High-frequency estimates on boundary integral operators for the Helmholtz exterior Neumann problem

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We study a commonly-used second-kind boundary-integral equation for solving the Helmholtz exterior Neumann problem at high frequency, namely the Regularized Combined Field Integral Equation (RCFIE) introduced in [1]. Writing Γ for the boundary of the obstacle, this integral operator map $L^2(\Gamma)$ to itself, contrary to its non-regularized version.

We prove new frequency-explicit bounds on the norms of both the RCFIE and its inverse. The bounds on the norm are valid for piecewise-smooth Γ and are sharp, and the bounds on the norm of the inverse are valid for smooth Γ and are observed to be sharp at least when Γ is curved.

Together, these results give bounds on the condition number of the operator on $L^2(\Gamma)$; this is the first time $L^2(\Gamma)$ condition-number bounds have been proved for this operator for obstacles other than balls [2].

REFERENCES

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