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The MH^{2}M Method

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In this work, we propose and analyze a variant of the Multiscale Hybrid-Mixed (MHM for short) method for the Darcy equation on complex domain [2, 3]. The construction of the Multiscale Hybrid-Hybrid-Mixed method (MH^{2}M for short) starts from a double hybridization of the primal formulation of the Poisson equation (see [1]) with rough coefficients. Like in the MHM's original idea, the exact solution splits into local solutions of Neuman's problems and a global problem defined on the skeleton of the partition. On the other hand, the leading Lagrange multiplier represents the trace of the exact solution on the skeleton of the partition, whereas it corresponds to the flux in the original MHM method. Consequently, the MH^{2}M method needs fewer degrees of freedom to reach an error threshold than the MHM method. Also, those degrees of freedom result from a positive definite algebraic global system. We prove that the nonconforming MH^{2}M method preserves conservation properties at the local level, is well-posed under compatibility conditions between the Lagrange multiplier spaces, and provides some best approximation results. Numerical tests validate the theoretical findings.

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