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Maximum-order data-driven Weighted Essentially Non-Oscillatory (WENO) schemes

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Weighted Essentially Non-oscillatory (WENO) schemes have been successfully applied in computational fluid dynamics (CFD), especially for compressible flows, due to their high-order shock-capturing ability [1]. With the steady rise of machine learning methods in CFD, novel combinations of data-driven and traditional numerical schemes are being explored [2]. Among others, data-driven weighted essentially non-oscillatory schemes have been developed [3]. Challenges faced with such schemes are to ensure maximum-order convergence and the ENO property. We propose to use a neural network as a weighting function in the WENO scheme and address aforementioned shortcomings [4]. Based on the input stencil, the neural network calculates a convex combination of local interpolation polynomials. We use a Galilean invariant embedding in the input layer and introduce an additional loss on the reconstruction weights, such that the WENO scheme inherently recognizes a smooth input function and achieves maximum-order convergence. We demonstrate the performance of the resulting WENO3-NN scheme for one- and two-dimensional test cases, including strong shocks and shock-density wave interactions. The WENO3-NN scheme shows very good generalizability across all benchmark cases and different resolutions, and exhibits a performance similar to or better than the classical WENO5-JS scheme. Finally, we discuss extensions of our approach to WENO-NN schemes of higher order.

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