

## A Cahn–Hilliard Keller–Segel model for tumor growth with angiogenesis

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We present a Cahn–Hilliard Keller–Segel multiphase field model for tumor growth with angiogenesis, derived from a diffuse–interface mixture model composed by a viable tumor phase, a necrotic tumor phase, a liquid phase and an angiogenetic phase. The mixture dynamics is coupled with the evolution of massless chemicals, comprising a nutrient species and an angiogenetic factor, through source, consumption and chemotactic terms.

We apply the model to describe the glioblastoma multiforme (GBM) tumour growth on a patient–specific basis, where multimodal magnetic resonance neuro images of a patient affected by GBM are integrated into a computational environment to provide anatomical and structural informations of the patient brain, as well as informations about the tumor spatial distribution at different stages of disease evolution. The parameters in the model are estimated in an optimised way through sensitivity analysis[1].

We present recent advancements on the main image processing steps to reconstruct quantitative informations from raw neuroimaging data. We also present a numerical approximation of the proposed model which deals with the highly nonlinear and degenerate model equations, and show numerical results on a clinical test case.

The proposed computational model can be useful to assist medical doctors in evaluating patient-specific therapeutic options and outcomes. The presence of an angiogenetic phase in the model is particularly useful to characterize the drug delivery towards the tumor and its response to radiotherapy [2].

## REFERENCES

- [1] Agosti, A., Ciarletta, P., Garcke, H., Hinze, M., *Learning patient-specific parameters for a diffuse interface glioblastoma model from neuroimaging data. Math Meth Appl Sci.*, Vol. **43**(15), pp. 1–35, 2020.
- [2] Hormuth, D. A., 2nd, Phillips, C. M., Wu, C., Lima, et al., *Biologically-Based Mathematical Modeling of Tumor Vasculature and Angiogenesis via Time-Resolved Imaging Data. Cancers*, Vol. **13**(12), 3008, 2021.