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Multistage DPG time-marching scheme for semi-linear problems

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The main idea of the Discontinuous Petrov-Galerkin (DPG) method [1] is to select optimal test functions that realize the supremum in the inf-sup condition in order to guarantee discrete stability. Recently, we applied the DPG method only in the time variable in order to obtain stable DPG-based time-marching schemes for linear PDEs [3, 4]. In this work, we extend this construction to transient semilinear problems. We first semidiscretize the PDE in space by a classical Bubnov-Galerkin method. Then, we approximate the nonlinear term by a polynomial in time employing known values of the solution from previous stages. Considering an ultraweak variational formulation of the linearized problem we calculate the optimal test functions analytically, which are exponential related functions. Finally, we obtain a time-marching scheme that locally computes the solution in the element interiors and performs post-processing for the trace variables. The equation we obtain for the traces is equivalent to the so-called variation-of-constants formula in exponential integrators [2]. Therefore, our method is equivalent to exponential integrators for the traces and we can additionally approximate the solution in the element interior. With this variational construction, we can naturally apply (goal-oriented) adaptive strategies and develop posteriori error estimation.

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