

Multi-GPU speedup of an iterative time-harmonic wave solver

Christiaan C. Stolk¹

¹ University of Amsterdam, Korteweg-de Vries Institute for Mathematics, P.O.Box
94248, 1090 GE Amsterdam, `c.c.stolk@uva.nl`

Keywords: *Helmholtz equation, wave equation, GPU computing*

In this paper we study the simulation of 3-D high-frequency time-harmonic waves. These problems leads to large, indefinite linear systems. As direct methods are often too expensive, iterative methods have been studied extensively for this problem.

Several of these methods proceed via the time domain. For example, in exact controllability, periodicity of solutions of a discrete time-domain wave equation with periodic forcing is enforced by optimization [1]. Recently there has been renewed interest in this class of methods, for one reason because of the good parallel scalability [2].

In this talk I continue the development of so called time-domain preconditioners [3]. In this type of method, the limiting-amplitude principle is applied on the discrete level. Starting from a discrete time-harmonic wave equation $HU = F$, $H \in \mathbb{C}^{N \times N}$, $U, F \in \mathbb{C}^N$, a matrix recurrence relation is derived, such that as time goes to infinity, the exact solution U (times a phase factor) is obtained. By iterating a large but finite number times, a preconditioner is obtained, that is attractive compared to other time-domain methods because of its simplicity and the absence of time-discretization errors.

In this work we will explore this idea further. We obtain new results in several directions. First we will consider different spatial discretizations than used previously, namely summation-by-parts finite differences. These exist of various orders and have good energy conservation properties. Secondly we consider the use of PML layers in combination with the time-domain preconditioner. Thirdly, we will show that considerable speed-ups can be obtained by performing the computations on a multi-GPU system.

REFERENCES

- [1] M.-O. Bristeau, R. Glowinski, and J. Périaux. Controllability methods for the computation of time-periodic solutions; application to scattering. *Journal of Computational Physics*, Vol. **147** pp.265–292, 1998.
- [2] M. J. Grote, F. Nataf, J. H. Tang, and P.-H. Tournier. Parallel controllability methods for the Helmholtz equation. *Computer Methods in Applied Mechanics and Engineering*, Vol. **362** 112846, 2020.
- [3] C. C. Stolk. A time-domain preconditioner for the Helmholtz equation. *SIAM Journal on Scientific Computing*, Vol. **43** pp. A3469–A3502, 2021.