

# IDENTIFYING STABILITY CONSTRAINTS OF HIGH-ORDER METHODS ON DISTORTED MESHES THROUGH A VON-NEUMANN ANALYSIS FRAMEWORK

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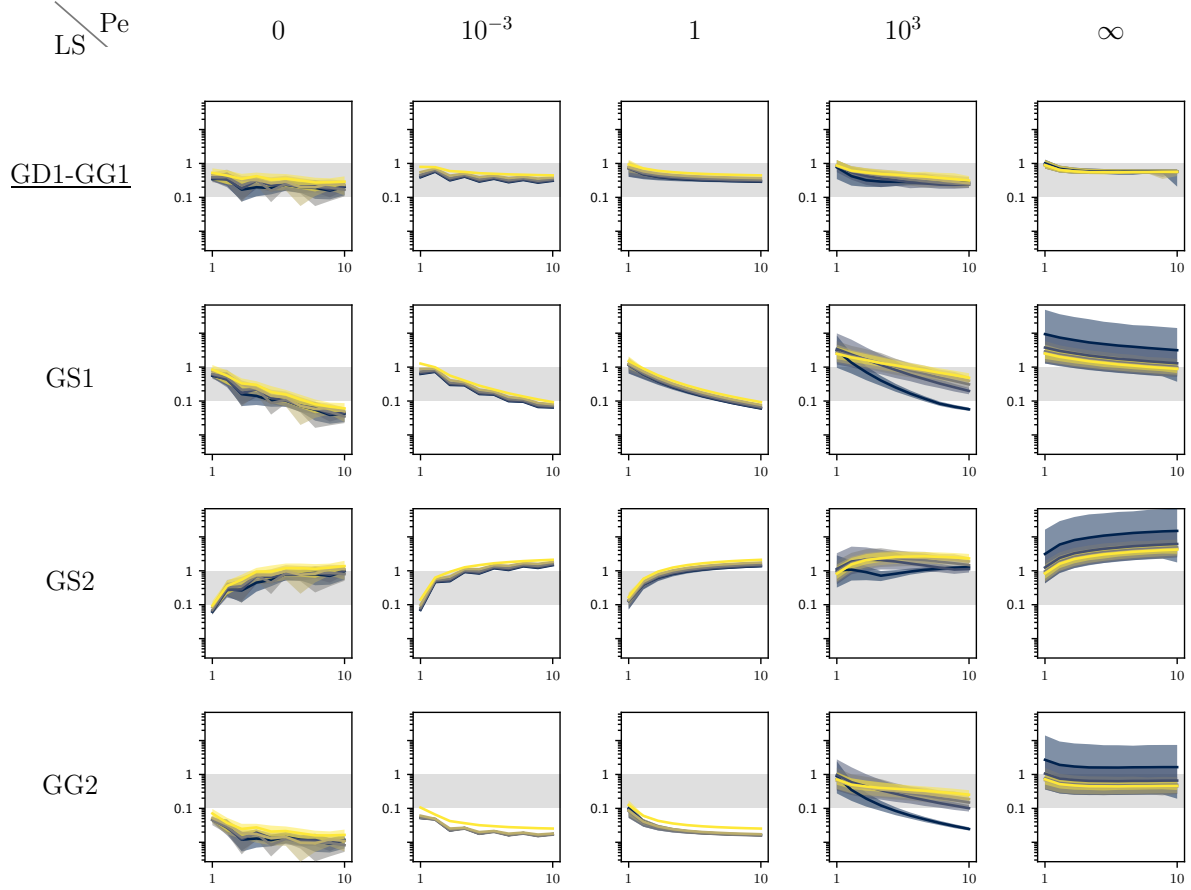
**Keywords:** *length-scale, high-order, CFL, timestep, curved mesh, distorted mesh*

The maximum stable explicit timestep for high-order methods exhibits a dependence on the polynomial-order  $p$  and the cell-shape [1, 2, 3]. This manifests itself in the form of large fluctuations in the stable CFL number, depending on the case,  $p$ , and mesh. For industrial cases, this amounts to a lot of trial-and-error for achieving stability, leading to wasted time and computational effort.

In this work, we identify patterns in the variation of the cell-local length-scale that results in the maximum stable timestep with minimum variation of the CFL number. This is done by constructing a von-Neumann analysis (VNA) framework on distorted linear and curved meshes. By offloading the mesh- and  $p$ -dependence onto the length-scale, we free the user from having to find the optimal CFL number for each new case. Based on the identified patterns, we propose a strategy to compute said length-scales. This strategy is then verified through extensive VNA on a vast range of mesh-skewness, polynomial-orders and flow-physics.

## REFERENCES

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**Figure 1:** Maximum CFL number in log-scale (ordinate of each sub-plot) plotted against increasing polynomial-orders  $p \in [1, 10]$  (abscissa of each sub-plot). All sub-plots share the same limits on the axes. The grey-shaded area marks the region of CFL lying between 0.1 and 1, which is a practically reasonable range. Colors represent meshes going from less skewed (yellow) to more skewed (blue). Color-shaded areas depict variation of CFL with wave-number and angle-of-attack of the input signal, while color-lines plot the average values of CFL. Plot-rows indicate different length-scale strategies (our strategy is “GD1-GG1”) and plot-columns vary the Peclet number  $Pe$ . The proposed strategy achieves stability with minimum variation of the CFL number as compared to other strategies.