The 8th European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS Congress 2022
5-9 June 2022, Oslo, Norway

POD-Galerkin ROMs and physics-informed neural networks for solving inverse problems for the Navier–Stokes equations.

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Keywords: *Navier-Stokes equations, Proper Orthogonal Decomposition, Inverse Problems, Physics-Based Machine Learning*

This work presents a model which merges the Proper Orthogonal Decomposition (POD)-Galerkin reduction methodology with Physics Informed Neural Networks (PINNs) for the sake of solving inverse problems involving the Navier–Stokes equations (NSE).

The model constructs a POD-Galerkin ROM for the NSE (or the modified turbulent NSE modeled by the RANS or LES approaches) resulting in a reduced dynamical system. Then, Physics Informed Neural Networks (PINNs) are employed for solving the reduced order system produced by the POD-Galerkin ROM. The inputs of the PINNs are time and the parameters, while their output is the vector of all reduced quantities, namely, the reduced velocity, pressure, turbulent (if applicable) and convective terms.

The PINNs are then trained by minimizing a loss function which corresponds to the (weighted) sum of the data loss and the reduced equation losses. PINNs features allow for the inference of unknown physical quantities that appear in the reduced equations. The model proposed, named *POD-Galerkin PINN ROM*, is also able to perform accurate forward computations for the input parameter despite the uncertainty in the problem.

The model is tested on two benchmark cases which are the steady case of the flow past a backward step and the flow around a surface mounted cubic obstacle. In both tests, the model is employed for the approximation of unknown parameters such as the physical viscosity. The POD-Galerkin PINN ROM shows accuracy in solving the inverse problems.

REFERENCES

[1] S. Hijazi, N. Landwehr and M. Freitag, On the use of POD-Galerkin ROMs and physics-informed neural networks for solving inverse problems for the Navier–Stokes equations. In preparation, 2021.

[2] M. Raissi, P. Perdikaris and G. Karniadakis, Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations, Journal of Computational Physics 378 (2019) 686–707.