

Forming a Well-Posed Model of Poroelasticity in the Presence of Fractures

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Accurate modeling of the interplay between flow and mechanics in fractured, porous media is important for a range of geological applications, including geothermal energy storage, CO₂ sequestration, and hydraulic fracturing. In this work, we propose a modeling framework for poroelasticity in the presence of fractures with three distinctive properties.

First, the fractures are represented as embedded, lower-dimensional manifolds. This leads to a geometry of intersecting subdomains of different dimension. Using concepts from exterior calculus [1], we propose a canonical way to define the relevant variables, specifically the bulk displacement and stress, as well as the fluid flux and pressure. By directly including the coupling equations, a system of coupled *mixed-dimensional* partial differential equations is formed.

Secondly, we focus on admissible non-linearities within the model. With a particular interest in the frictional contact mechanics at the fractures, we present a class of constitutive laws that leads to a well-posed problem. We then identify the resulting system as an *evolutionary equation*[2]. In turn, this allows us to obtain well-posedness by exploiting the symmetry of the problem as well as the maximal monotonicity of the constitutive laws[3].

Third, our model is defined in the continuous setting and does not rely on any discretization tools. Through the established well-posedness of the continuous system, we provide a rigorous basis for the development of discretization methods and their subsequent convergence analysis.

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