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Atmospheric radiation boundary conditions for the wave equation in helioseismology

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Helioseismology aims at studying the solar interior by the analysis and interpretation of the solar oscillations. The first step is to solve the forward problem, which requires to solve a PDE for the 3D wave displacement $\boldsymbol{\xi}$. Neglecting rotation and advection by flows, $\boldsymbol{\xi}$ is solution of the Galbrun equation

$$-\rho_0\sigma^2\,\boldsymbol{\xi} \,-\, \nabla\left[\rho_0\, c_0^2 \nabla\cdot\boldsymbol{\xi}\right] \,+\, \boldsymbol{\mathcal{L}_g}\,\boldsymbol{\xi} \,=\mathbf{s}, \tag{1}$$

where $\mathbf{s}$ is a stochastic source term and the operator $\boldsymbol{\mathcal{L}_g}$ is necessary to understand the surface and internal gravity waves. Here, ρ_0 is the density, c_0 the sound speed and σ the (complex) frequency. Neglecting gravity, one can find a scalar equation satisfied by $\psi = \rho_0 c_0^2 \nabla\cdot\boldsymbol{\xi}$

$$-\frac{\sigma^2}{\rho_0 c_0^2}\psi - \nabla\cdot\left(\frac{1}{\rho_0}\nabla\psi\right) = f. \tag{2}$$

One difficulty in solving Eqs. 1 and 2 is the absence of a solid boundary at the surface of the Sun, and thus a boundary condition needs to be derived in order to truncate the computational domain. Here, we present families of radiation boundary conditions for Eqs. 1 and 2 when the density is exponentially decaying and the sound speed remains contant in the atmosphere. We study the boundary conditions for the vector problem derived in [1] and compare them to the ones of the scalar problem that were analyzed from a physical perspective in [2]. For high-frequency waves, the solution of both equations is propagating in the atmosphere but the presence of gravity should be taken into account to obtain accurate boundary conditions. Moreover, due to the presence of gravity, a new propagative region appears at low frequencies which is numerically challenging. Detailed numerical results as well as a comparison with observed quantities will be presented.

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