to $+e_1$, the other connecting $+e_1$ to $-e_1$) far away enough from each other is also asymptotically stable. This is proved under suitable assumptions on the external magnetic field which formalize the fact that the two domain walls must move away from each other and that the interactions between them must be negligible.

REFERENCES

[1] G. Carbou and S. Labbé, *Stability for static walls in ferromagnetic nanowires*. Discrete Contin. Dyn. Syst. Ser. B 6, no. 2, pp. 273—290, 2006.

[2] Y. Gou, A. Goussev, J. M. Robbins and V. Slastikov, *Stability of precessing domain walls in ferromagnetic nanowires*, Phys. Rev. B, vol. 84-10, 104445